Three new Insights on Quantum Criticality

Beijing, Nov. 11th, 2012

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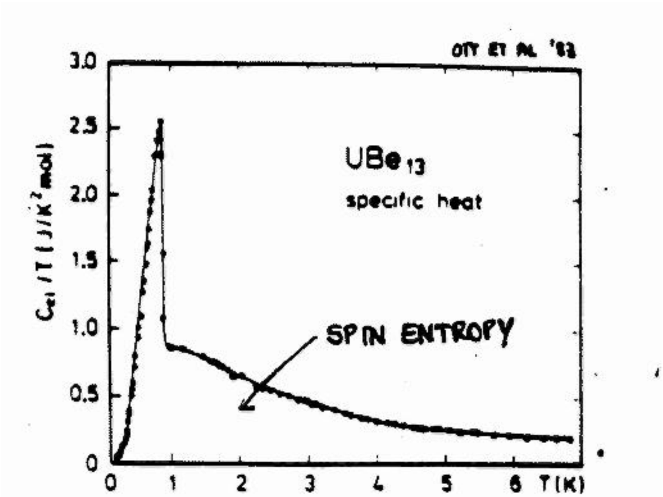
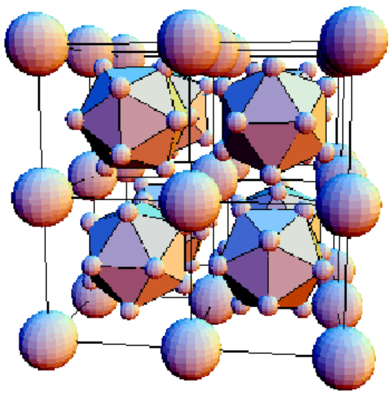
M. Norman **I. Paul**

C. Pépin **(IPhT, CEA-Saclay)** **IIP, Natal**

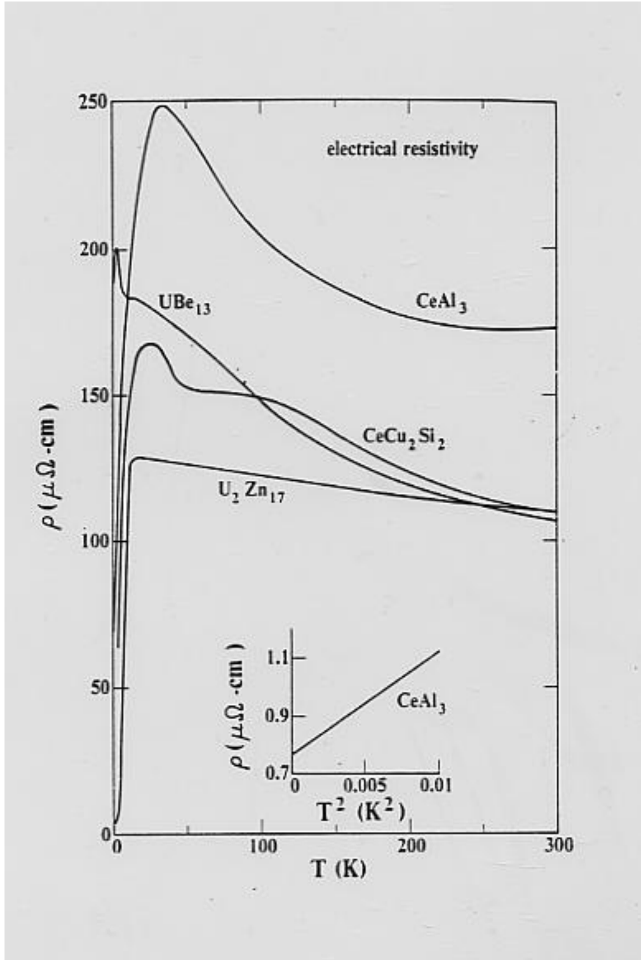
Heavy Fermion Metals: Extreme Limit of Mass Renormalization.





UBe_{13}

$$\frac{m^*}{m} \sim 1000$$



Ce : $4f^1$

S=1/2 L=3
Spin Orbit : J= |L-S|= 5/2

Yb : $4f^{13}$

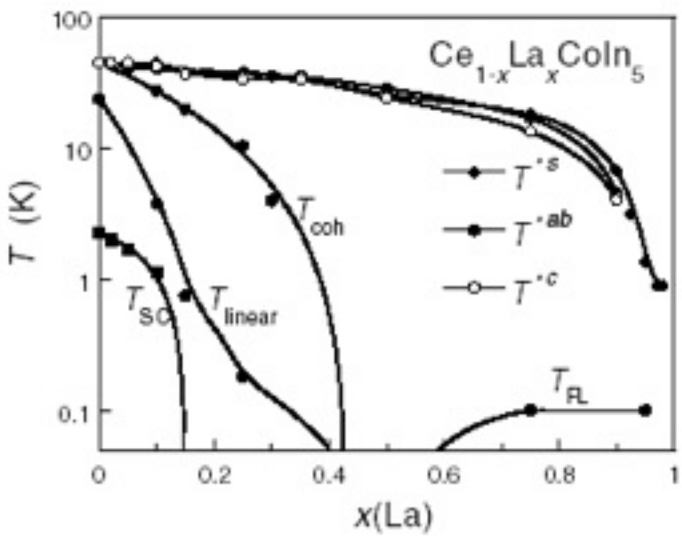
S=1/2 L=3
S O : J= |L+S|= 7/2

U : $5f^2$

S=1 L=3+2
S O : J= |L-S|= 4

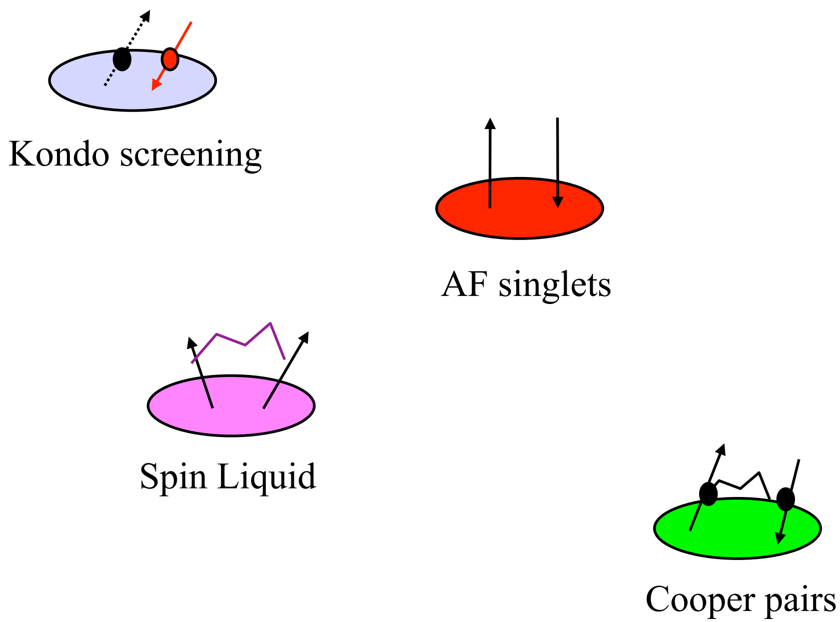
Crystal Electric Field effects split the big moments and compete with Hunds rules

- → Ferromagnetic fluctuations
- → valence fluctuations
- → multiple stage screening ?



Nakatsuji 03

Entropic considerations

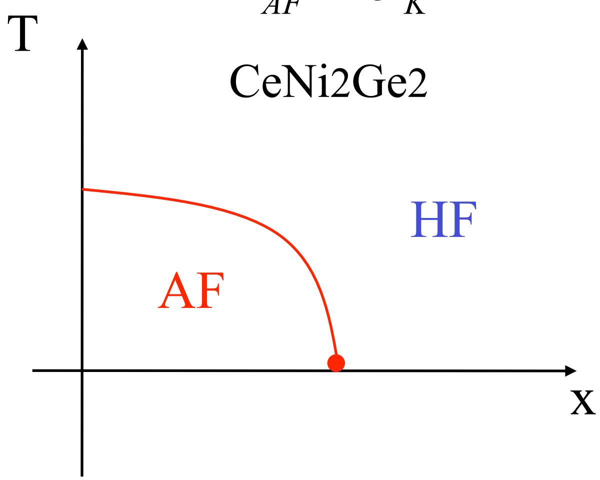


Two scenarios

Spin Density Wave

$$E_{AF} \leq J_K$$

CeNi2Ge2



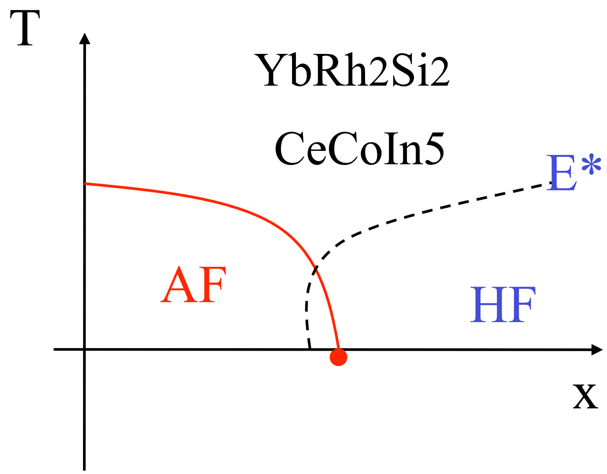
SDW scenario:
big Fermi surface at the QCP

Kondo Breakdown

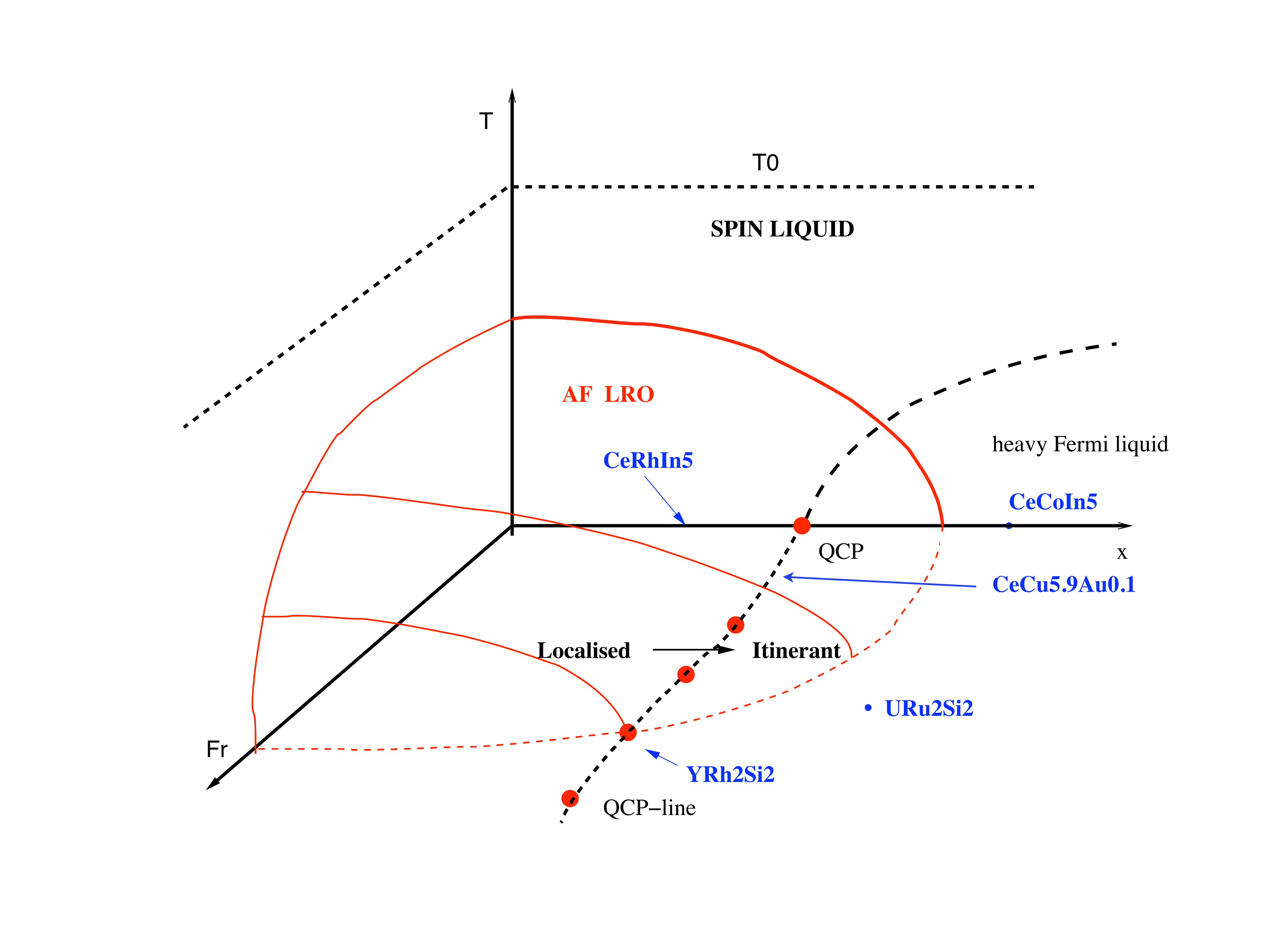
$$E_{AF} \geq J_K$$

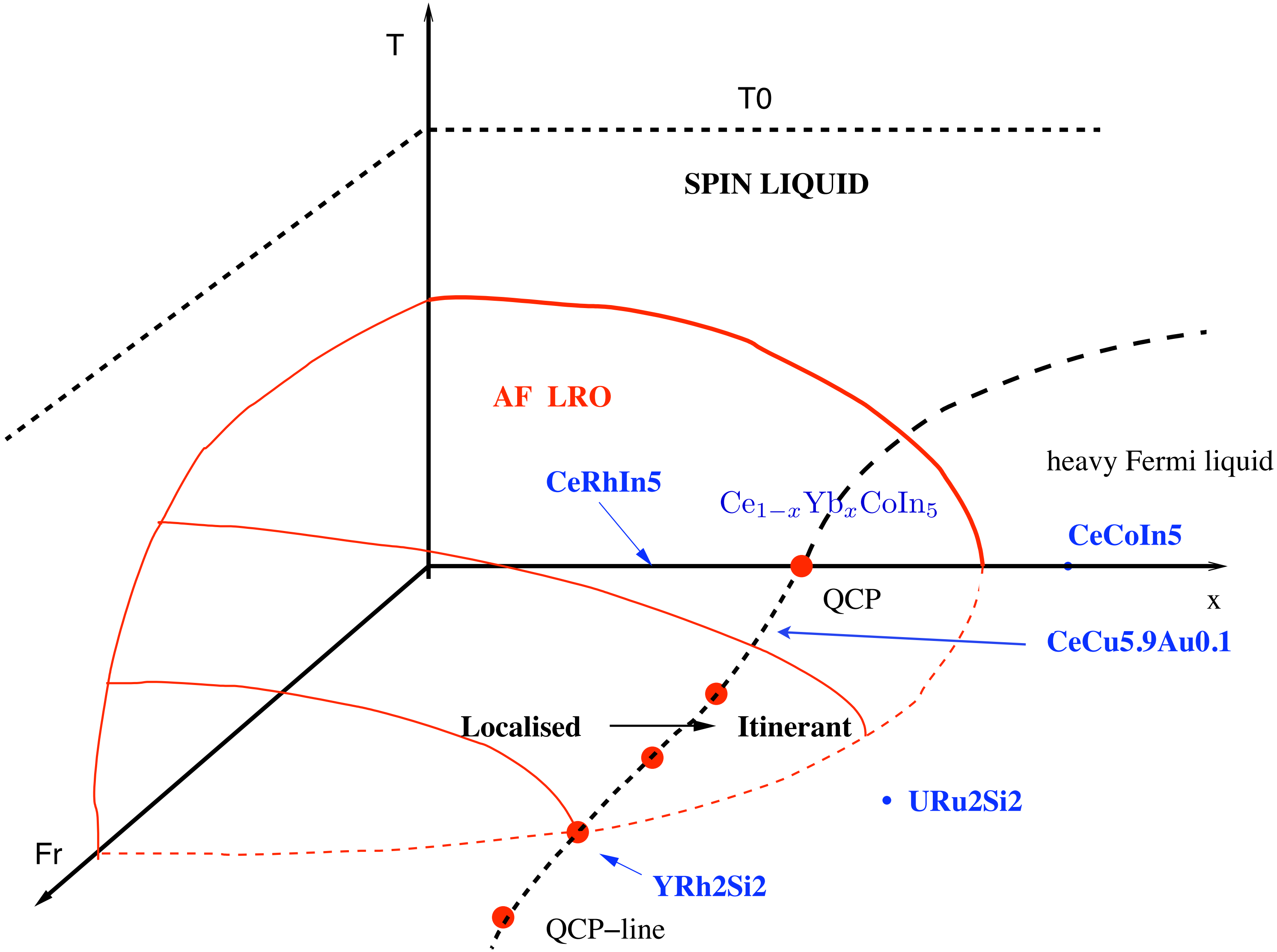
YbRh2Si2

CeCoIn5

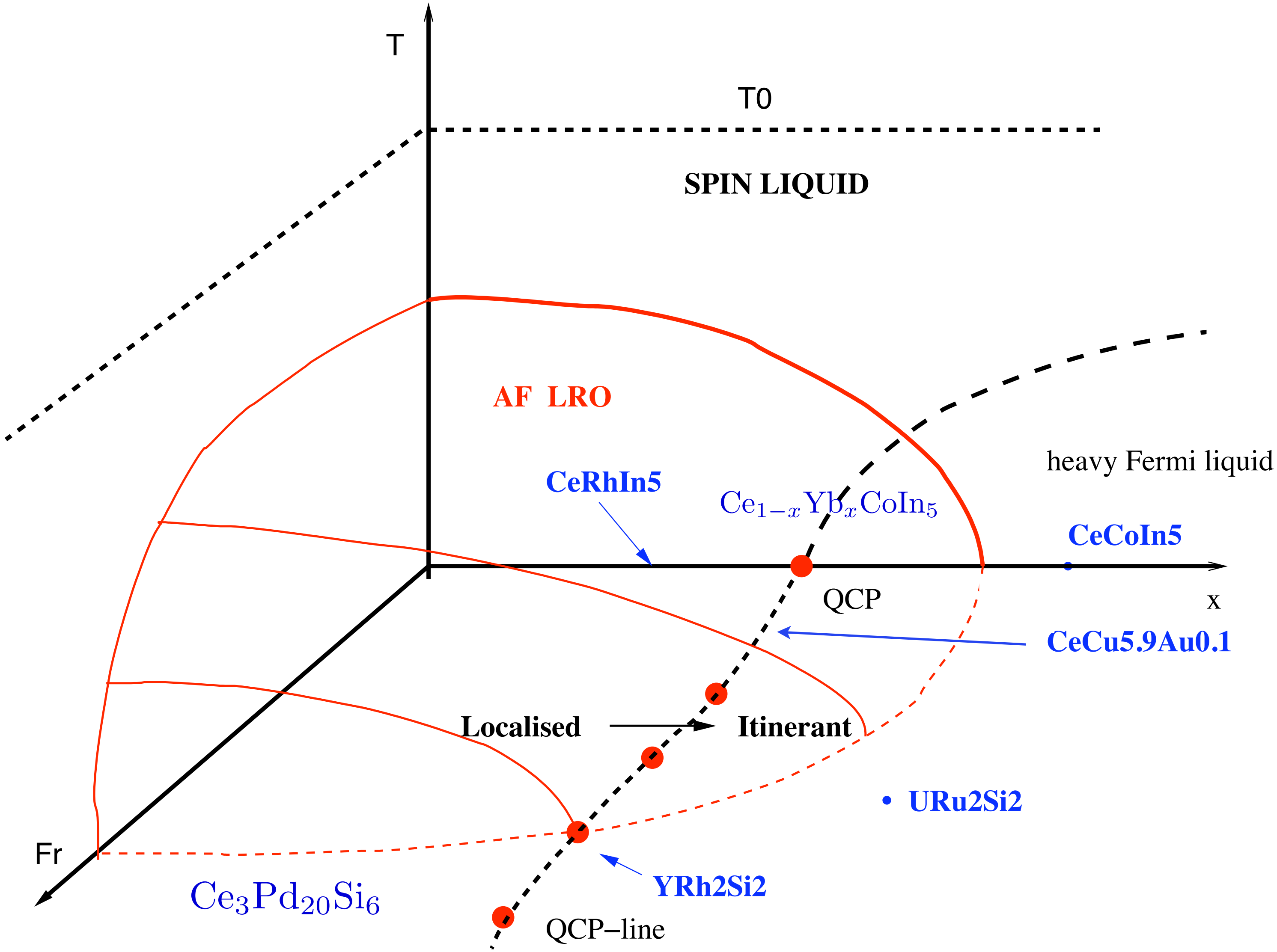


QCP with fractionalization


T
T0
SPIN LIQUID
AF LRO
CeRhIn5
heavy Fermi liquid
CeCoIn5
QCP
x
CeCu5.9Au0.1
Localised
Itinerant
URu2Si2
Fr
YRh2Si2
QCP−line



dimanche 11 novembre 12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x
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Localised
Itinerant
URu2Si2
Fr
YRh2Si2
$Ce_3Pd_{20}Si_6$
QCP−line

Breakdown of the Kondo effect associated with a Mott transition on the f-electrons

P. Coleman (Schroder 2000)
deconfinement, fractionalization

Zhu, Martin, PNP (09)
modulations in Kondo breakdown

Q. Si, Nature (02-)
S. Kirchner (06,08)
locally quantum critical

Burdin, Grempel, Georges (98)
breakdown by exhaustion

Pines, Yang, Fisk (08)
two fluids model

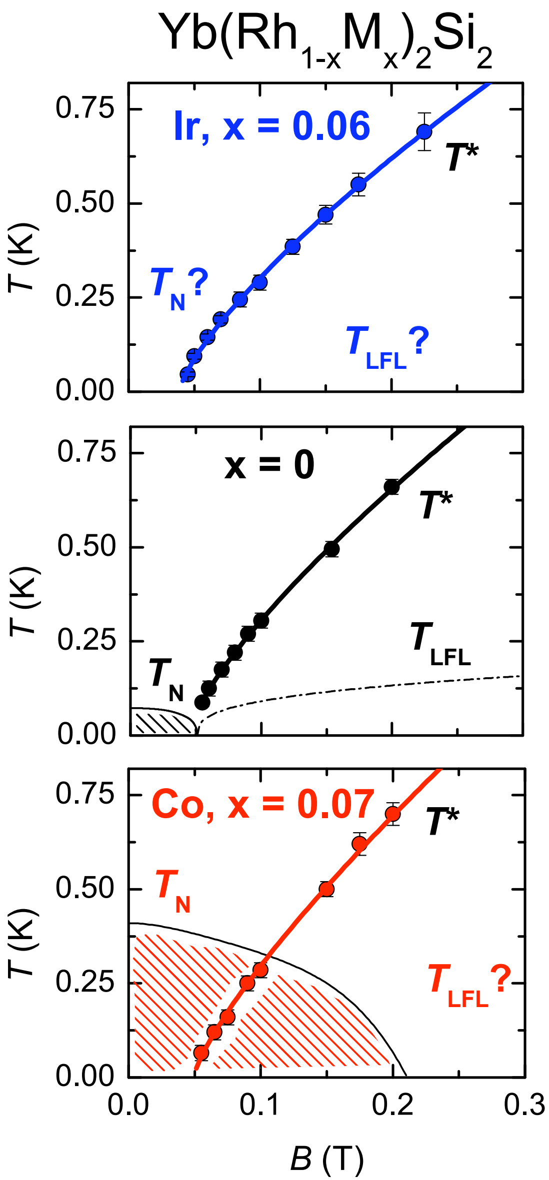
B. Jones (2010)
RG on Kondo Breakdown

Continentino (09) Vekhter + Seo+ CP (10) Paul ,Norman (10)
SC quantum critical point

CP, Norman ,Paul (07)
selective Mott transition, z=3 regime of fluctuations

Senthil, Sachdev ,Vojta (04)
model for fractionalization, spin liquid

Doping plays the role of pressure



Dresden group (10)

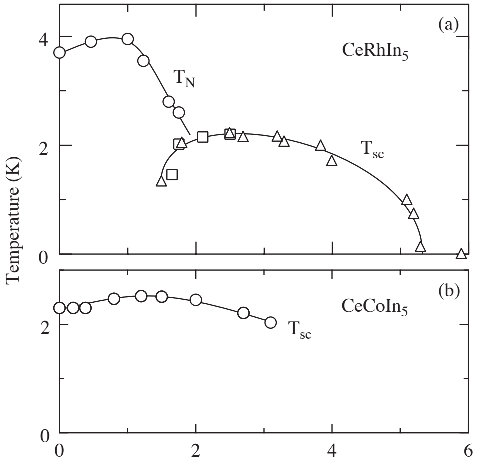
Is there a spin liquid on the left of T* ?

Differentiate the scenarios where the KB is tight to AFM transition (Si et al.) from the ones where the KB is alone

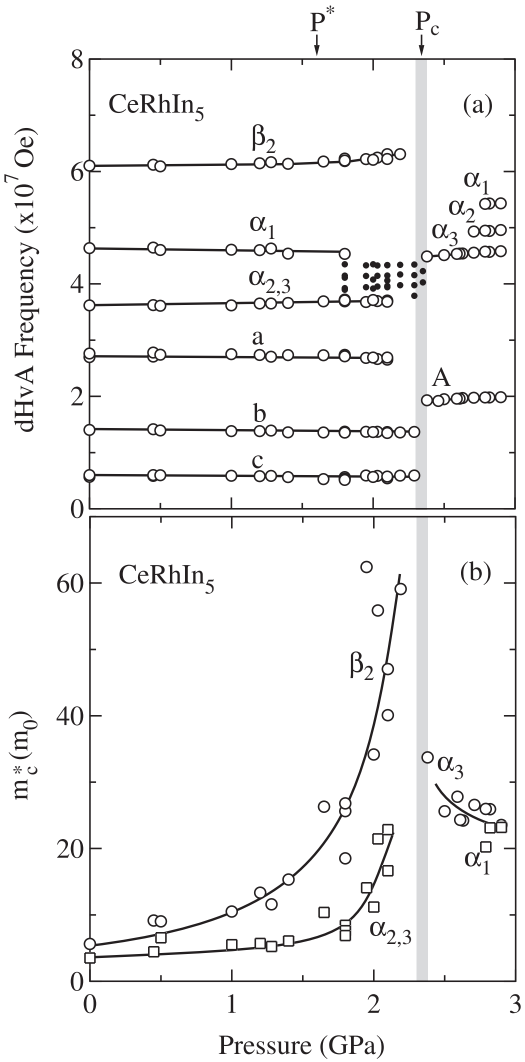
T* related to a Lifchitz transition

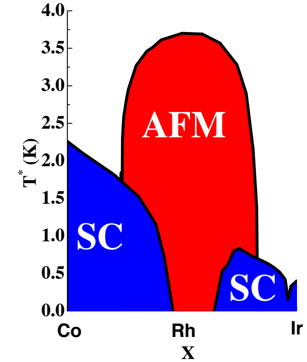
Vojta, Benlagra ('12)

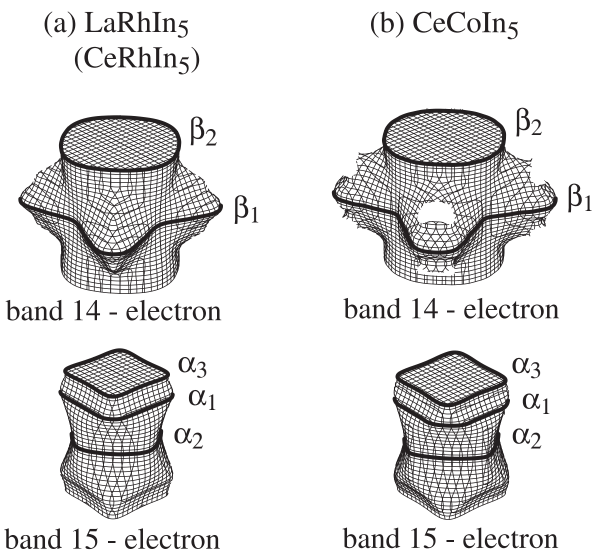
Pressure induced SC and Fermi surface reconfiguration in 115



similar to UGe2
but AFM vs FM

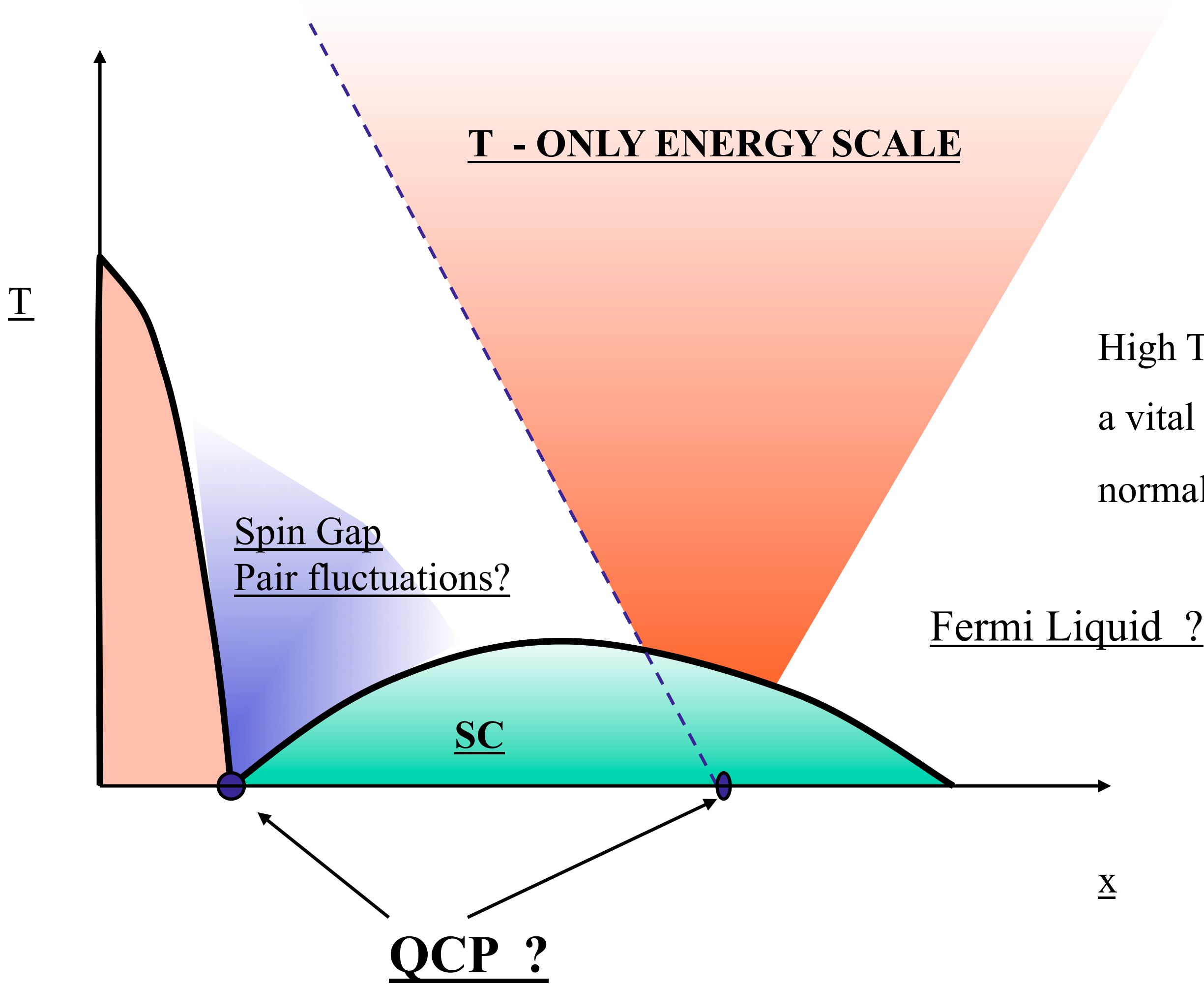






Onuki's group (05)

Quantum criticality



High Tc: QCP may play a vital role in setting the normal state properties.

Do we understand QCPs
where the order parameter is uncontraversial?

Theoretical approaches

Low energy properties ⟷ Universality

UV

High energy physics – microscopic hamiltonian

Integrate out the "fast" degrees of freedom

IR

Low energy, slow, universal part

Universality

Low energy properties ⟷ Universality

UV

High energy physics – microscopic hamiltonian

Integrate out the "fast" degrees of freedom

IR

Low energy, slow, universal part

Universality

What is observed around some QCP in heavy fermions

$\rho(T) \sim T$,

$\chi(T) \sim T^{-\alpha}$, with $\alpha \leq 1$

$\gamma_p(T) = \frac{C_P}{T} \sim T^{-\beta}$, with $\beta \leq 1$.

Universal Too!

Landau Fermi liquid theory verified by ``all '' conductors above 1D

$\rho(T) \sim T^2$,

$\chi(T) \sim \mu_0^2 \rho(\epsilon_F)$,

$\gamma_p(T) = \frac{C_P}{T} \sim \frac{k_B^2 \pi^2}{3} \rho(\epsilon_F)$.

Universal exponents

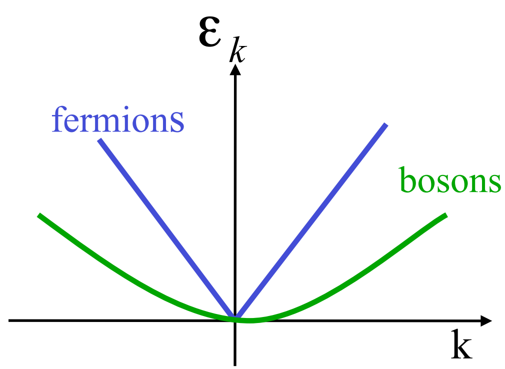
Can we integrate the fermions out of the partition function?

φ^4 effective bosonic theory

For example z=2

$$D^{-1}(q,\Omega) = \frac{|\Omega|}{E_F} + \frac{q^2}{k_F^2}$$

fermions are mass-less but fast compared to bosons?



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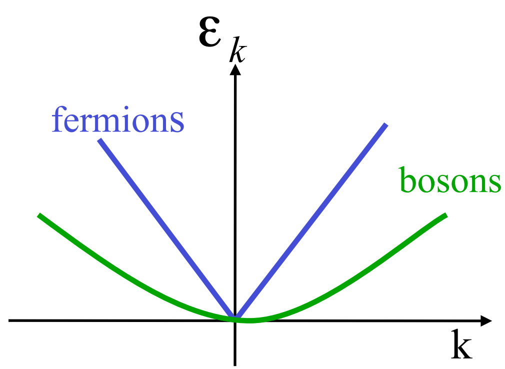
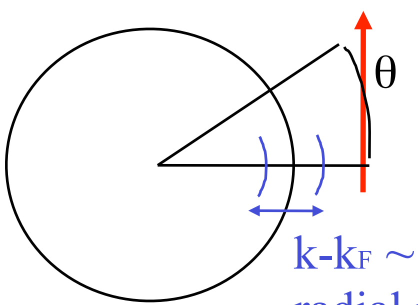


transverse modes are slow!
(controlled by boson fluctuations)



$k - k_F \sim T$
radial modes are fast

$$q_{radial} \sim T$$
$$q_{transverse} \sim \sqrt{T}$$

Eliashberg theory around itinerant ferromagnetism, or U(1) gauge theory coupled to matter

$$H_{sf} = \sum_{k,\alpha} \epsilon_k c^{\dagger}_{k,\alpha} c_{k,\alpha} + \sum_{q} \chi^{-1}_{s,0}(q) \mathbf{S}_q \mathbf{S}_{-q}$$

$$+ g \sum_{k,q,\alpha,\beta} c^{\dagger}_{k,\alpha} \boldsymbol{\sigma}_{\alpha\beta} c_{k+q,\beta} \cdot \mathbf{S}_q,$$

$$\chi_{s,0}(q,\Omega) = \frac{\chi_0}{\xi^{-2} + q^2 + A\Omega^2 + O(q^4,\Omega^4)}.$$

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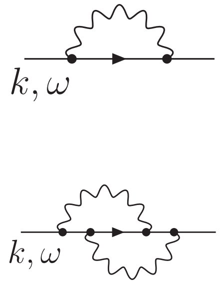
d=2



$\omega_0^{1/3} \; \omega^{2/3}$

$\downarrow \quad \left(\frac{\omega_0}{\omega}\right)^{1/3}$

$\omega_0^{2/3} \; \omega^{1/3}$

Rech, CP, Chubukov (06)

The bare power counting diverges in $d \leq 3$

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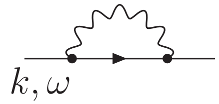

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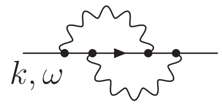

$\omega_0^{2/3}\ \omega^{1/3}$

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The bare power counting diverges in $d \leq 3$

- neglect vertex corrections
- dressed propagators (self-energy)

$$\alpha \sim \frac{\bar{g}^2}{\gamma v_F^3} \sim \frac{\bar{g}}{N E_F} \ll 1 \qquad \beta \sim \frac{m\bar{g}}{\gamma v_F} \sim \frac{m_B}{Nm} \ll 1.$$

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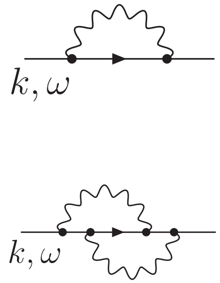

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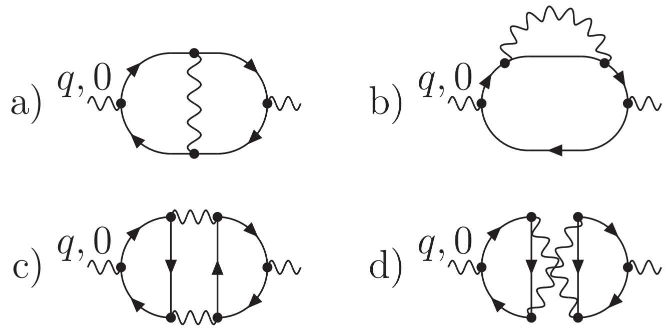

$$|q| \rightarrow d = 2$$

$$q^2 \log q \rightarrow d = 3$$

Belitz, Vojta, Kirkpatrick(03), Chubukov, Maslov (07)
Green, ben Simon(11)

And the culprit is ...

$2k_F$- scattering processes

the back -scattering

affecting AFM, nematic, and Ferro

- FS deformed at the hot spots
- anomalous exponents

$2k_F$- scattering processes

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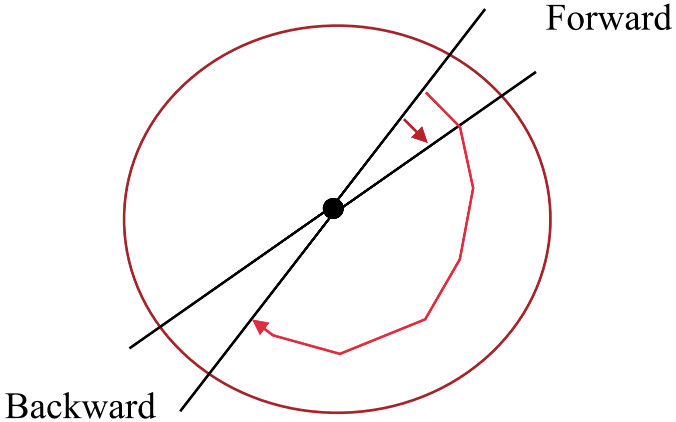
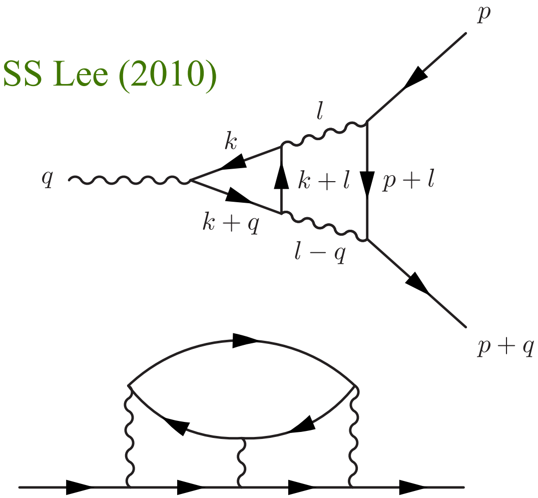


Fermi liquid

$2k_F$- scattering processes

the back -scattering

SS Lee (2010)



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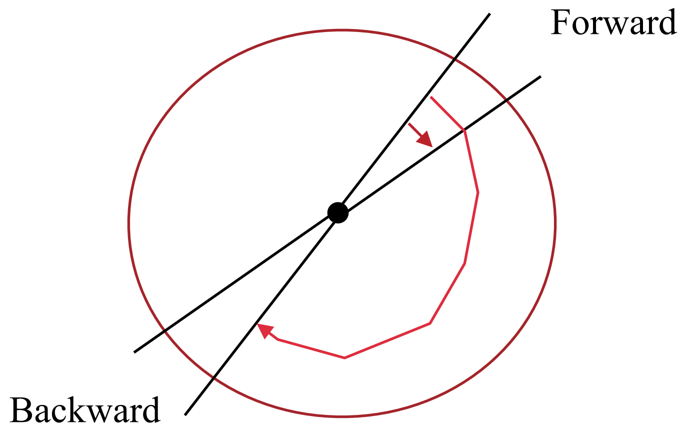

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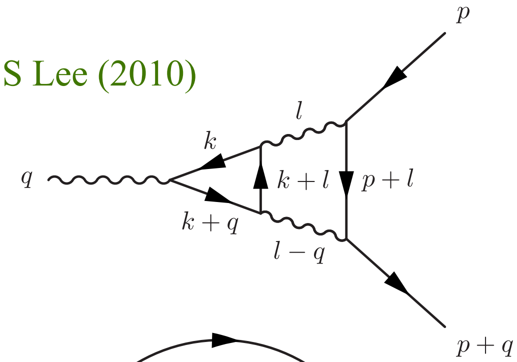

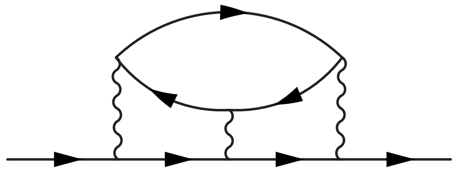

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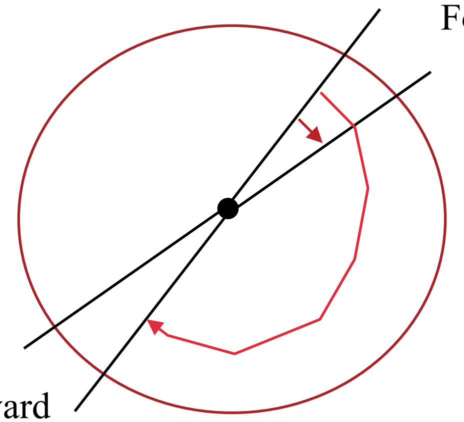

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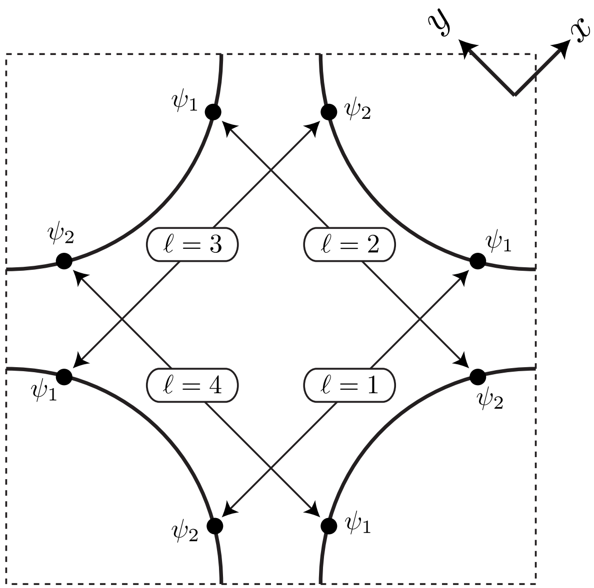

Metlitski, Sachdev (2010)

Pseudo-gap from quantum criticality

K.B.Efetov, H.Meier, C.P.

AFM QCP in d=2

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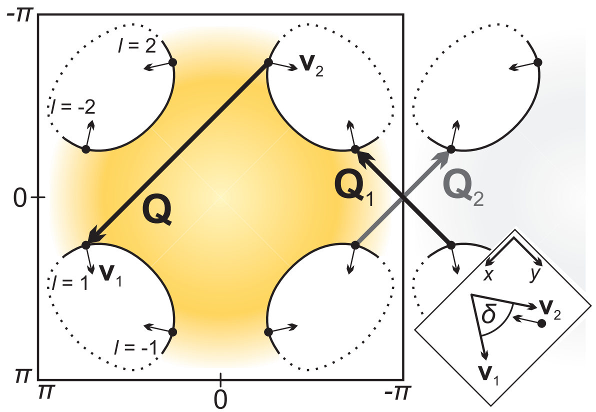
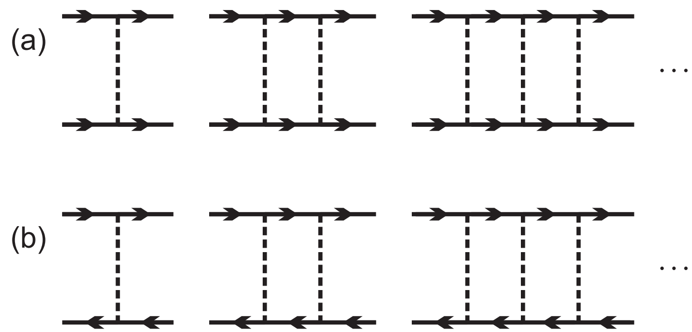
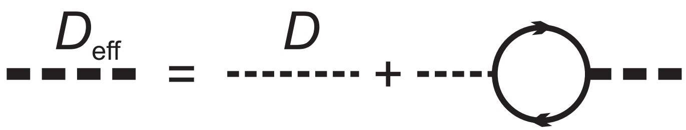


$$c_{\mathbf{p}}^{\mathrm{pp}} \left\langle (i\sigma_2)_{\alpha\beta}\, \psi_{\alpha,\mathbf{p}} \psi_{\beta,-\mathbf{p}} \right\rangle + c_{\mathbf{p}}^{\mathrm{ph}} \left\langle \delta_{\alpha\beta} \psi_{\alpha,\mathbf{p}} \psi^{*}_{\beta,-\mathbf{p}} \right\rangle ,$$





AFM QCP in d=2

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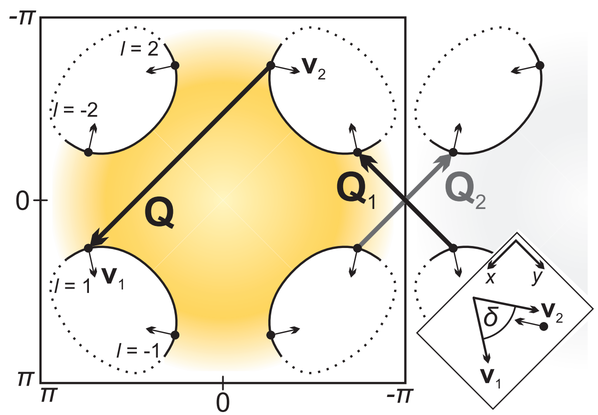
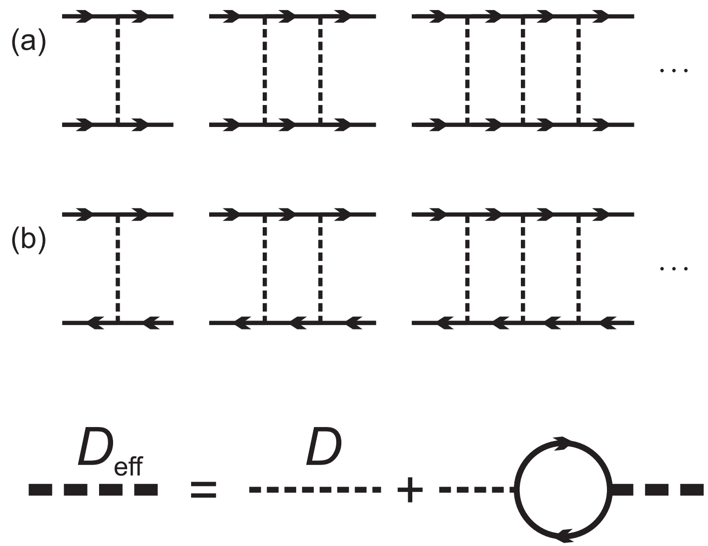
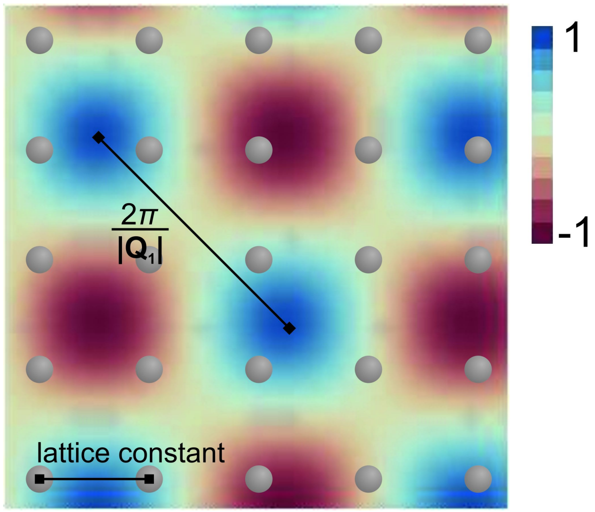


$$c^{\rm pp}_{\mathbf{p}} \left\langle (i\sigma_2)_{\alpha\beta}\, \psi_{\alpha,\mathbf{p}}\psi_{\beta,-\mathbf{p}} \right\rangle + c^{\rm ph}_{\mathbf{p}} \left\langle \delta_{\alpha\beta}\psi_{\alpha,\mathbf{p}}\psi^*_{\beta,-\mathbf{p}} \right\rangle ,$$





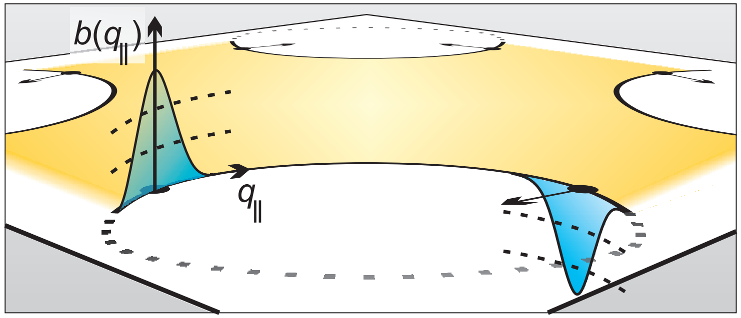
AFM QCP in d=2

Pseudo-Gap equations around the QCP are universal

$$\bar{b}\left(\bar{\varepsilon}\right)=\bar{T}\sum_{\bar{\varepsilon}'}\frac{\cos\Theta\left(\bar{\varepsilon}'\right)}{\sqrt{\bar{\Omega}\left(\bar{\varepsilon}-\bar{\varepsilon}'\right)}},$$

$$\bar{f}\left(\bar{\varepsilon}\right)=\bar{\varepsilon}+\bar{T}\sum_{\bar{\varepsilon}'}\frac{\sin\Theta\left(\bar{\varepsilon}'\right)}{\sqrt{\bar{\Omega}\left(\bar{\varepsilon}-\bar{\varepsilon}'\right)}},$$

$$\bar{\Omega}\left(\bar{\omega}\right)=2\pi\bar{T}\sum_{\bar{\varepsilon}}\sin^2\left(\frac{\Theta\left(\bar{\varepsilon}+\bar{\omega}\right)-\Theta\left(\bar{\varepsilon}\right)}{2}\right)$$



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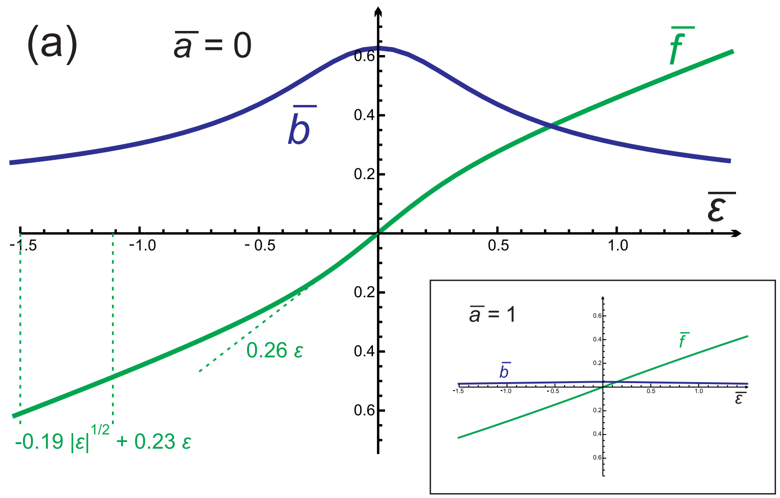
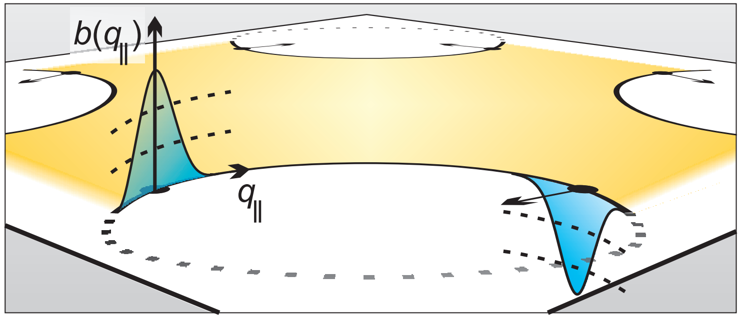
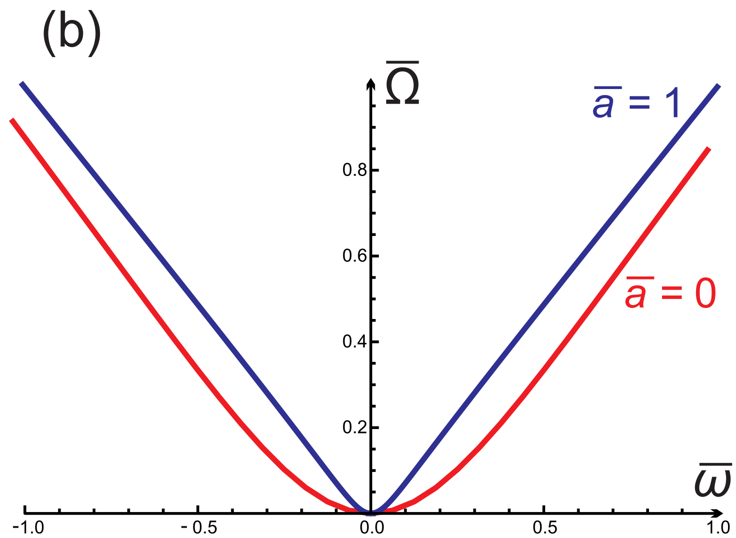
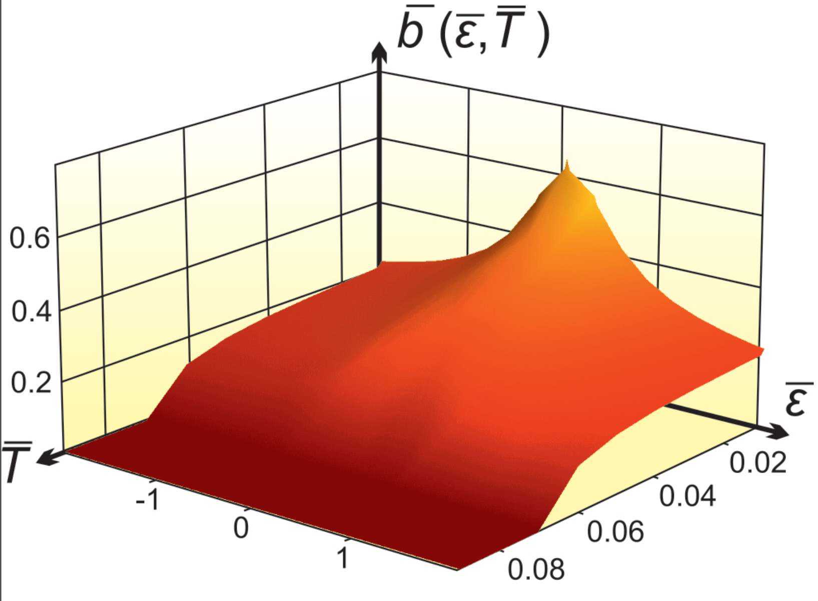
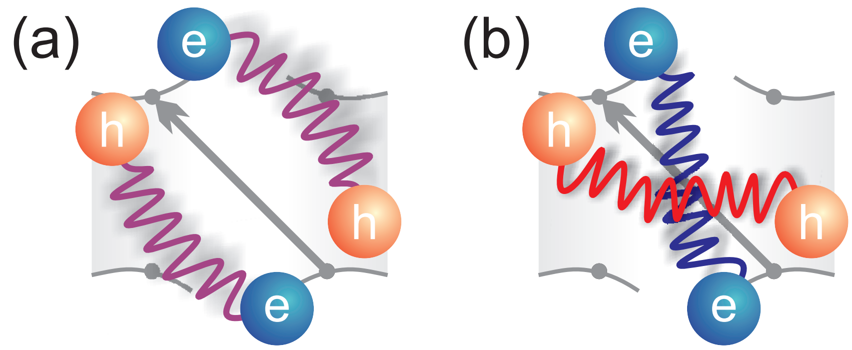
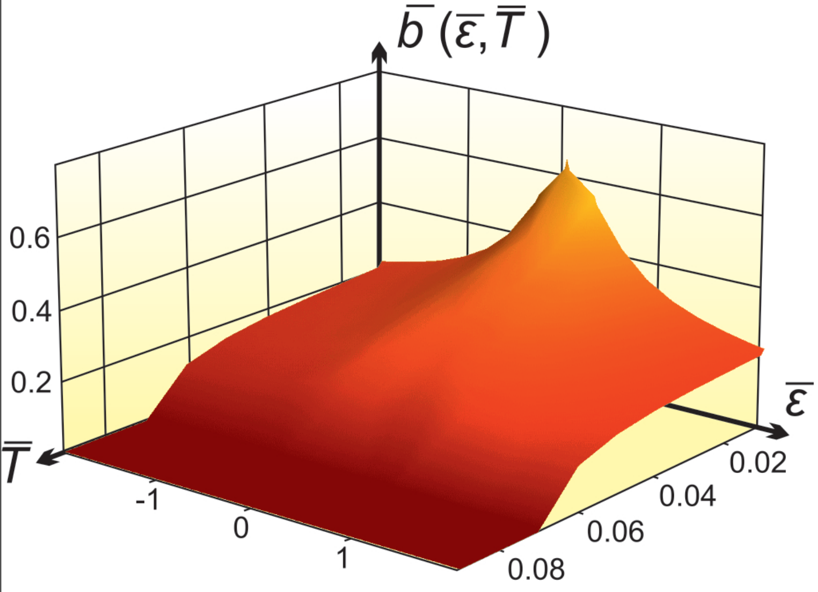




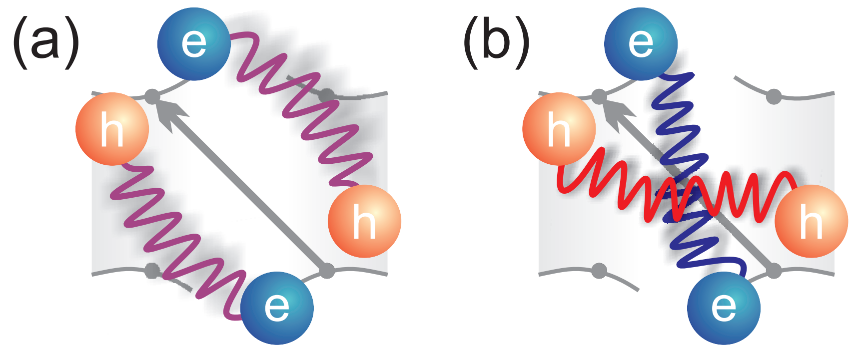



$\bar{b}(\bar{\varepsilon},\bar{T})$
0.6
0.4
0.2
$\bar{T}$
-1
0
1
$\bar{\varepsilon}$
0.02
0.04
0.06
0.08

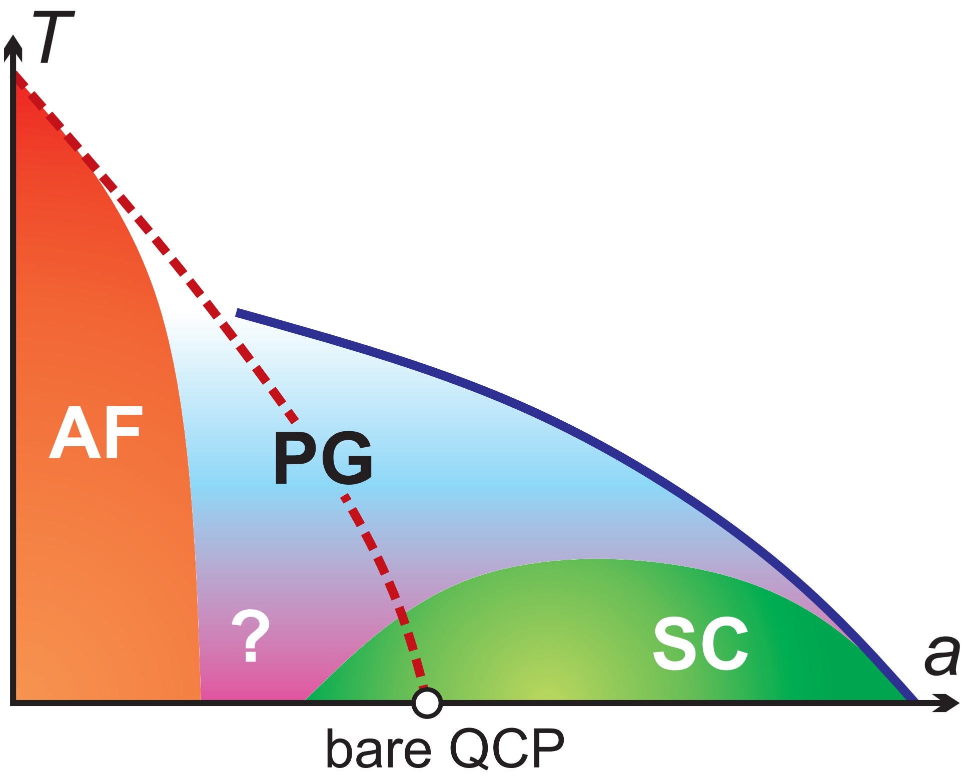

(a)
e
h
h
e
(b)
e
h
h
e



$\bar{b}\,(\bar{\varepsilon},\bar{T}\,)$
0.6
0.4
0.2
$\bar{T}$
-1
0
1
$\bar{\varepsilon}$
0.02
0.04
0.06
0.08



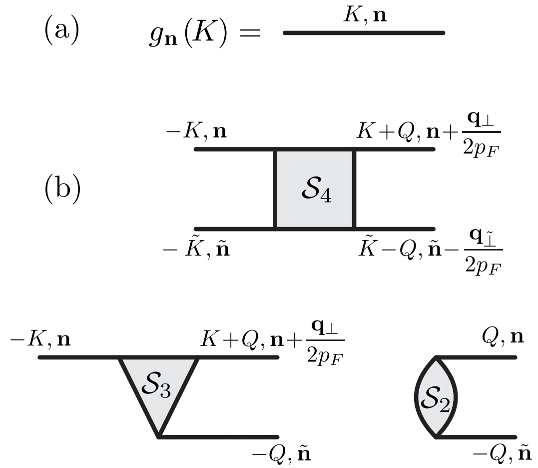
(a)
e
h
h
e
(b)
e
h
h
e



T
AF
PG
?
SC
a
bare QCP

Recent susy-bosonization in high dimensions : application to Fermi liquid theory

Hendrik Meier,CP, Efetov



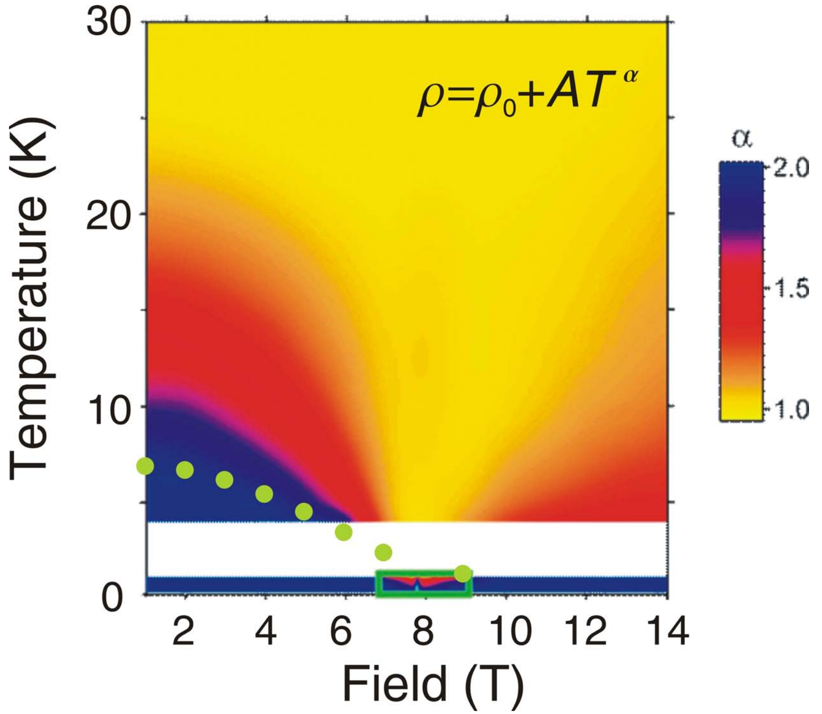
- Re-summation of the BS processes
- Curvature effects : charge and spin channels are coupled
- Re-summation of all non analyticities for the FL theory

$$\delta\Omega = \frac{\zeta(3)T^3}{\pi v_F^2}\left\{\frac{\ln^2(1+\gamma_\pi^{\mathrm{I}}L)}{L^2} + 3\,\frac{\ln^2(1+\gamma_\pi^{\mathrm{II}}L)}{L^2}\right\}$$

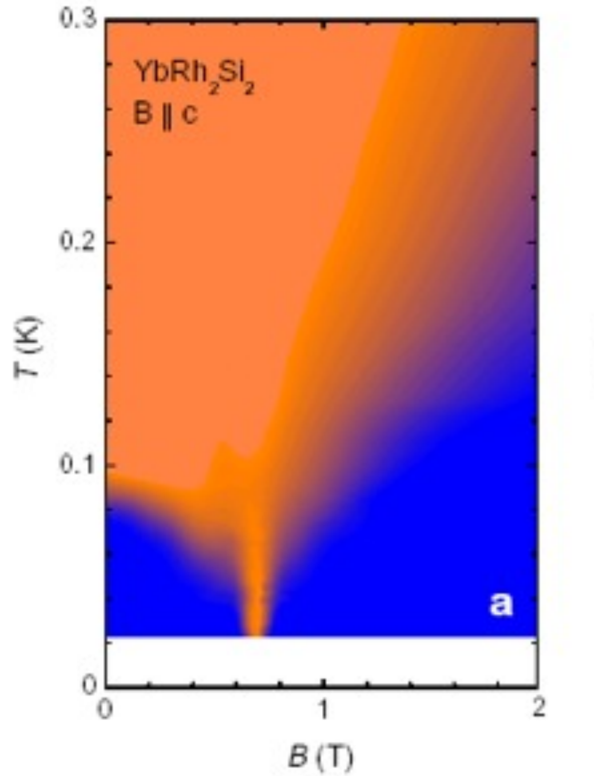
$$\gamma_I = \gamma_c - 3\gamma_s$$
$$\gamma_{II} = \gamma_c + \gamma_s$$

* K.B. Efetov, C. Pepin, H. Meier,
Exact bosonization for an interacting Fermi gas in arbitrary dimensions
Phys. Rev. Lett. 103,186403 (2009); PRB 82,235120 (2010), preprint 2011

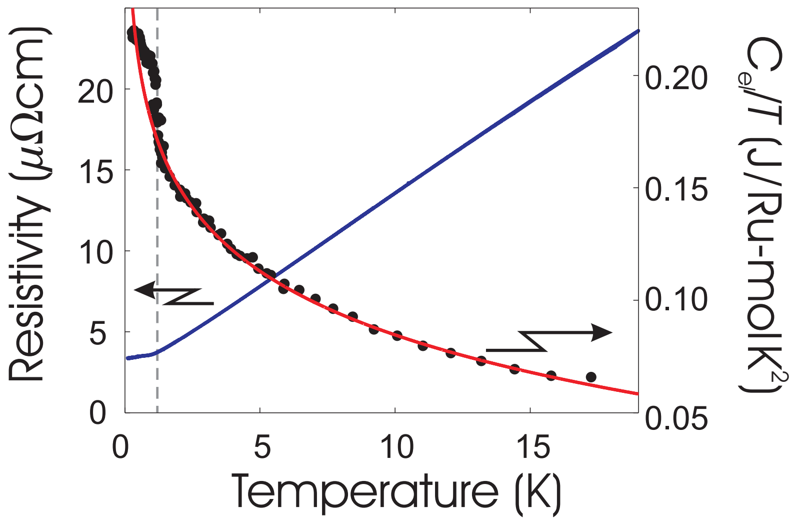
Sr3Ru2O7



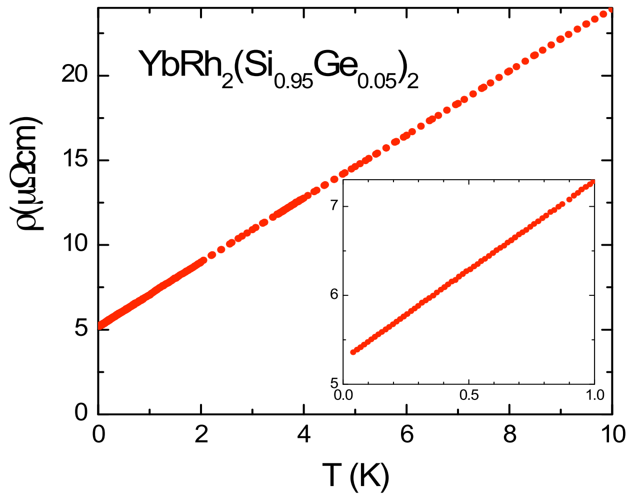
$\rho=\rho_0+AT^{\alpha}$
α
2.0
1.5
1.0
30
20
10
0
Temperature (K)
2
4
6
8
10
12
14
Field (T)



YbRh2Si2
B ∥ c
a
0.3
0.2
0.1
0
T (K)
0
1
2
B (T)



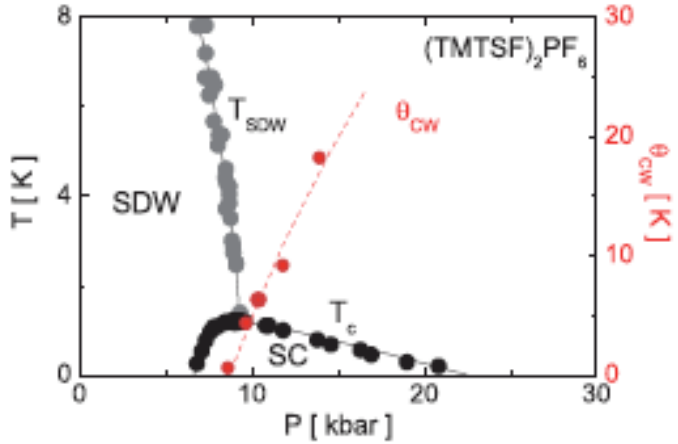
Resistivity (μΩcm)
20
15
10
5
0
0
5
10
15
Temperature (K)
0.20
0.15
0.10
0.05
C_{el}/T (J/Ru-molK^2)

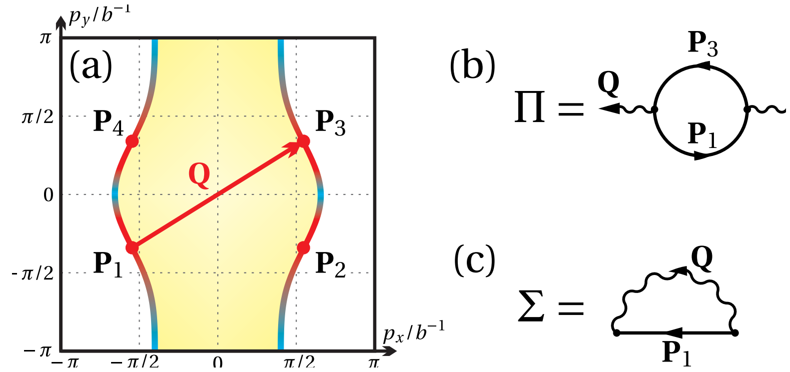


YbRh2(Si0.95Ge0.05)2
ρ(μΩcm)
20
15
10
5
0
0
2
4
6
8
10
T (K)
7
6
5
0.0
0.5
1.0

Organic superconductors

H. Meier, P. Auban-Senzier, D. Jerome, C.P.





$$\Pi_{\omega,\mathbf{q}} = \frac{g_3|q_\perp|}{4\pi^2 v} \ln\left\{\frac{(2b_4 q_\perp^4)^2}{\omega^2+\xi_{\mathbf{q}}^2}\right\}$$

$$\mathrm{Im}\ \Sigma^R(\varepsilon) \simeq \pi T\ \frac{\ln\left(p_F^{-2}\mu + \varepsilon^2/\varepsilon_F^2\right)}{\ln\left(\varepsilon^2/\varepsilon_F^2\right)}$$

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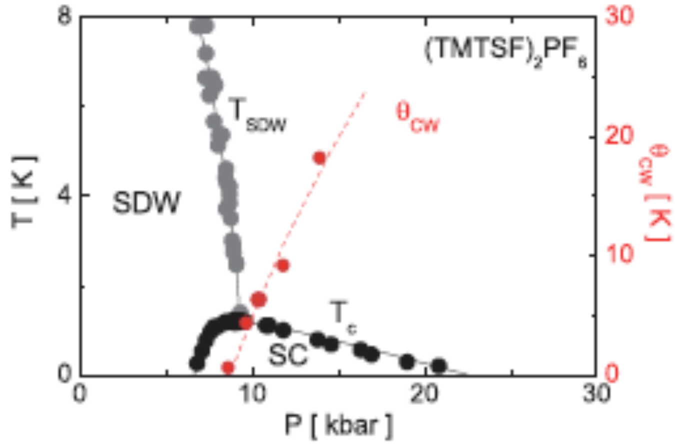

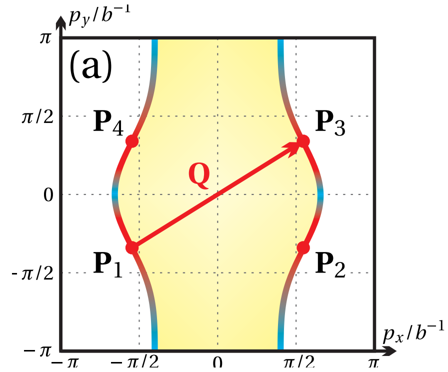

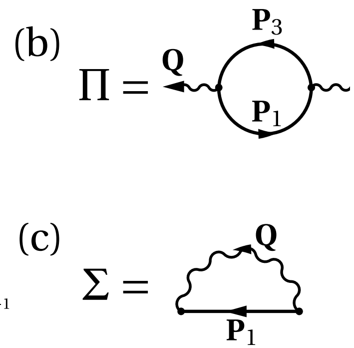

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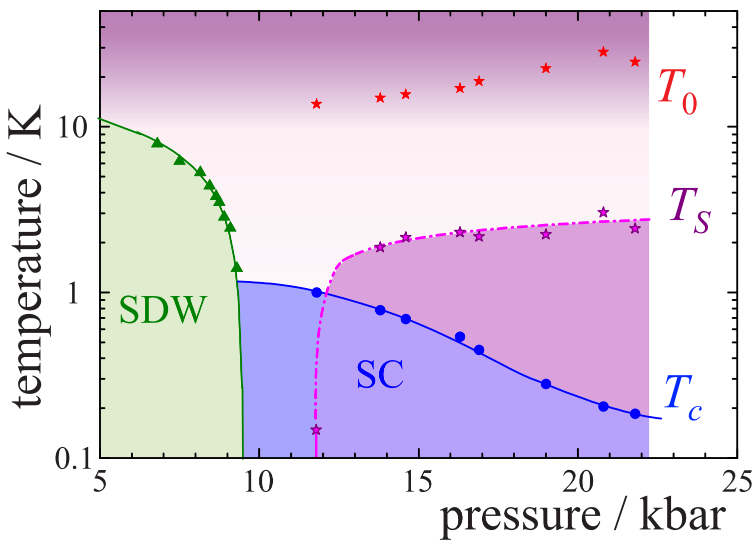

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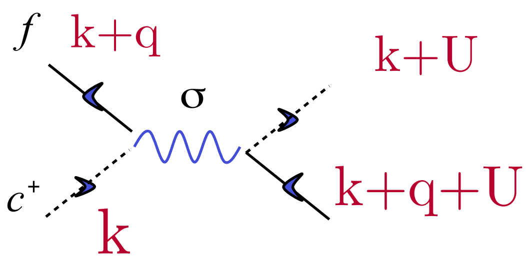
$$\sigma(T) = \frac{v_h}{\rho_0 + \rho_{\mathrm{hot}}(T)} + \frac{1 - v_h}{\rho_0 + \rho_{\mathrm{cold}}(T)}$$

Two types of fermions and anomalous hybridization fluctuations

I. Paul, M. Norman, C.P.



U : Umklapp

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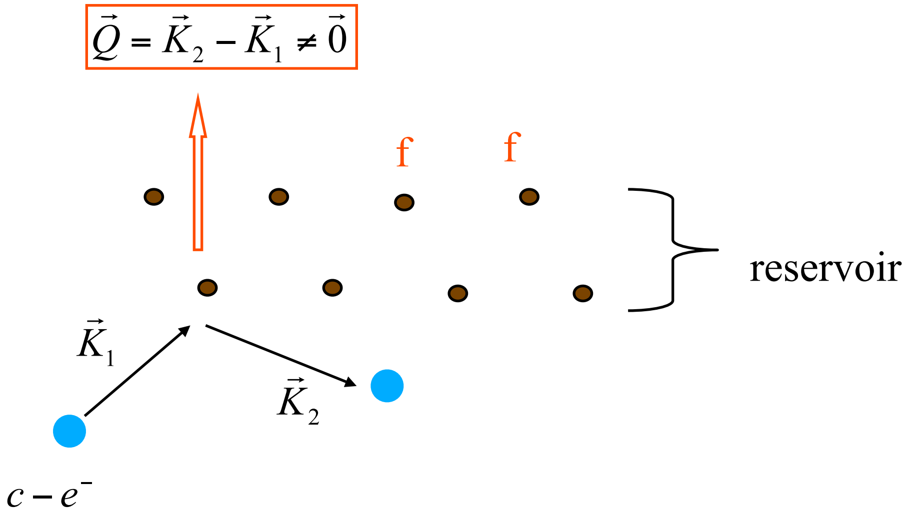

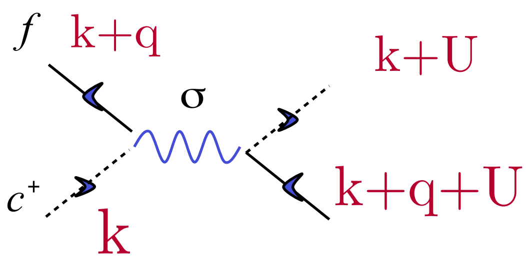

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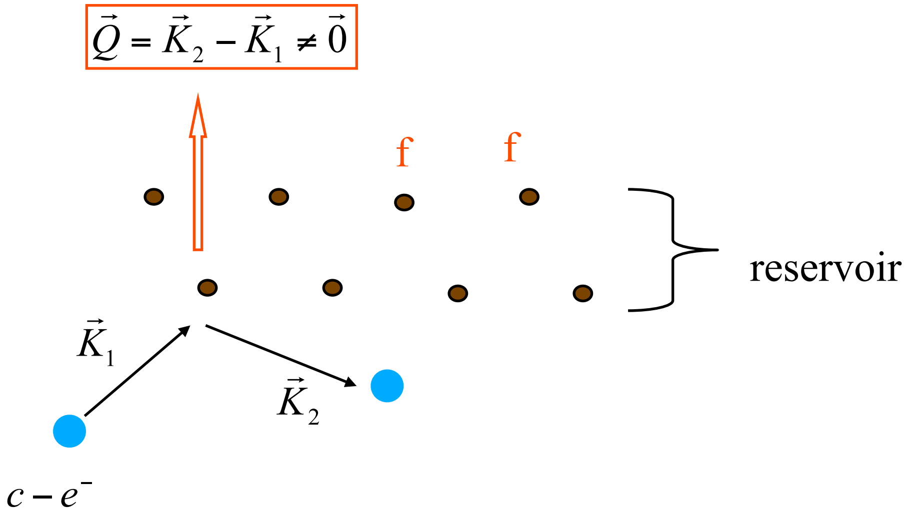
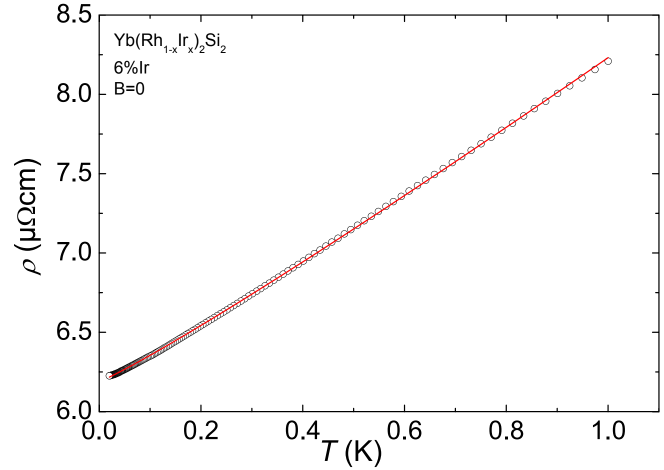
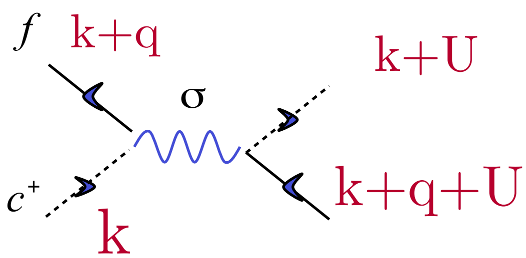


…Quasi-linear in Temperature above E*



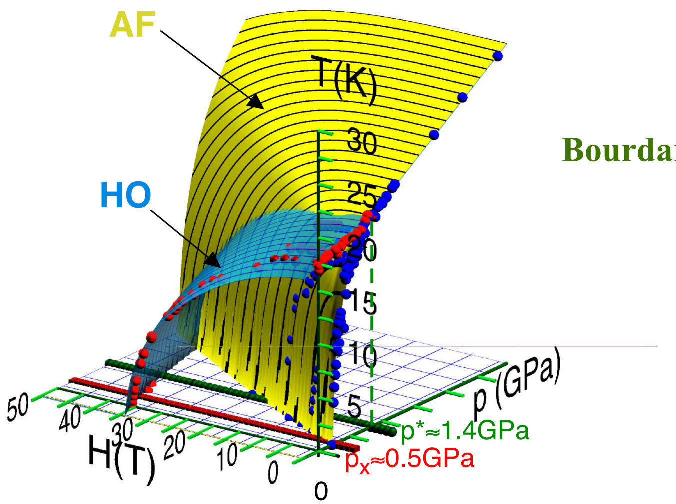


U : Umklapp

$$\rho(T) - \rho(0) \equiv \delta\rho(T) \propto \begin{cases} T \log(T/E^*) & E^* < T < \alpha E_F \\ T^2 & T < E^* \end{cases}$$

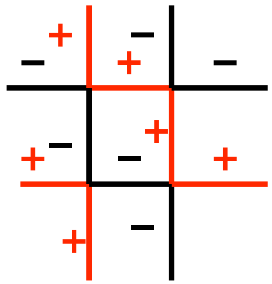
$$\frac{1}{\tau_{cf}} \propto \mathrm{Im}\Sigma_c$$

The Modulated Spin Liquid = Hidden Order in URu2Si2

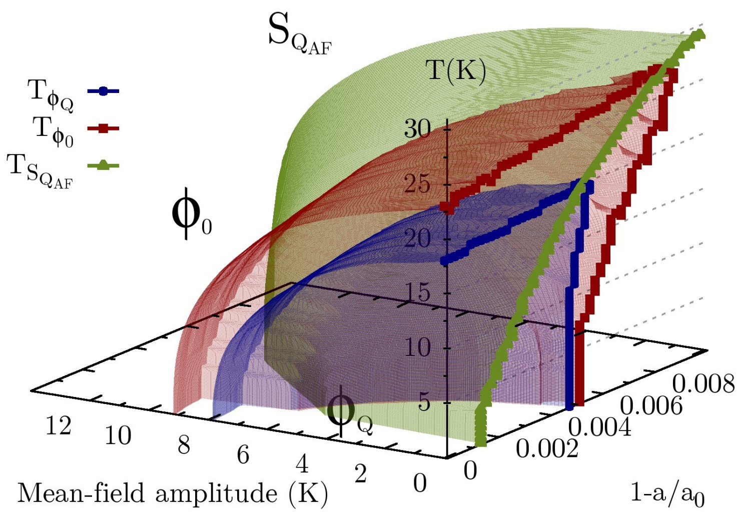


Bourdarot et al, PRB (2011)

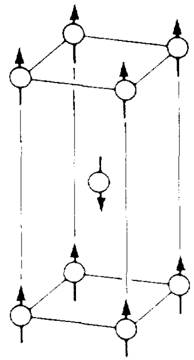
MSL valence bound



Thomas, Burdin, Ferraz and CP, Preprint (2012)



AFM Order



Conclusions

- Strong experimental evidence for anomalous quantum criticality in HF compounds

- Breakdown of the conventional techniques which integrate out the fermions for (almost all?) models below d=3.

- Many mysteries remain : linear in T resistivity, pseudo-gap regimes.

- Three insights

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  - quasi 1d QCP in organics has linear in T regime
  - multiple species in heavy fermions : key to anomalous resistivity