# **Evolution of the superconducting** critical temperature in Yb-substituted **CeCoIn5**

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Nov10, 2012

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#### **Thermal conductivity**

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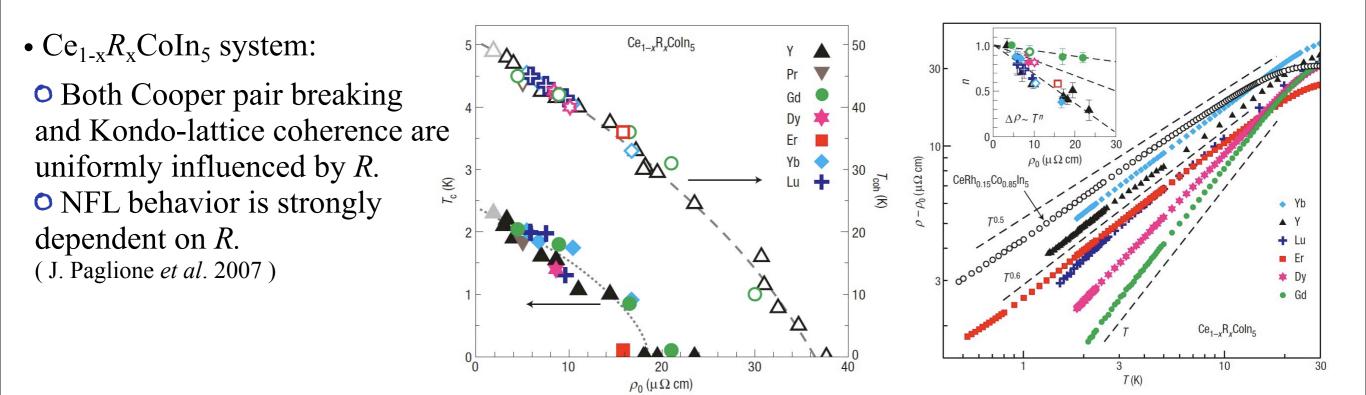
### Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub> – motivation

• CeCoIn<sub>5</sub>:

1. unconventional HF superconductor ( $T_c=2.3$  K at ambient pressure),

2. NFL behavior,

- 3. magnetic field-induced QCP.
- YbCoIn<sub>5</sub>: conventional nonmagnetic metal (1 K 300 K).



• Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub> system:

- 1. Electron-hole analogy between the  $Ce^{3+}(4f^{1})$  and  $Yb^{3+}(4f^{13})$ .
- 2. Unstable valence of Ce( $3 + \le v_{Ce} \le 4 +$ ) and Yb( $2 + \le v_{Yb} \le 3 +$ ).
- 3. Yb is expected to become more magnetic under pressure.

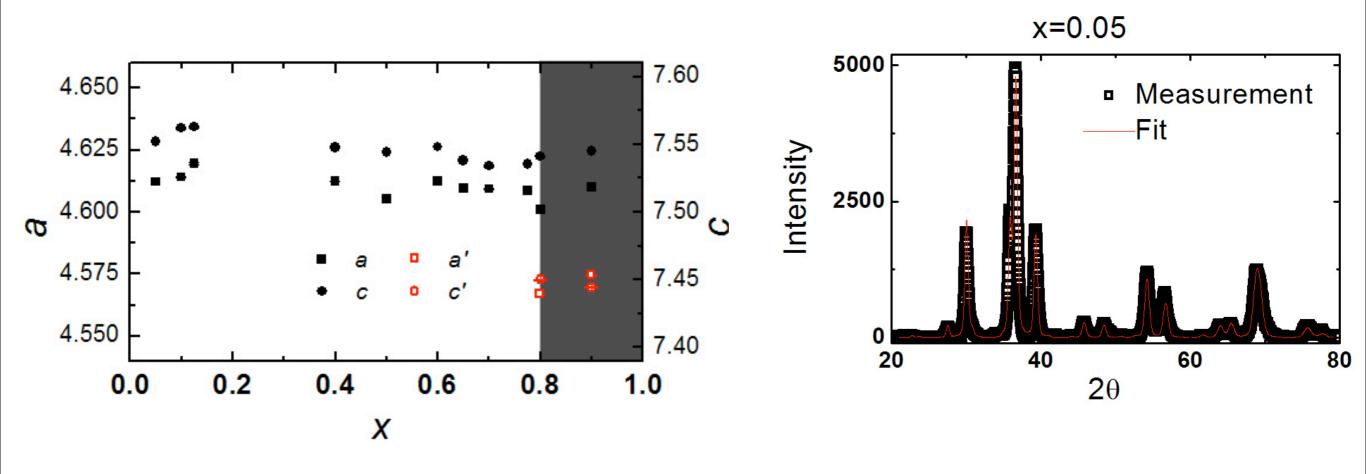
### Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub> – Organization of talk

- Review characteristics of the Yb-stabilized correlated electron state in Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub>
- Efforts to determine Ce and Yb valences as a function of x
- Evidence for changes in valence and physical properties near x = 0.2
- Unusual  $T_c$  vs x phase boundary ( $T_c \propto Ce$  composition!)
- High pressure experiments to probe the normal and SC'ing states

Publications reporting research of other groups on Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub>:

- C. Capan et al., EPL 92, 46004 (2010)
- C. H. Booth et al., PRB 83, 235117 (2011)
- A. Polyakov et al., PRB 85, 245119 (2012)
- M. Shimozawa et al., PRB 86, 144526 (2012)

Lattice parameters *a* and *c* 



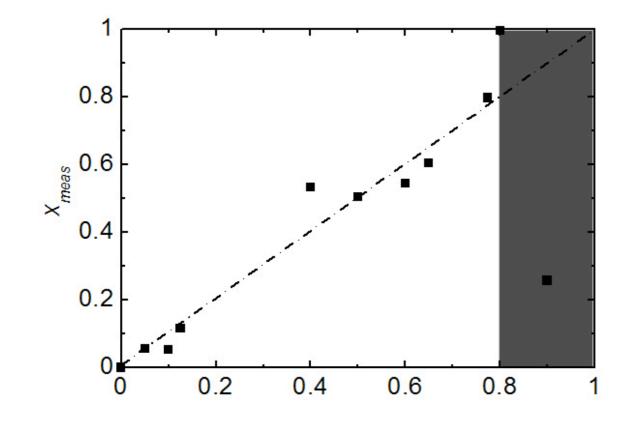
#### Vegard's Law:

*a* and *c* should decrease linearly with *x*, **if** there are no changes in

- the valence of the Ce and Yb ions,
- or bonding due to variation in the electron concentration

Ce and Yb ions do not retain  $v_{Ce} = 3+$  for x=0  $v_{Yb} = 2+$  for x=1

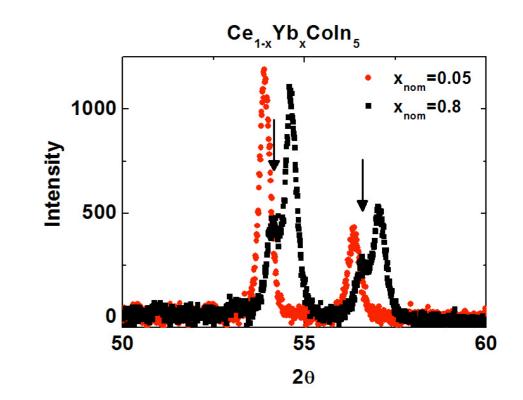
#### Measured Yb concentration $x_{meas}$ from EDX vs. nominal x



7.60 4.650 7.55 4.625 <sup>7.50</sup> ပ 4.600 ð 4.575 7.45 C 4.550 7.40 0.2 0.8 0.4 0.6 0.0 1.0 X

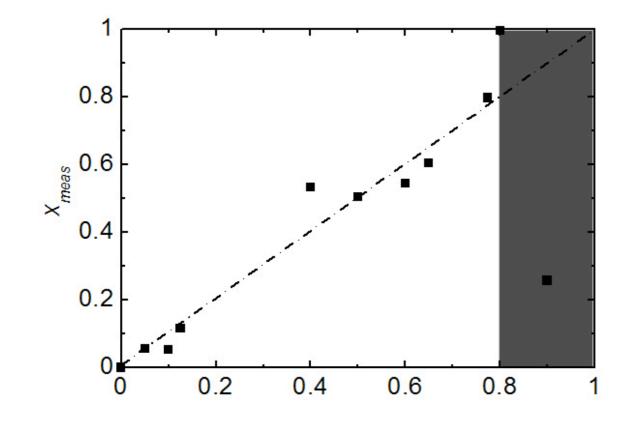
• The EDX data reveal crystals with the expected Yb concentration form for x < 0.8,

• For  $x \ge 0.8$ , each peak in XRD profile splits into two peaks.



Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

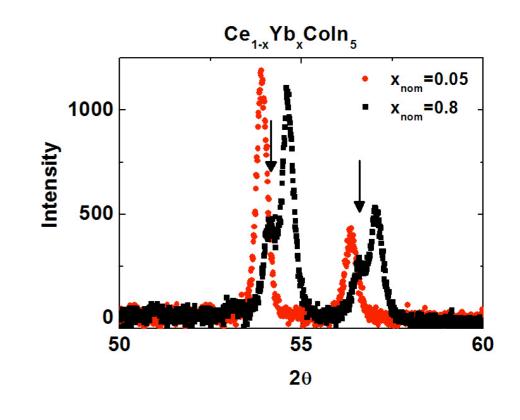
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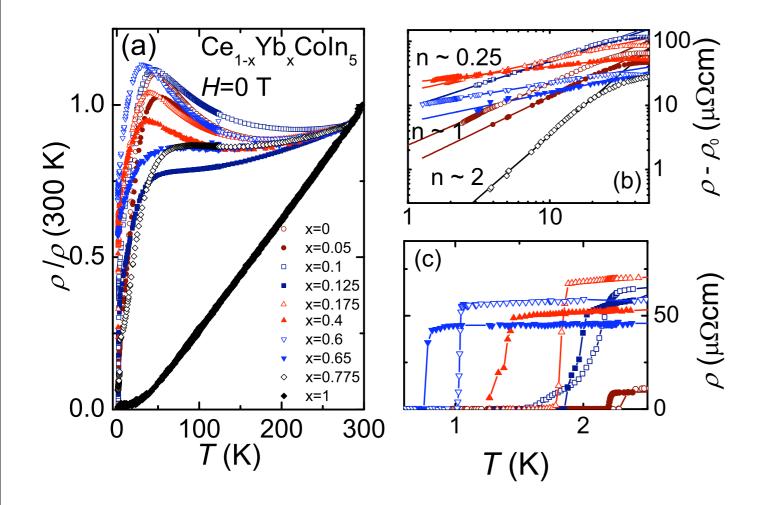
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Phase separation occurs for  $x \ge 0.8$ 

Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

#### Electrical Resistivity p



Normalized  $\rho$  curves for  $x \le 0.775$  are typical of many HF materials:

- weak *T* dependence at high *T*;
- a maximum or broad hump at *T*\*;
- followed by a decrease in  $\rho$  with decreasing *T*.
- $T^*$  remains roughly constant.

YbCoIn<sub>5</sub> does not exhibit correlated electron effects.

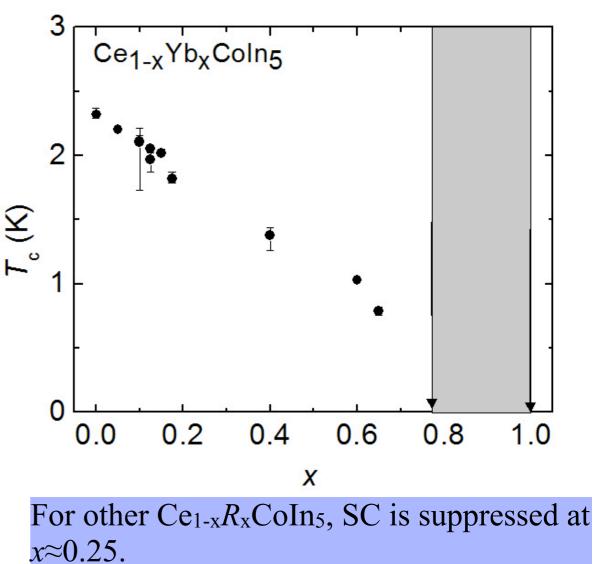
Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

For  $x \le 0.775$ : •  $o(T) = o_x + AT^n$  (7)

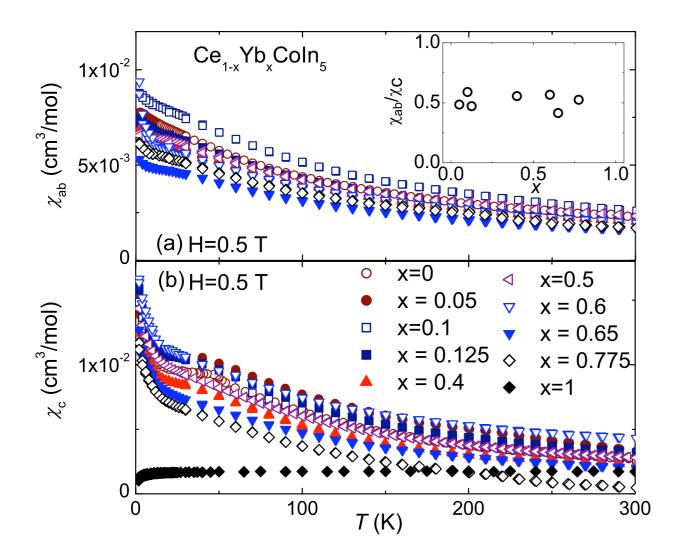
•  $\rho(T) = \rho_0 + AT^n_\rho \ (T_c < T < 25K)$ 

• Sub-*T*-linear transport scattering rate (NFL behavior).

SC transitions are clearly observed in  $\rho(T)$  for  $0 \le x \le 0.65$ .

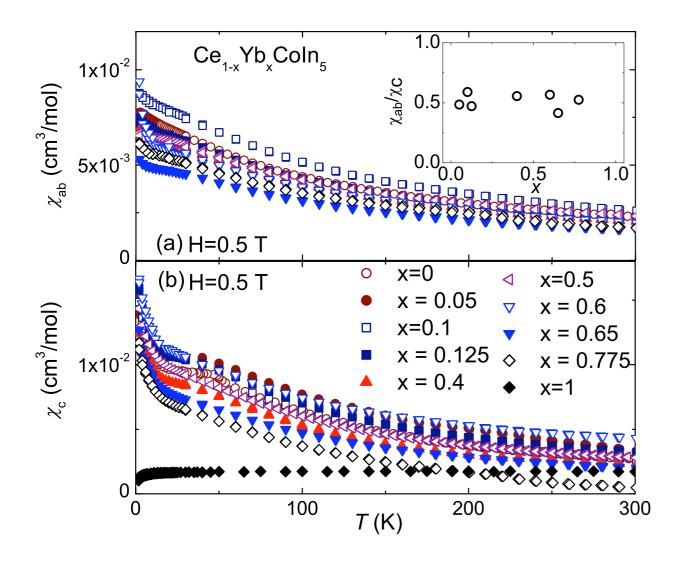


#### Magnetic susceptibility $\chi$



For x ≤ 0.775, χ(T) is nearly identical to that of x = 0:
1) Curie-Weiss behavior at high T.
2) χ(T) saturates below 50 K (consistent with the coherent behavior in ρ(T)).
3) χ(T) increases upon cooling below 20 K (intrinsic effect), contrary to the behavior of ideal HF compounds. χ<sub>c</sub> = χ<sub>c</sub>(0) + a/T<sup>n</sup><sub>χ</sub> (1.8 K < T < 20 K) NFL behavior</li>

#### Magnetic susceptibility $\chi$



Yb ions does not enter the lattice in the nonmagnetic divalent state, in which case  $\chi(T)$  should scale with (1-*x*).

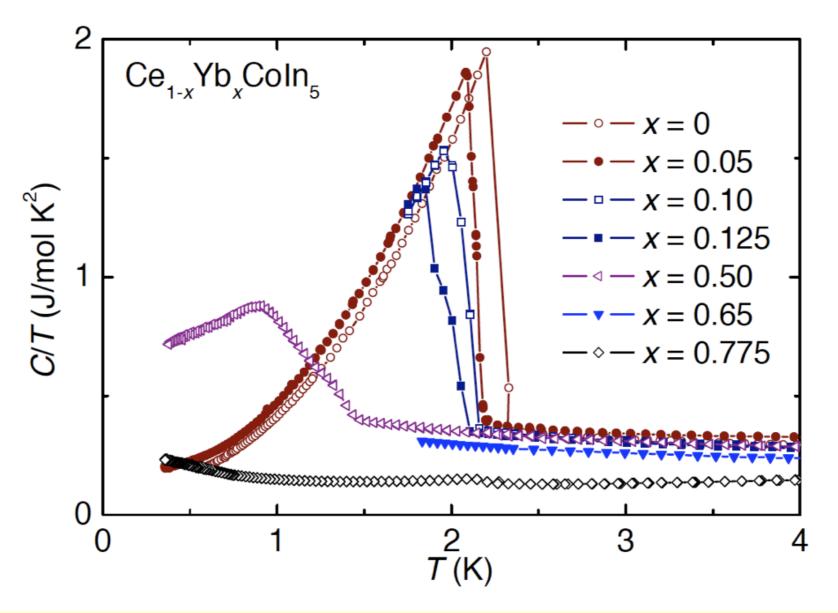
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#### Specific heat



For x  $\leq$  0.775, *C*/*T* tends to increase with decreasing *T* down to the SC transition (NFL)

#### For x = 0.775,

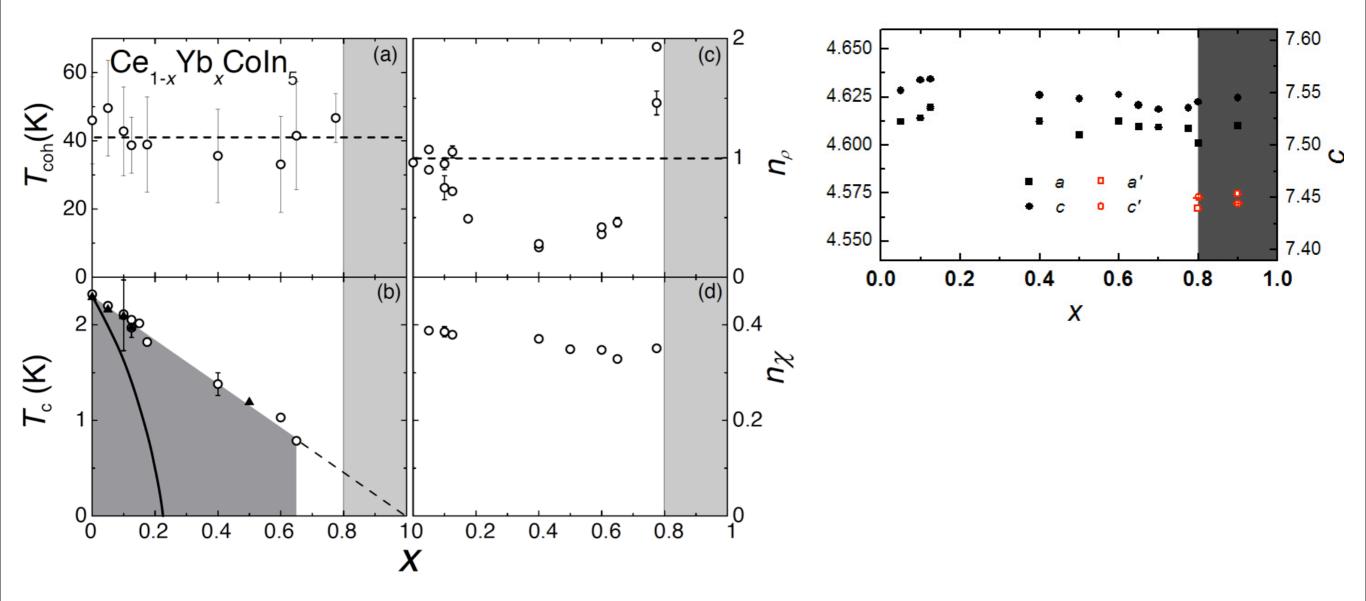
- $A \approx 0.036 \ \mu\Omega \text{cm/K}^2$  (ground state is a heavy Fermi liquid),  $\rho = \rho_0 + AT^2 (T_c \le T \sim 25 \text{K})$ .
- Kadowaki-Woods ratio  $R_{KW} = A/Y^2 = 1.86 \times 10^{-6} \,\mu\Omega \text{cm}(\text{mol-K/mJ})^2$ , intermediate between what is expected for Ce- and Yb- based heavy fermion compounds (Kadowaki and Woods 1986, Tsujii *et al.* 2005), emphasizing that **strong electronic correlations persist up to**  $x \approx 0.775$ .

Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

Our study reveals that:

- 1) *a* and *c* remain nearly constant for  $x \le 0.775$ , phase separation occurs when x > 0.775.
- 2)  $T_c$  is weakly suppressed with x. SC would disappear near x = 1 in the absence of phase separation.
- 3)  $T^*$  remains roughly constant up to x=0.775.  $T_c$  does not scale with  $T^*$ .
- 4) Strong electronic correlation persists up to x=0.775.

5) The NFL behavior is strongly influenced by x, a recovery of FL-like behavior is observed with increasing x. No apparent QCP.



Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

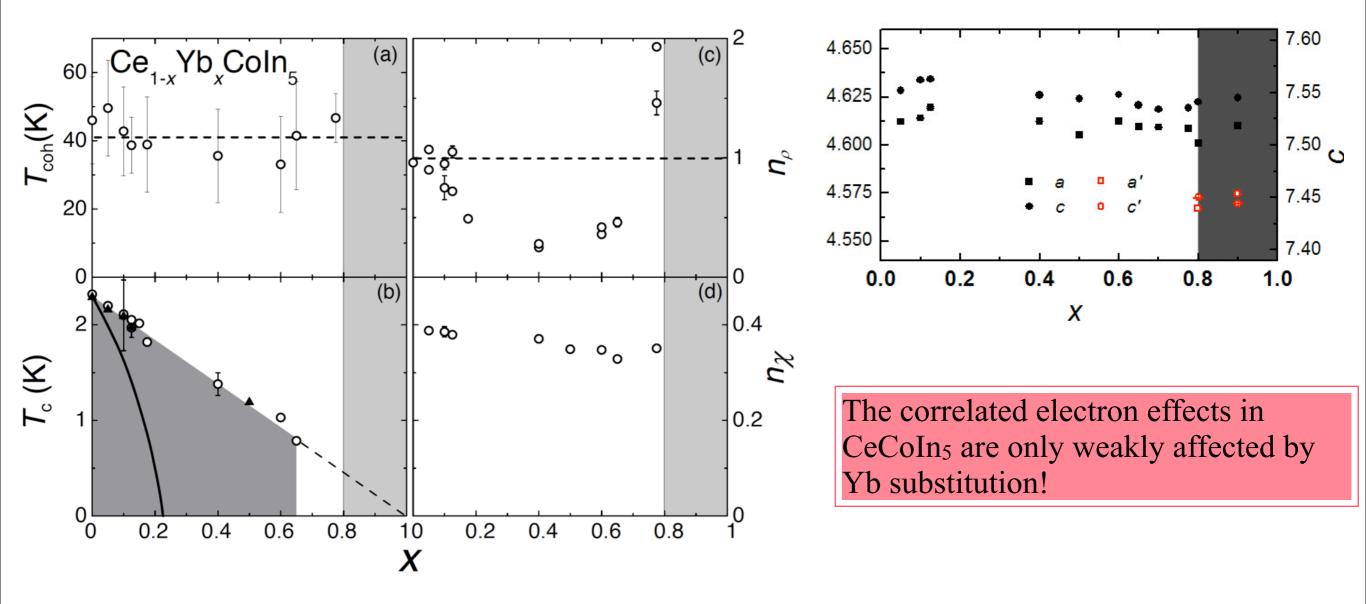
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Shu, Baumbach, Janoschek et. al. PRL 106 156403 (2011)

Ce and Yb cooperatively change their electronic states to preserve the Kondo-like lattice behavior and SC of CeCoIn<sub>5</sub>. NFL state is strongly susceptible to the introduction of Yb ions.

A possible explanation: valence fluctuations arising from a cooperative IV state formed by the Ce and Yb ions, which stabilizes the electronic properties of  $Ce_{1-x}YbCoIn_{5}$ . The cooperative IV state provides a mechanism that may drive the observed NFL physics.

Note: Quantum valence criticality yields NFL-like anomalies

Watanabe and Miyake. 2010 Okada and Miyake. 2011

$\beta$ -YbAlB <sub>4</sub> and YbRh <sub>2</sub> (Si <sub>0.95</sub> Ge <sub>0.05</sub> ) <sub>2</sub> : $C/T \sim -lnT$ (low T)	Custers et al. 2003
$\chi \sim T^{-n}_{\chi}, n_{\chi} = 0.5 - 0.6$	
$\Delta \rho \sim T$	Nakatsuji <i>et al</i> . 2008
$\Delta p \sim I$	

 $CeCu_2(Si_{1-x}Ge_x)_2$  Yuan *et al.* 2006

### Ce and Yb Valence vs x

### Valences of Ce and Yb in Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub>

L<sub>III</sub> edge XANES Ce:  $v_{Ce} \approx +3$  for all x Yb:  $v_{Yb} \approx +2.3$  for all  $0.2 \le x \le 1$ 

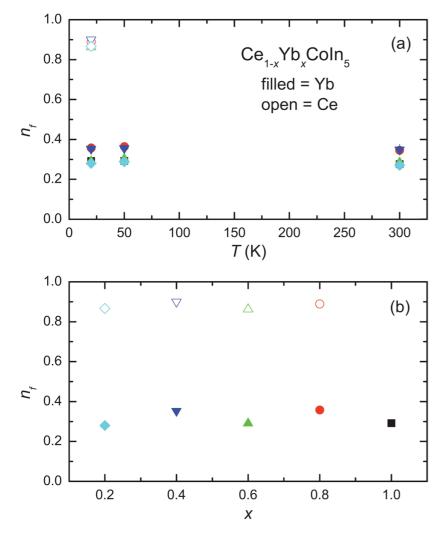
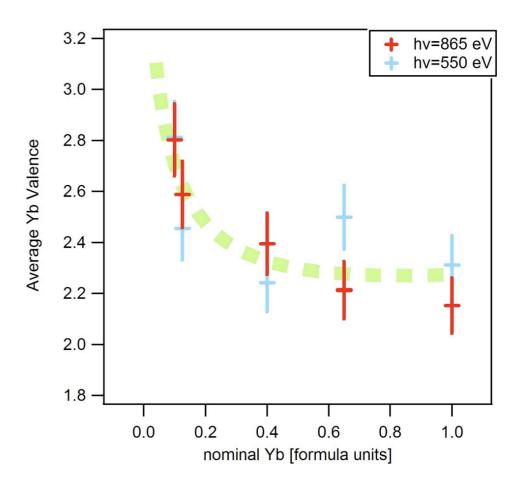


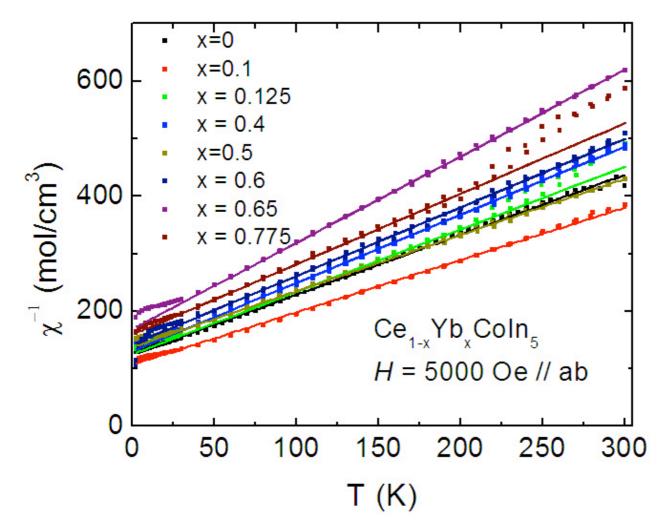
FIG. 8. (Color online) Fit results for  $n_f$  as a function of (a) temperature and (b) x, for both Ce and Yb orbitals. Note that  $n_f$  refers to the f-electron orbital occupancy for Ce and the f-hole orbital occupancy for Yb. The rare-earth valence is then  $v = n_f + 3$  for Ce and  $v = n_f + 2$  for Yb.

- XAS near Ce M<sub>4</sub> and M<sub>5</sub> edges Ce:  $v_{Ce} \approx +3$  for all x
- 4f XPS
  - Yb:  $v_{Yb}$  drops from ~+3 at x=0 to ~+2.3 at x=0.2, then remains ~constant to x=1
- Yb valence transition below  $x \approx 0.2$



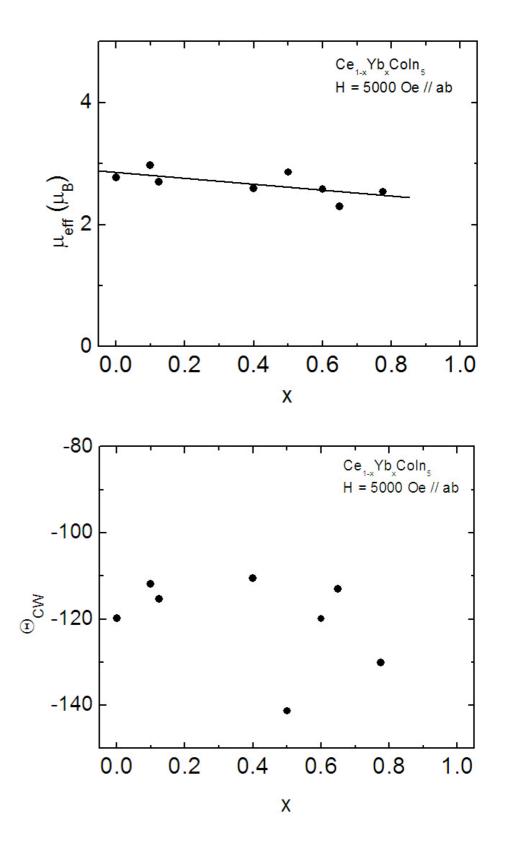
Dudy et al., manuscript in preparation (2012)

Booth et. al. PRB 83, 235117 (2011)



Straight lines above are the Curie-Weiss fits. Fitting parameters  $\mu_{eff}$  and  $\Theta$  are indicated in the figs on right side.

$$\chi = \frac{N_{\rm A} \mu_{\rm eff}^2}{3k_{\rm B}(T - \Theta_{\rm CW})}$$



- If  $\Theta_{\text{CW}}$  is constant, then

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x) \quad (1)$$

XPS, XANES measurements indicate
v<sub>Ce</sub> ≈ +3 for all x ⇒ μ<sub>Ce</sub> ≈ 2.54 μB
Solve Eq. (1) for μ<sub>Yb</sub>(x) from μ<sub>eff</sub>(x)

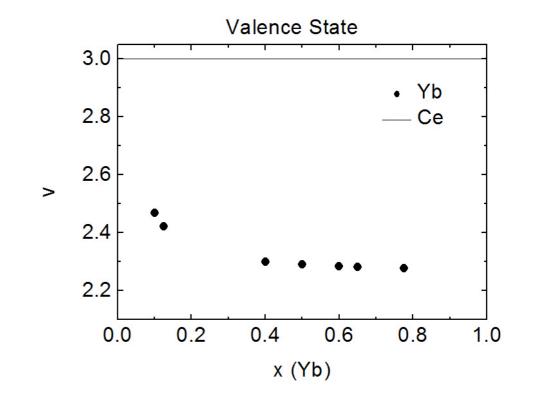
$$V_{\rm Yb}(x) = \mu_{\rm Yb}(x)^2 / \mu_{\rm Yb3+}^2 + 2$$
 (2)

- If  $\Theta_{\text{CW}}$  is constant, then

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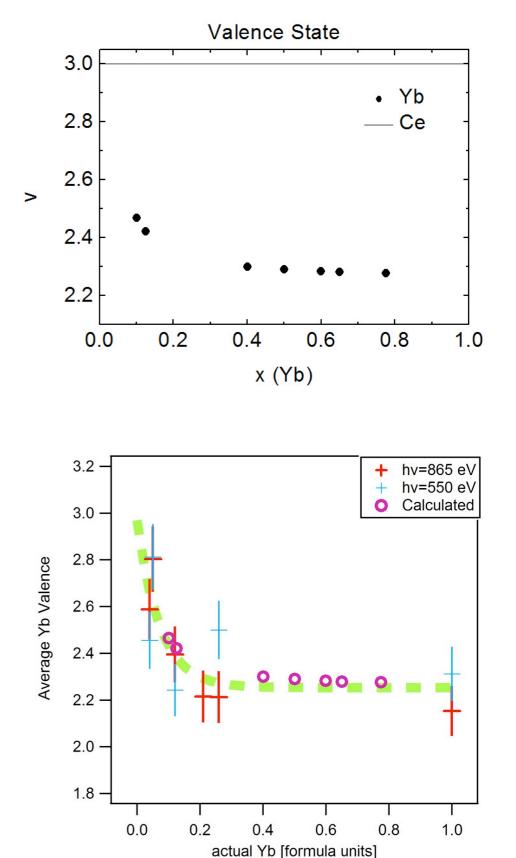
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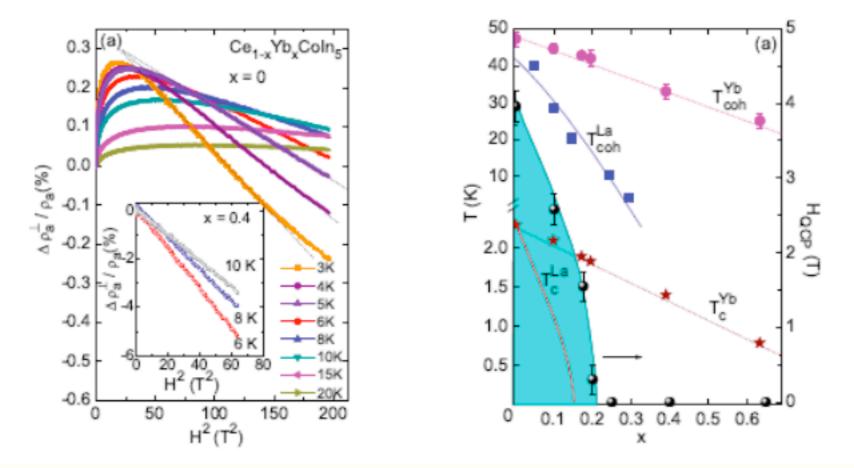
#### Comparison with XPS data of Dudy et al

Note: the analysis does not account for CEF and valence fluctuation or Kondo effect modifications of chi(*T*) explicitly. It only assumes we can analyze  $\chi(T)$  at high *T* in terms of a Curie-Weiss law and use effective moments to infer the valence. Could also be large errors (Curie-Weiss *T* are large in magnitude and some CEF splittings may be appreciable). Nonetheless, the analysis seems to be, at least, qualitatively consistent with XPS data.



### Change in electronic properties at $x \approx 0.2$

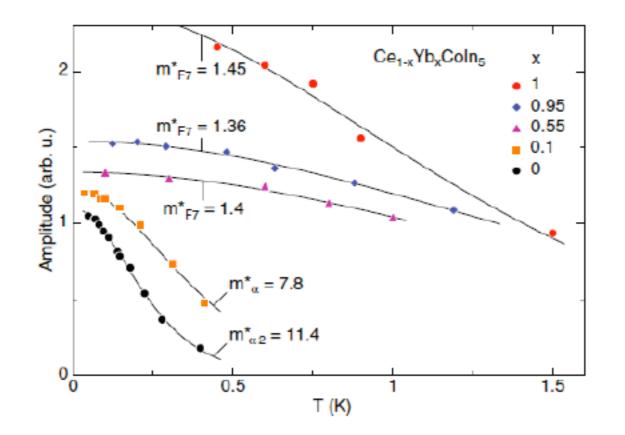
- Quantum critical field H<sub>QCP</sub>(x) determined from analysis of magnetoresistance
- H<sub>QCP</sub> → 0 at x ≈ 0.2 ⇒ NFL behavior and unconventional SC associated with (a) quantum criticality for x < ~0.2; (b) another mechanism for x > ~0.2 (VF's?)
- Character of NFL behavior in ρ(T) also changes near x ≈ 0.2



T. Hu, Y. P. Singh, L. Shu, M. Janoschek, M. Dzero, M. B. Maple, C. C. Almasan, archive: 1208.4308.

dHvA studies of Ce<sub>1-x</sub>Yb<sub>x</sub>Coln<sub>5</sub>

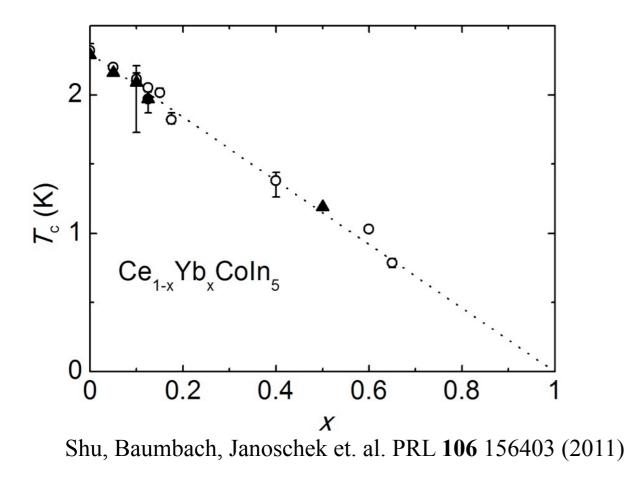
- x = 0.1: Slight change in heavy effective mass m\* of CeCoIn<sub>5</sub> (x = 0)
- x = 0.2: Changes in FS topology
- x = 0.55 and above: Drastic reconstruction in FS and small effective masses

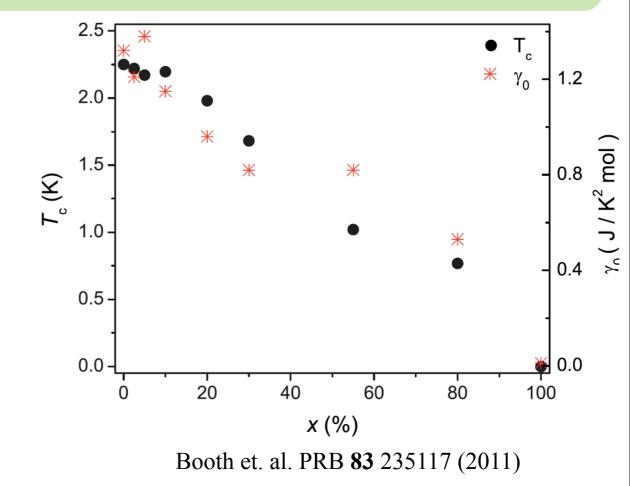


A. Polyakov, O. Ignatchik, B. Bergk, K. Götze, A. D. Bianchi, S. Blackburn, B. Prévost, G. Seyfarth, M. Coté, D. Hurt, C. Capan, Z. Fisk, R. G. Goodrich, I. Sheikin, M. Richter, J. Woznitza, PRB **85**, 245119 (2012)

### $T_c \propto Ce \text{ concentration in } Ce_{1-x}Yb_xCoIn_5$

### $T_{\rm c} \propto {\rm Ce\ concentration}$





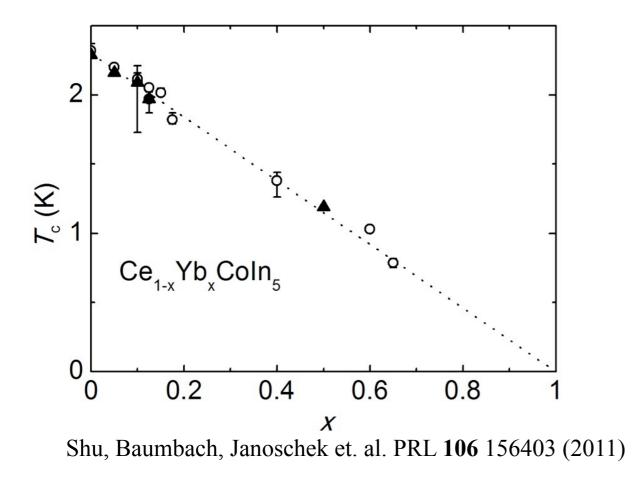
Tc ∝ Ce concentration
As if SC generated locally by Ce
No obvious feature in *T*c(x) near x ≈ 0.2
Could provide clues to origin of unconventional SC of CeCoIn<sub>5</sub>

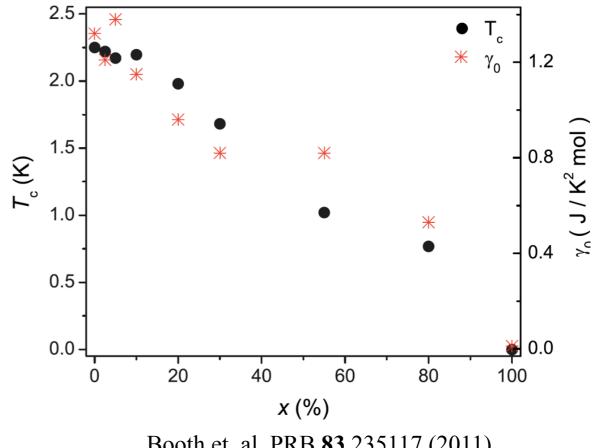
Each Ce atom provides a composite pair

 $T_c$  and  $\rho_s$  increase linearly with Ce concentration

Communication with P. Coleman

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Booth et. al. PRB 83 235117 (2011)

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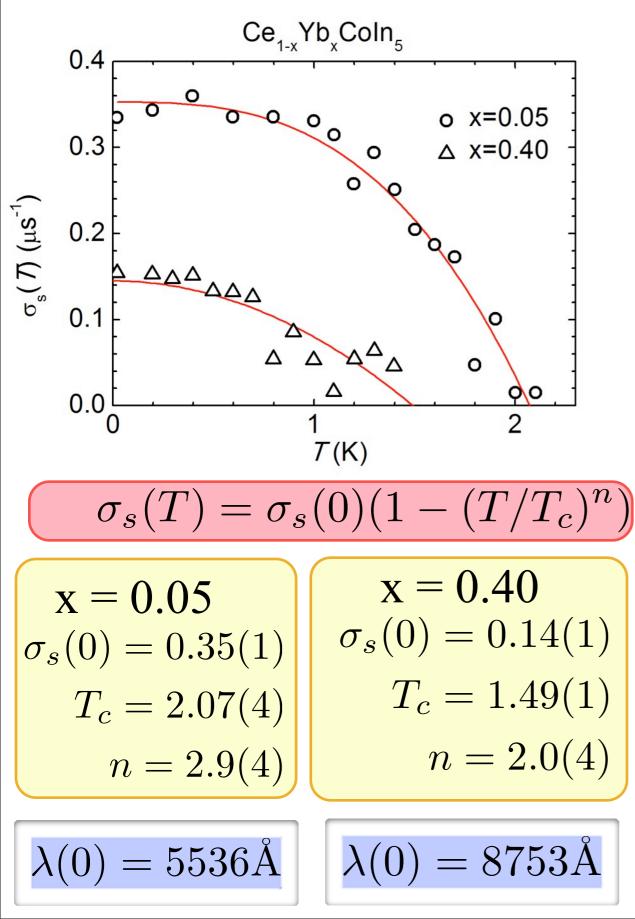
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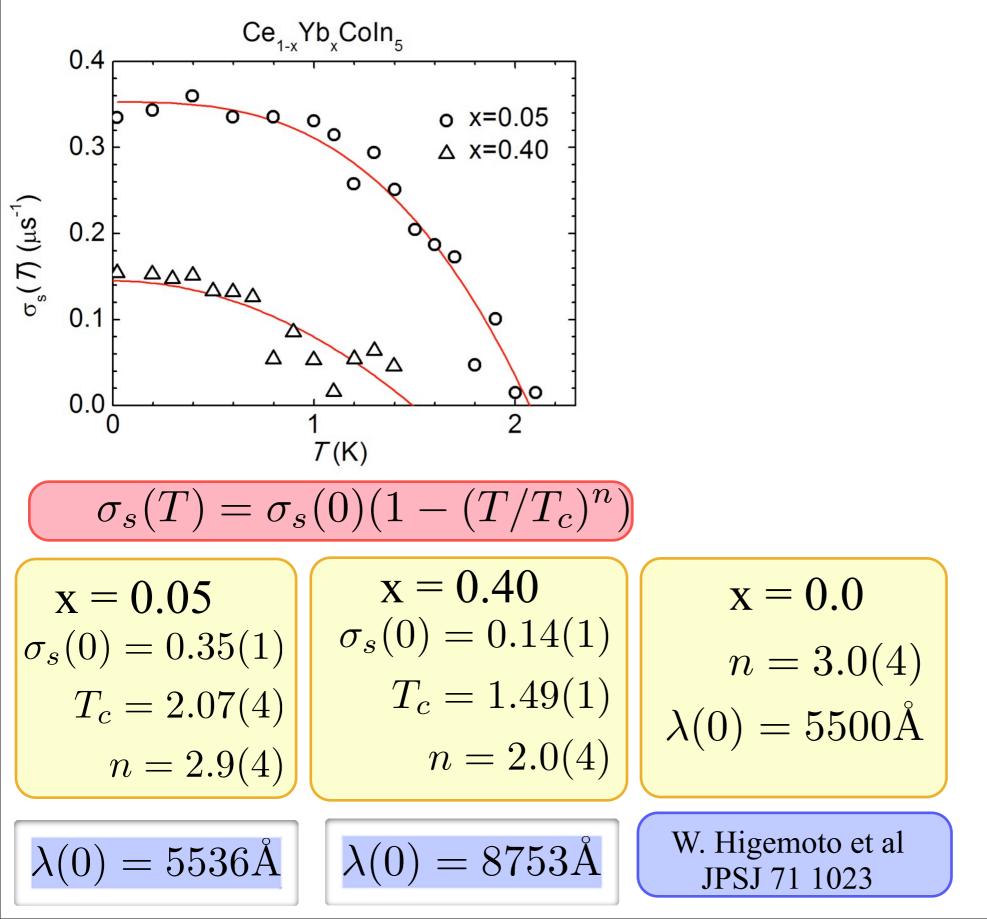
### TF-µSR

Measure absolute value of magnetic penetration depth  $\propto \rho_s^{-1/2}$ , to see if *T*c is controlled by  $\rho_s$  or whether  $\rho_s$ is roughly constant.

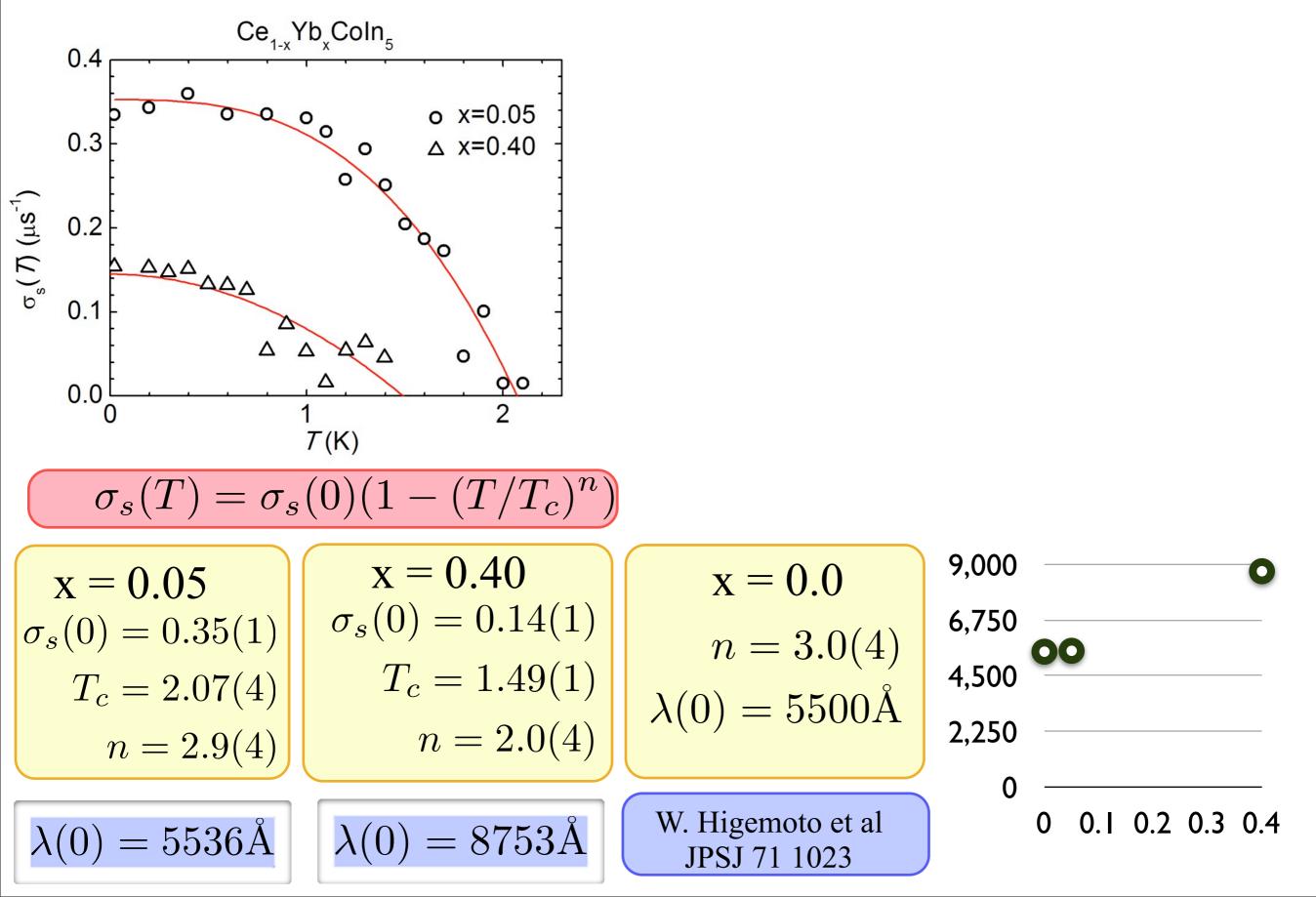
Muon's relaxation rate in the vortex state of Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub>



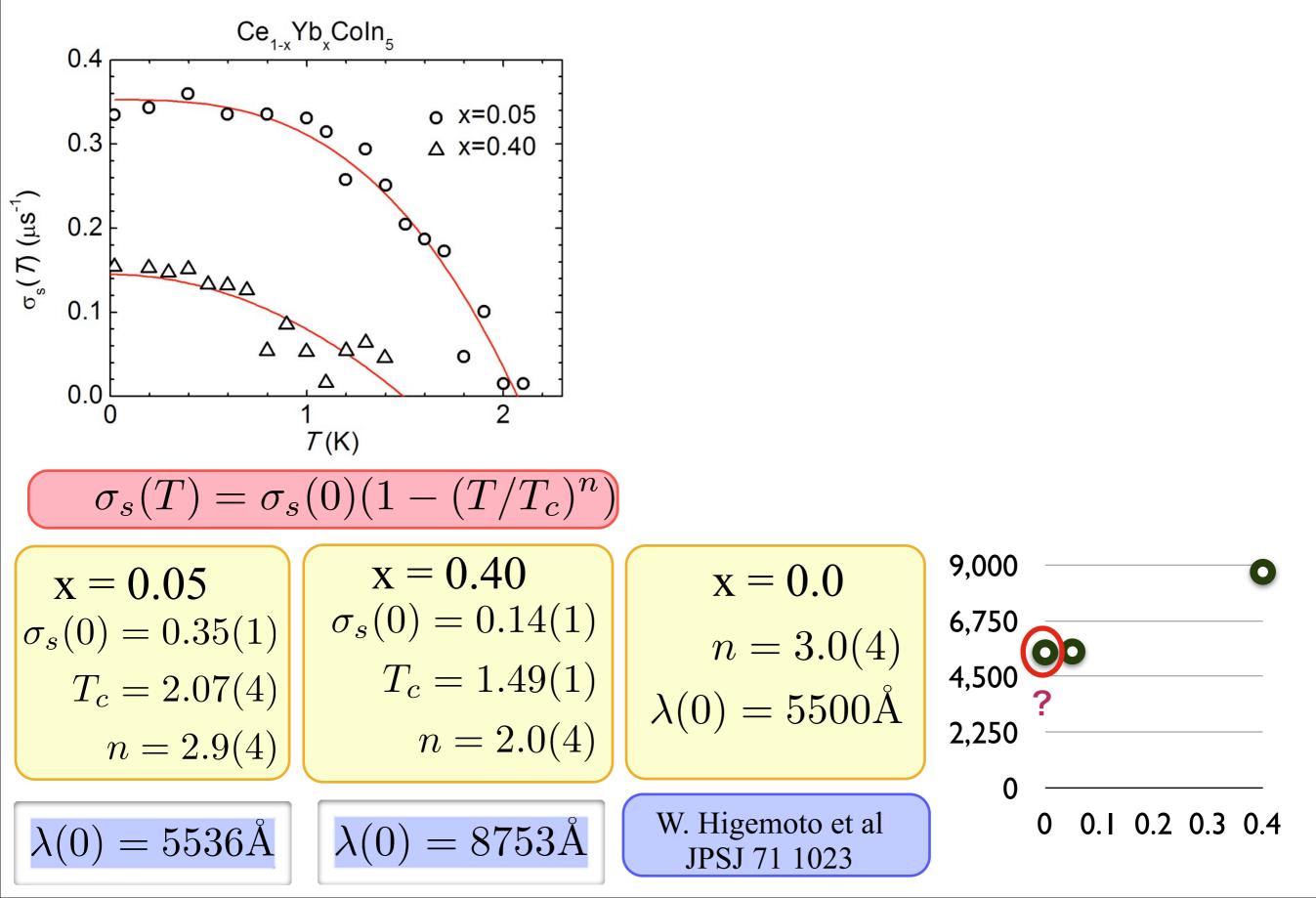
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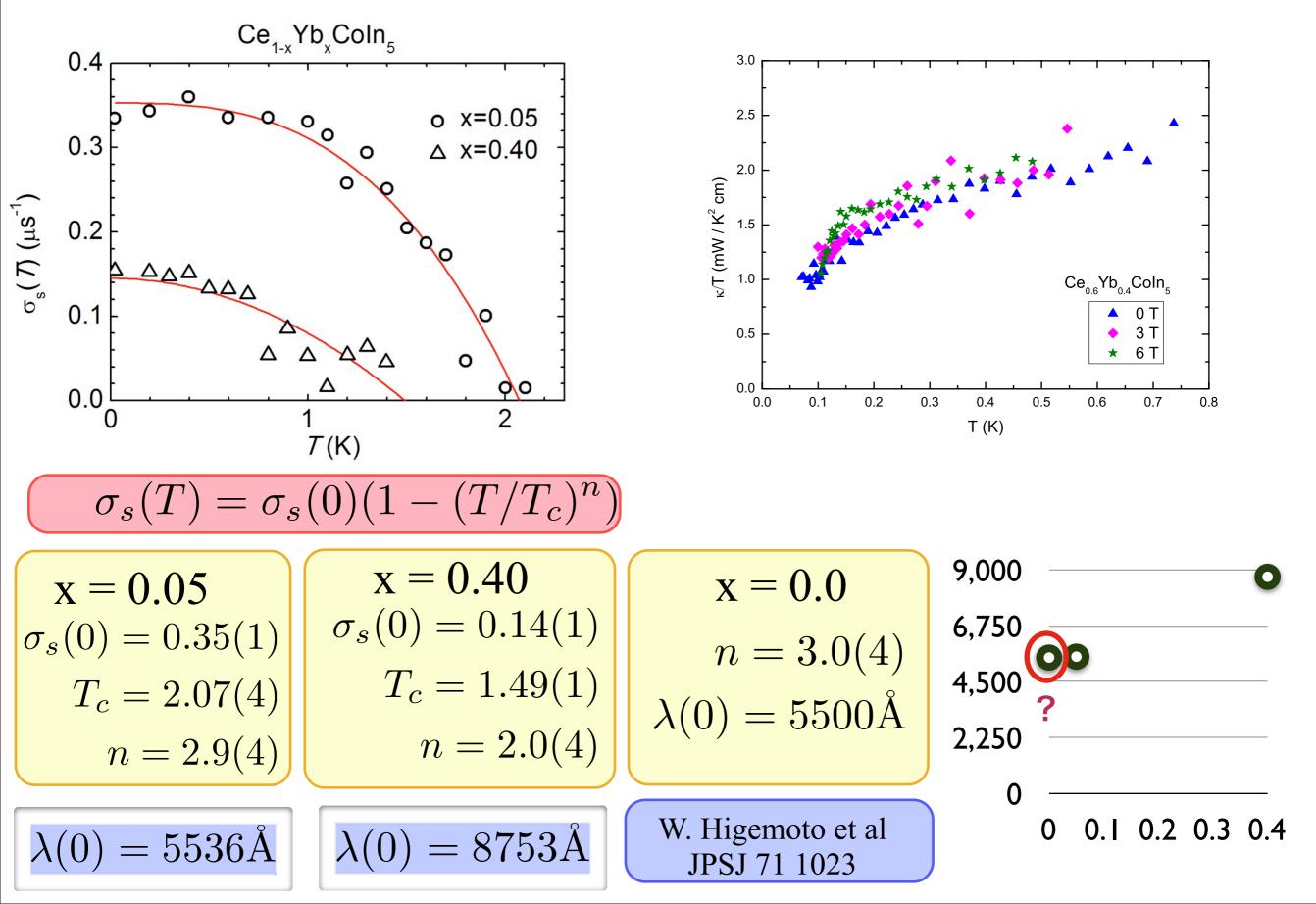


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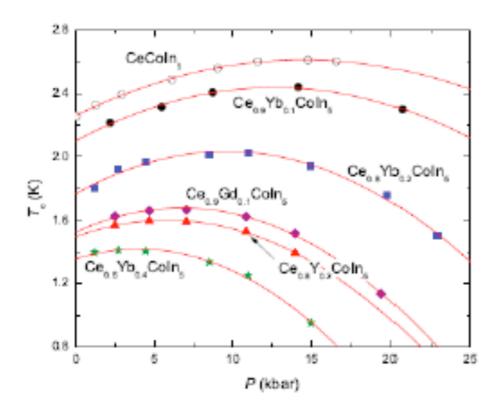
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Thermal conductivity



### Ce<sub>1-x</sub>Yb<sub>x</sub>CoIn<sub>5</sub>: Pressure dependence of $T^*$ and $T_c$

# Effect of pressure on superconductivity and Kondo-lattice coherence temperature in $Ce_{1-x}Yb_xCoIn_5$



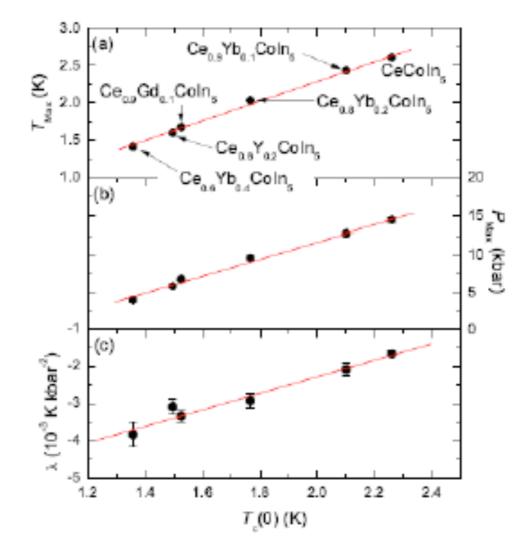
T<sub>c</sub>(P) data described by expression

 $T_c = T_{\rm Max} + \lambda \left( P - P_{\rm Max} \right)^2$ 

 $T_{Max}$ ,  $P_{Max}$ ,  $\lambda$  are linear functions of  $T_c(0)$ 

T\*(P) data described by expression

 $T^{*}(P) = T^{*}(0) + \xi P$ 



- Effect of pressure on Ce<sub>1-x</sub>R<sub>x</sub>CoIn<sub>5</sub> is independent of R and x in range 0 to 2.5 GPa
- Apparently, pressure does not change v<sub>Yb</sub> and primarily affects electronic state of Ce ion

B. D. White, J. J. Hamlin, K. Huang, L. Shu et al, Phys. Rev. B 86 100502(R) 2012

### Conclusions

• In the Ce1-xYbxCoIn5 system, Kondo coherence and SC are weakly dependent on x, while the NFL characteristics exhibit strong variations with x

- This may be due to cooperative behavior involving the unstable valences of Ce and Yb
- XPS, XANES, and magnetic susceptibility measurements indicate:
  - Ce valence is close to +3 for all x

- Yb valence decreases from ~+3 at  $x \approx 0$  to ~+2.3 at  $x \approx 0.2$  and then remains constant at ~ +2.3 to  $x \approx 1 \Rightarrow$ Yb VALENCE TRANSITION

• *T*c is proportional to Ce concentration!  $T_c$  does not scale with  $T^*$ 

• The pressure dependences of  $T^*$  and  $T_c$  in Ce<sub>1-x</sub>R<sub>x</sub>CoIn<sub>5</sub> are independent of *R* ion (Yb, Y, and Gd) in the pressure range 0 to 2.5 GPa  $\Rightarrow$ no change in v<sub>Yb</sub> in this *P* range, need higher pressure!

x : Yb concentration  $n_{Ce}^{3+}$  : number of Ce<sup>3+</sup>  $n_{Ce}^{4+}$  : number of Ce<sup>4+</sup>  $n_{Yb}^{3+}$  : number of Yb<sup>3+</sup>  $n_{Yb}^{2+}$  : number of Yb<sup>2+</sup>  $\delta V_{Ce}$  : the volume difference between each Ce<sup>3+</sup> and Ce<sup>4+</sup>, which should > 0  $\delta V_{Yb}$  : the volume difference between each Yb<sup>3+</sup> and Yb<sup>2+</sup>, which should < 0

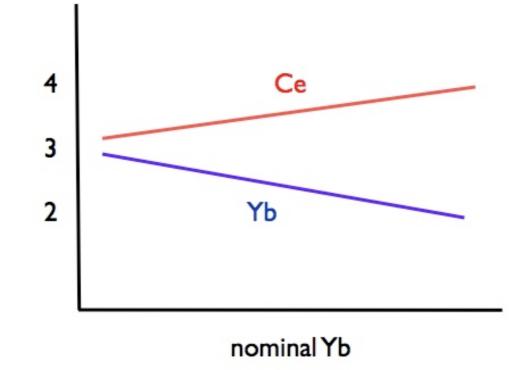
Ce and Yb ions adopt cooperative intermediate valence states, which means  $n_{Ce}^{3+}$ ,  $n_{Ce}^{4+}$ ,  $n_{Yb}^{3+}$ ,  $n_{Yb}^{2+}$  adjust cooperatively so that

$$(1-x)(n_{\mathrm{Ce}^{3+}} - n_{\mathrm{Ce}^{4+}})\delta V_{Ce} + x(n_{\mathrm{Yb}^{3+}} - n_{\mathrm{Yb}^{2+}})\delta V_{Yb} = 0$$

x : Yb concentration  $n_{Ce}^{3+}$  : number of Ce<sup>3+</sup>  $n_{Ce}^{4+}$  : number of Ce<sup>4+</sup>  $n_{Yb}^{3+}$  : number of Yb<sup>3+</sup>  $n_{Yb}^{2+}$  : number of Yb<sup>2+</sup>  $\delta V_{Ce}$  : the volume difference between each Ce<sup>3+</sup> and Ce<sup>4+</sup>, which should > 0  $\delta V_{Yb}$  : the volume difference between each Yb<sup>3+</sup> and Yb<sup>2+</sup>, which should < 0

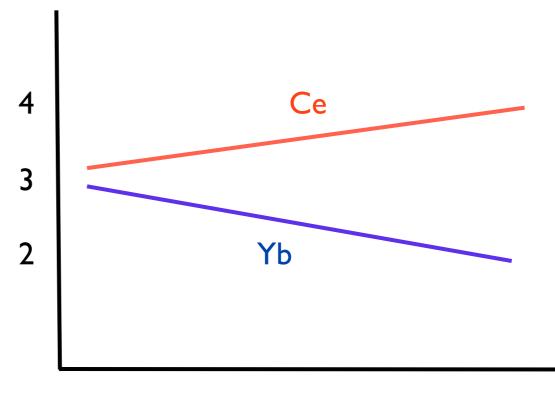
Ce and Yb ions adopt cooperative intermediate valence states, which means  $n_{Ce}^{3+}$ ,  $n_{Ce}^{4+}$ ,  $n_{Yb}^{3+}$ ,  $n_{Yb}^{2+}$  adjust cooperatively so that

$$(1-x)(n_{\mathrm{Ce}^{3+}} - n_{\mathrm{Ce}^{4+}})\delta V_{Ce} + x(n_{\mathrm{Yb}^{3+}} - n_{\mathrm{Yb}^{2+}})\delta V_{Yb} = 0$$



The pair-breaking effect in unconventional superconductors arises via

- potential (non-magnetic) Abrikosov-Gor'kov model ( $T_c \rightarrow 0$ , when
- $I_{mfp} \rightarrow \xi$ )
- spin-flip scattering
  - $\Delta T_{c} \propto \mathcal{J}^{2}D_{J}$
  - $\mathcal{J}$  is small (even-parity)  $D_{\rm J}: 0.18 \ {\rm Ce}^{3+}(4f^1)$ 
    - $0.32 \text{ Yb}^{3+}(4f^{13})$



nominal Yb

- *f*-electron occupancy for Ce:  $n^{Ce_f}$
- *f*-hole occupancy for Yb:  $n^{Yb_f}$
- valence of Ce:  $v_{Ce}$
- valence of Yb:  $v_{Yb}$
- effective moment of Ce:  $\mu_{Ce}$
- effective moment of Yb:  $\mu_{Yb}$ , then we have the relation:

$$\nu_{Ce} = 3n_f^{Ce} + 4(1 - n_f^{Ce})$$

$$n_f^{Ce} = 4 - \nu_{Ce}$$

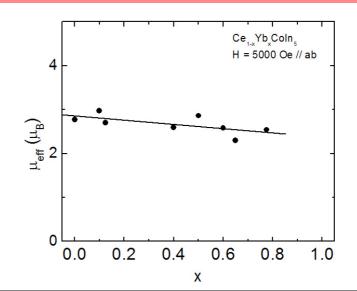
$$n_f^{Ce} = \frac{\mu_{Ce}^2}{\mu_{Ce3+}^2}$$

$$\nu_{Yb} = 3n_f^{Yb} + 2(1 - n_f^{Yb})$$

$$n_f^{Yb} = \nu_{Yb} - 2$$

$$n_f^{Yb} = \frac{\mu_{Yb}^2}{\mu_{Yb3+}^2}$$

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x)$$



- *f*-electron occupancy for Ce:  $n^{Ce_f}$
- *f*-hole occupancy for Yb:  $n^{Yb_f}$
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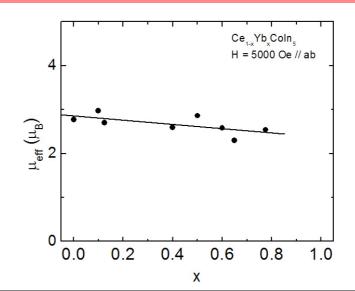
$$n_f^{Ce} = \frac{\mu_{Ce}^2}{\mu_{Ce3+}^2}$$

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$$n_f^{Yb} = \nu_{Yb} - 2$$

$$n_f^{Yb} = \frac{\mu_{Yb}^2}{\mu_{Yb3+}^2}$$

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x)$$



- *f*-electron occupancy for Ce:  $n^{Ce_f}$
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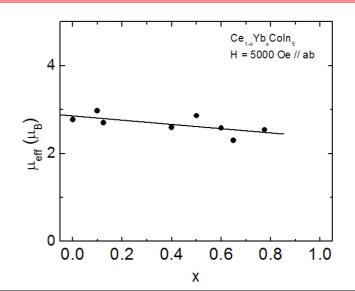
$$n_f^{Ce} = \frac{\mu_{Ce}^2}{\mu_{Ce3+}^2}$$

$$\nu_{Yb} = 3n_f^{Yb} + 2(1 - n_f^{Yb})$$

$$n_f^{Yb} = \nu_{Yb} - 2$$

$$n_f^{Yb} = \frac{\mu_{Yb}^2}{\mu_{Yb3+}^2}$$

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x)$$



- *f*-electron occupancy for Ce:  $n^{Ce_f}$
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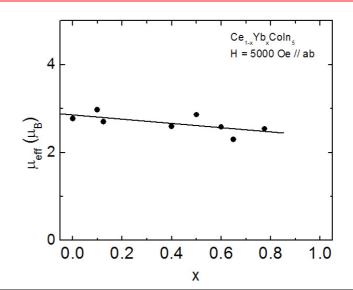
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$$\nu_{Yb} = 3n_f^{Yb} + 2(1 - n_f^{Yb})$$

$$n_f^{Yb} = \nu_{Yb} - 2$$

$$n_f^{Yb} = \frac{\mu_{Yb}^2}{\mu_{Yb3+}^2}$$

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x)$$



- *f*-electron occupancy for Ce:  $n^{Ce_f}$
- *f*-hole occupancy for Yb:  $n^{Yb_f}$
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$$n_f^{Ce} = \frac{\mu_{Ce}^2}{\mu_{Ce3+}^2}$$

$$\nu_{Yb} = 3n_f^{Yb} + 2(1 - n_f^{Yb})$$

$$n_f^{Yb} = \nu_{Yb} + 2$$

$$n_f^{Yb} = \frac{\mu_{Yb}^2}{\mu_{Yb3+}^2}$$

$$\mu_{eff}^2(x) = \mu_{Ce}^2(1-x) + \mu_{Yb}^2(x)$$

