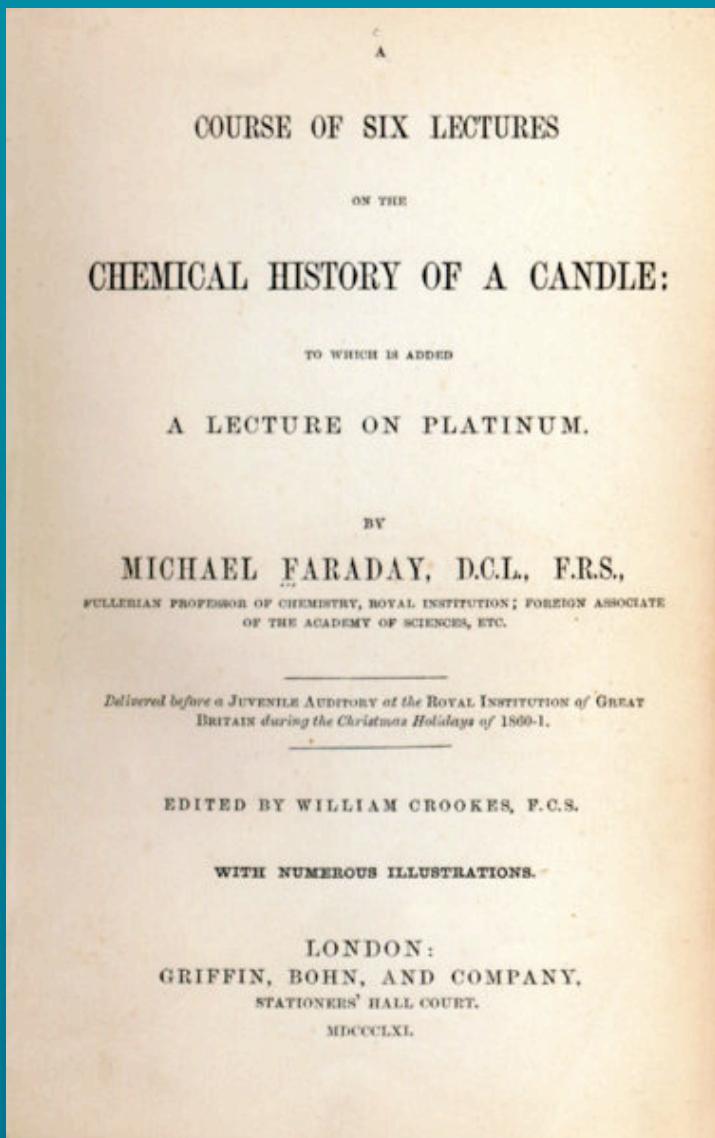


Heavy Fermion Physics – Perspectives from look-alikes

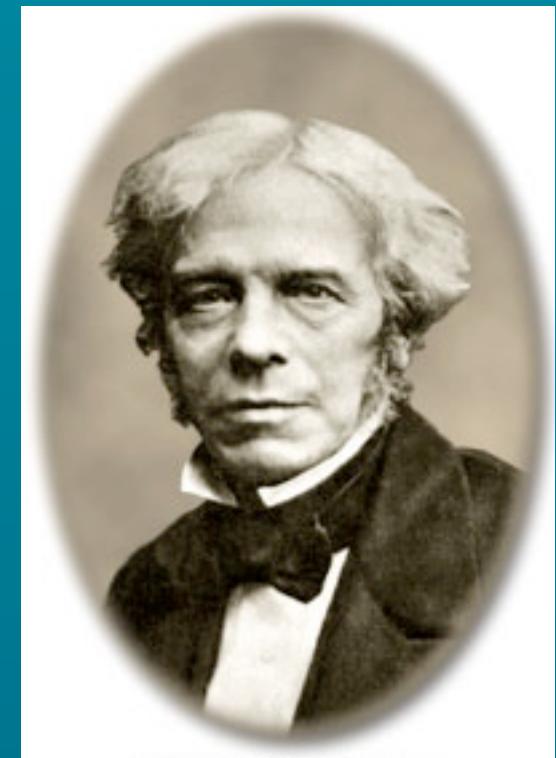
Andy Schofield
University of Birmingham



The chemical history of a candle



“There is no better, there is no more open door by which you can enter into the study of natural philosophy than by considering the physical phenomena of a candle.”
Michael Faraday 1860



Heavy fermions – a universe of correlated electron physics

“There is no better, there is no more open door by which you can enter into the study of **correlated electrons** than by considering the physical phenomena of the **heavy fermions**.”

- Fermi liquids
- Novel Superconductors
- Quantum critical metals – conventional and unconventional
- “Hidden order” phases
- Significant challenges
- Insights from look-alikes – can we build simple analogues?
- Future Trends:
 - Analogue heavy fermion systems
 - Semiconductors; cold atoms
 - New experimental techniques

The basic Hamiltonian

- The Anderson lattice model:

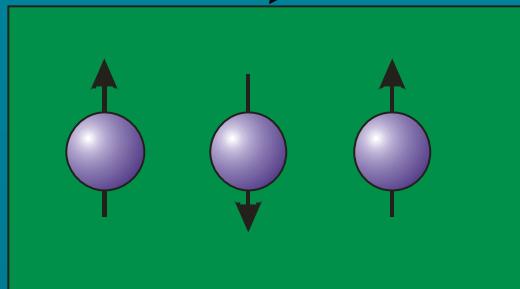
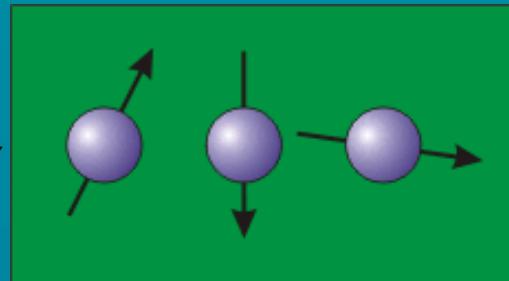
$$H = \sum_{ij\sigma} \epsilon_{ij} c_{i\sigma}^\dagger c_{j\sigma} - \sum_{ij\sigma} [V_{ij} c_{i\sigma}^\dagger f_{j\sigma} + V_{ij}^* f_{i\sigma}^\dagger c_{j\sigma}] - \Delta \sum_{i\sigma} f_{i\sigma}^\dagger f_{i\sigma} + U \sum_i f_{i\sigma}^\dagger f_{i\sigma} f_{i\bar{\sigma}}^\dagger f_{i\bar{\sigma}}$$

- Essential ingredients
 - Non-interacting conduction electrons (c)
 - Strongly interacting f electrons – usually considered in the limit of large U and Δ (so valence fluctuations small) with no direct f-f hybridization
 - Weak hybridization between f and c.
- Reduces to:
 - Local moments with spin degrees of freedom interacting via magnetic exchange with the conduction electrons

How is the spin entropy of the f moments lost as T->0?

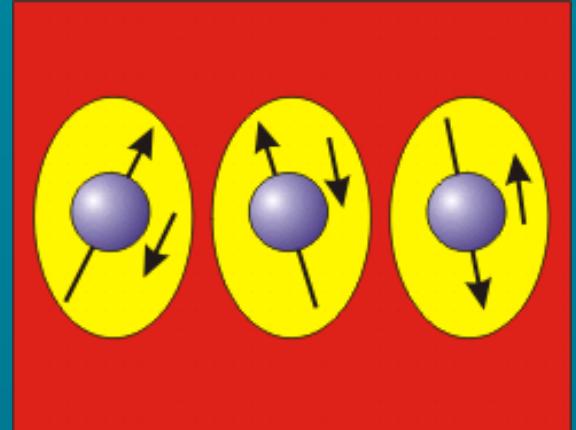
P. Coleman, C. Pépin, Q. Si & R. Ramazashvili; J. Phys. C. **13** R723 (2001)

High temperature
free magnetic moments
+ conduction electrons
 $N_e = N_c$



Low temperatures
Free spins order, and
 $N_c = N_c$ (small Fermi volume)

OR

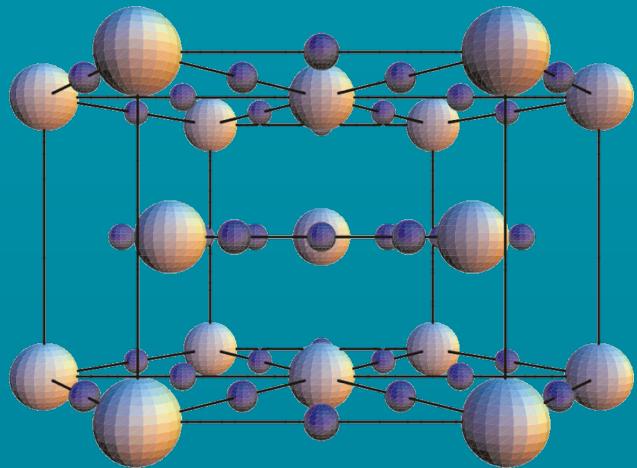
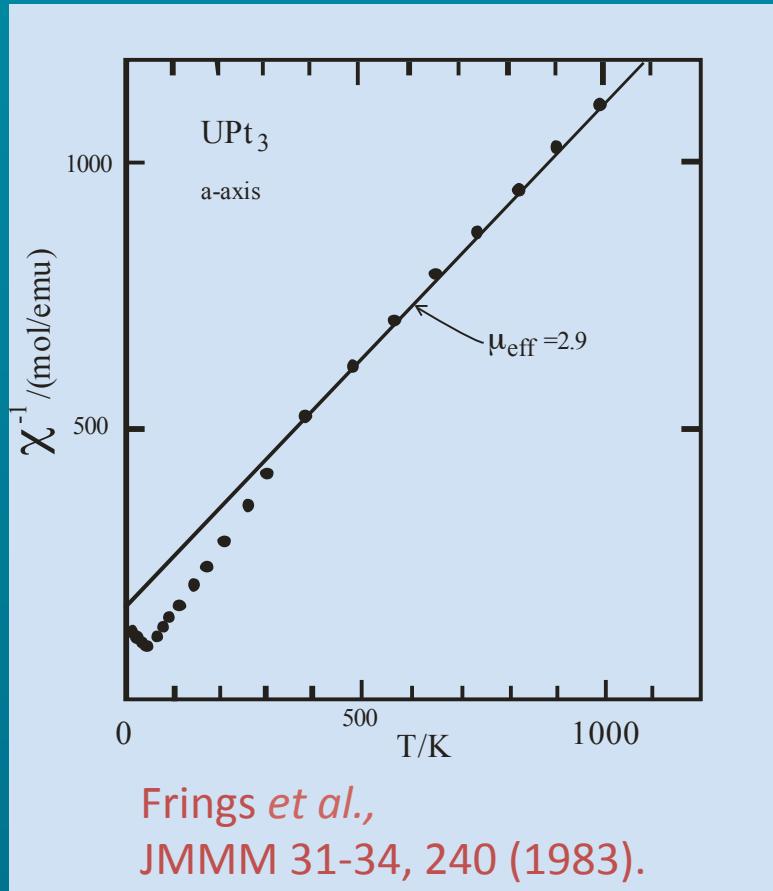


Low temperatures
No free spins but very heavy
electrons ($m \sim 10^3 m_e$) and
 $N_e = N_c + N_f$ (large Fermi volume)

The remarkable Fermi liquid: UPt₃

A heavy Fermi liquid: $U \sim 5f^3$

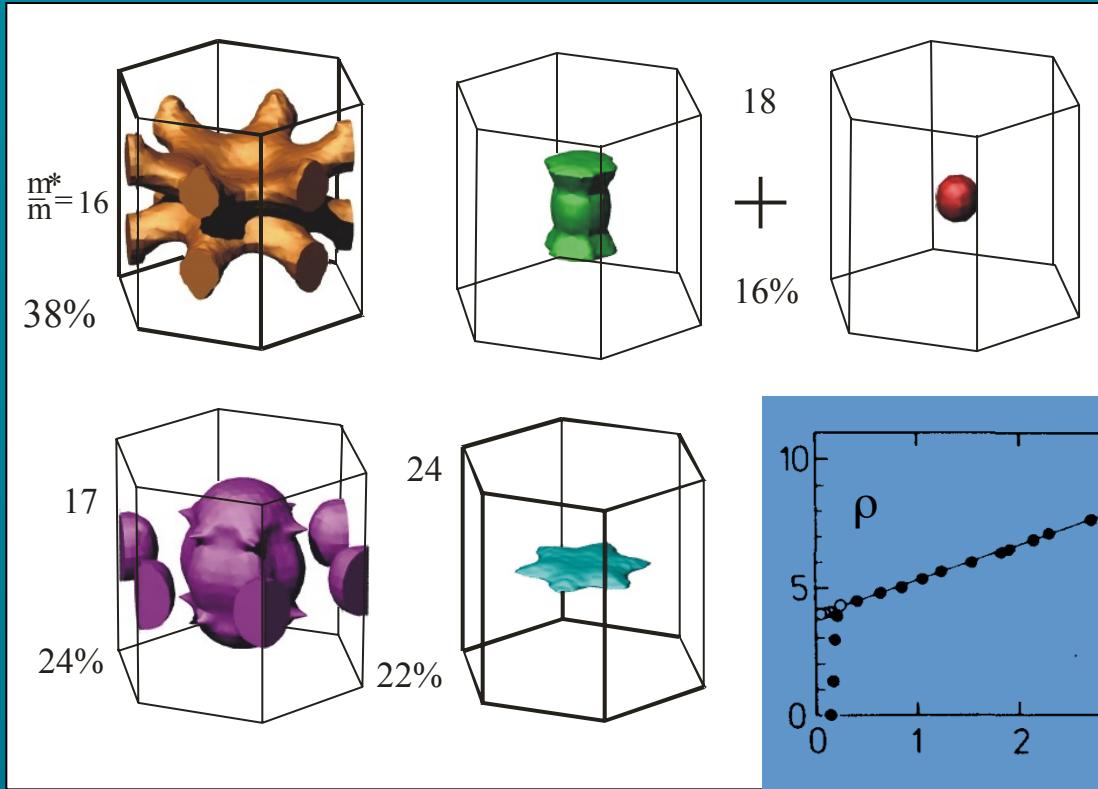
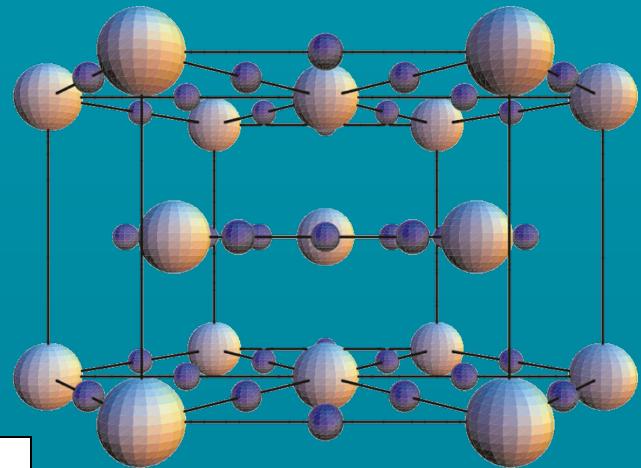
High temp: f electrons bound to form
a local moment via Hund's rule ($\sim 1\text{eV}$):



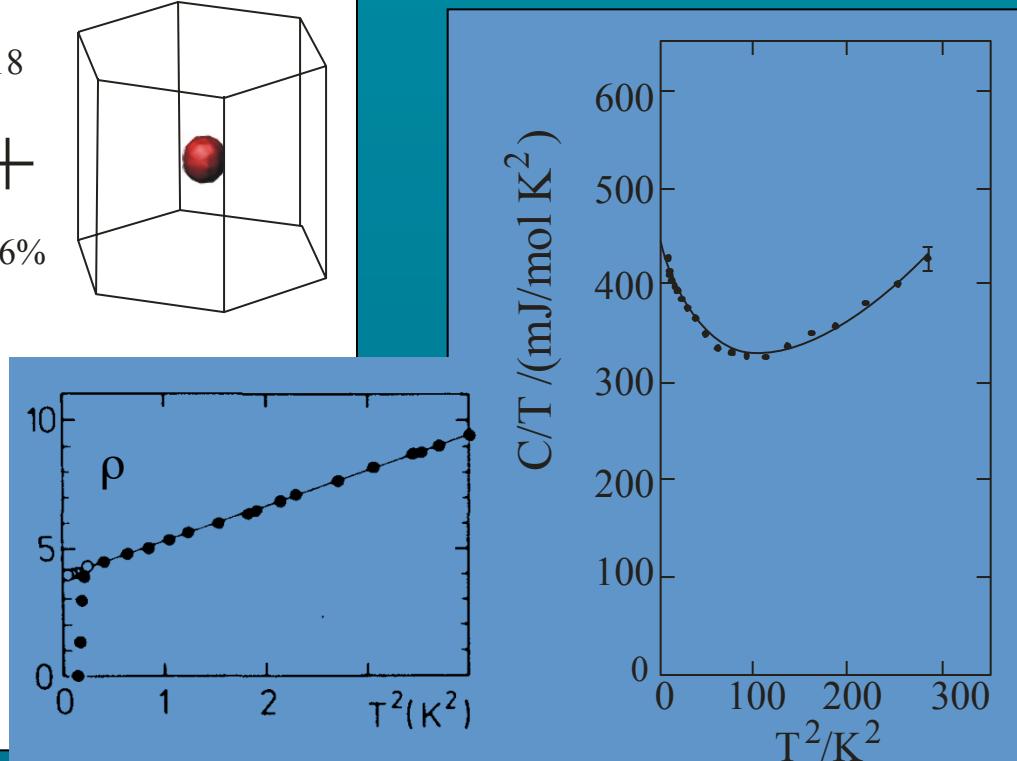
The remarkable Fermi liquid: UPt₃

A heavy Fermi liquid: $U \sim 5f^3$

S.R. Julian and G. McMullan,
unpublished (1998)



Low temp: f electrons delocalize to make up a
heavy Fermi liquid



G.R. Stewart *et al.*,
PRL, 52, 679 (1984)

A Fermi liquid susceptible to superconductivity

Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu_2Si_2

F. Steglich

Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz

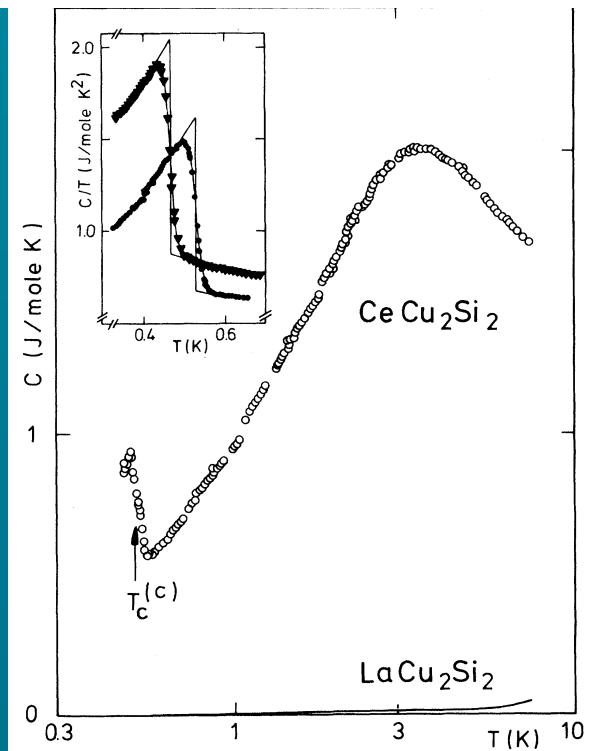
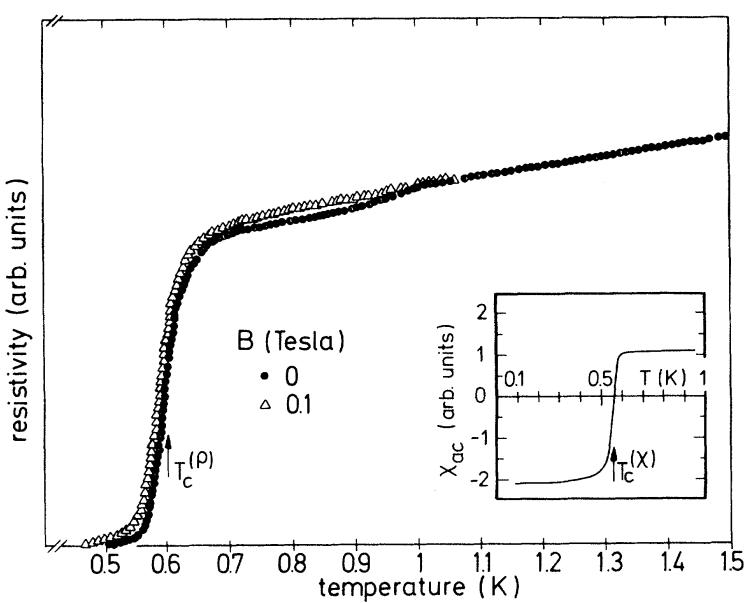
II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

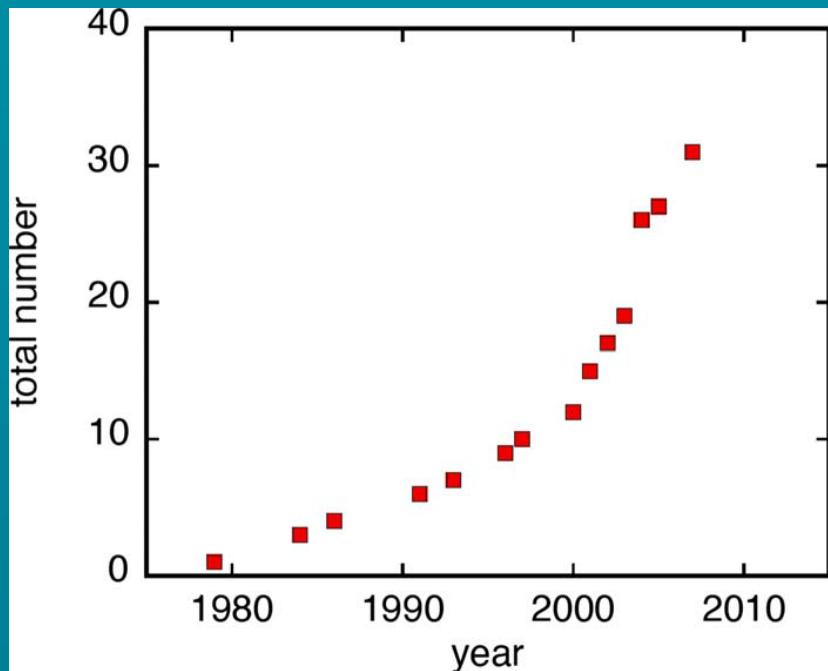
H. Schäfer

Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

(Received 10 August 1979; revised manuscript received 7 November 1979)



A universe of new superconductors



Christian Pfleiderer

Rev. Mod. Phys. 81, #4, 1551-1624 (2009).

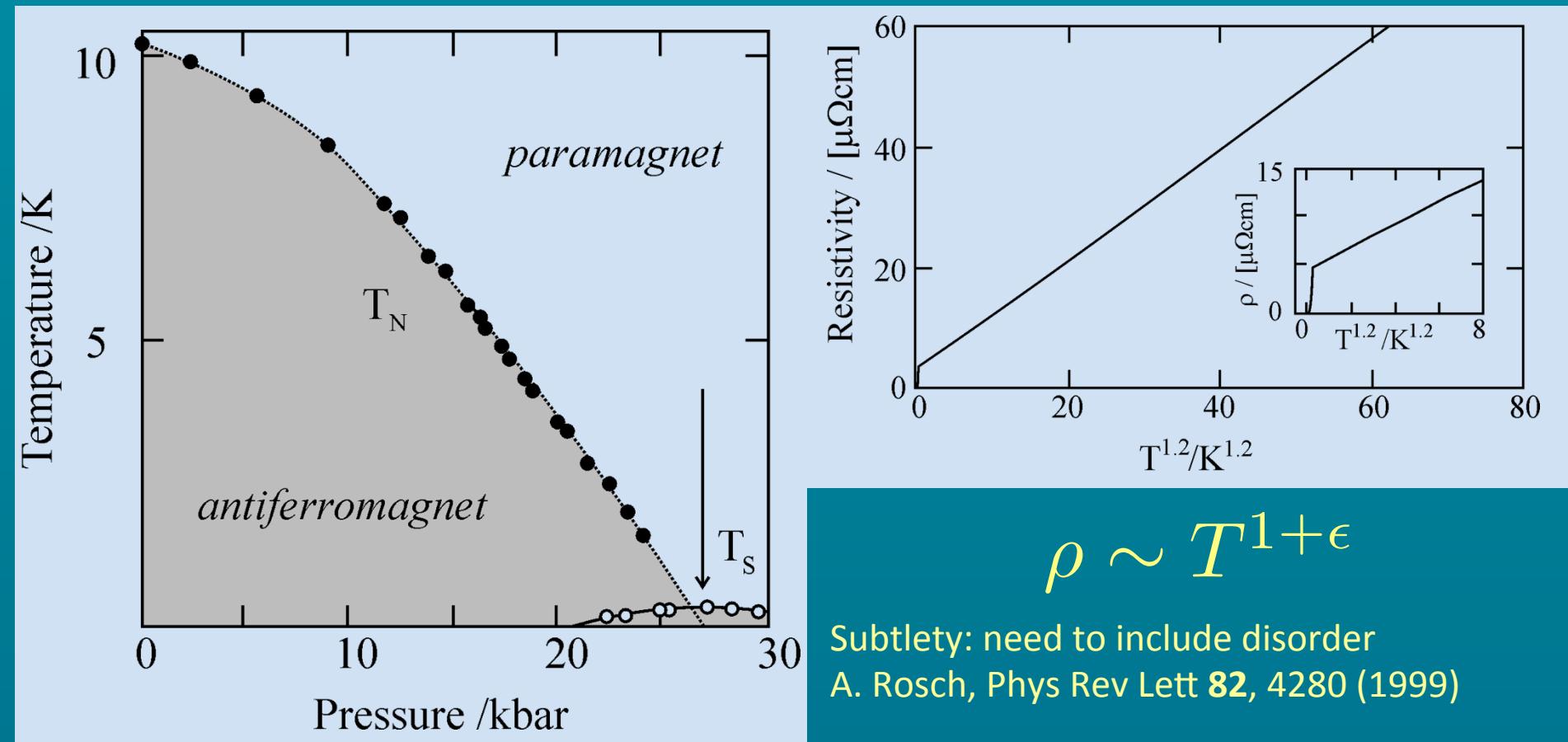
Superconductors that require new rules:

- On the border of magnetism
 - AFM: CePd_2Si_2
 - FM: UCoGe
- Coexisting with magnetism
 - AFM: UPt_3
 - FM: UGe_2
- Non-centrosymmetric
 - CePt_3Si
- “High temperature” superconductors
 - PuCoGa_5

Exemplars of quantum criticality

H. v. Löhneysen, T. Pietrus, G. Portisch, H. G. Schlager, A. Schröder, M. Sieck and T. Trappmann Phys. Rev. Lett. **72**, #20, 3262-3265 (1994).

e.g. pressure tuned CePd₂Si₂

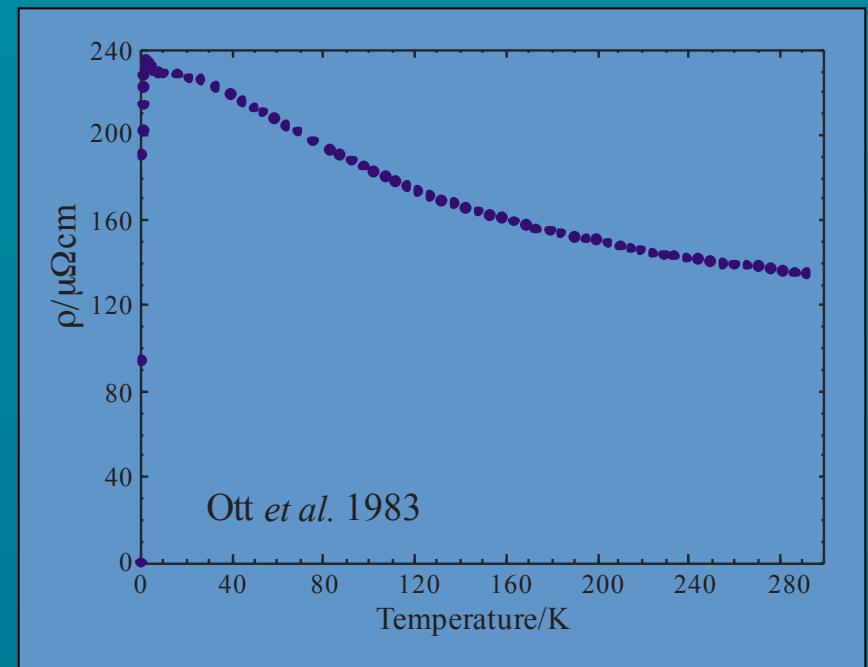
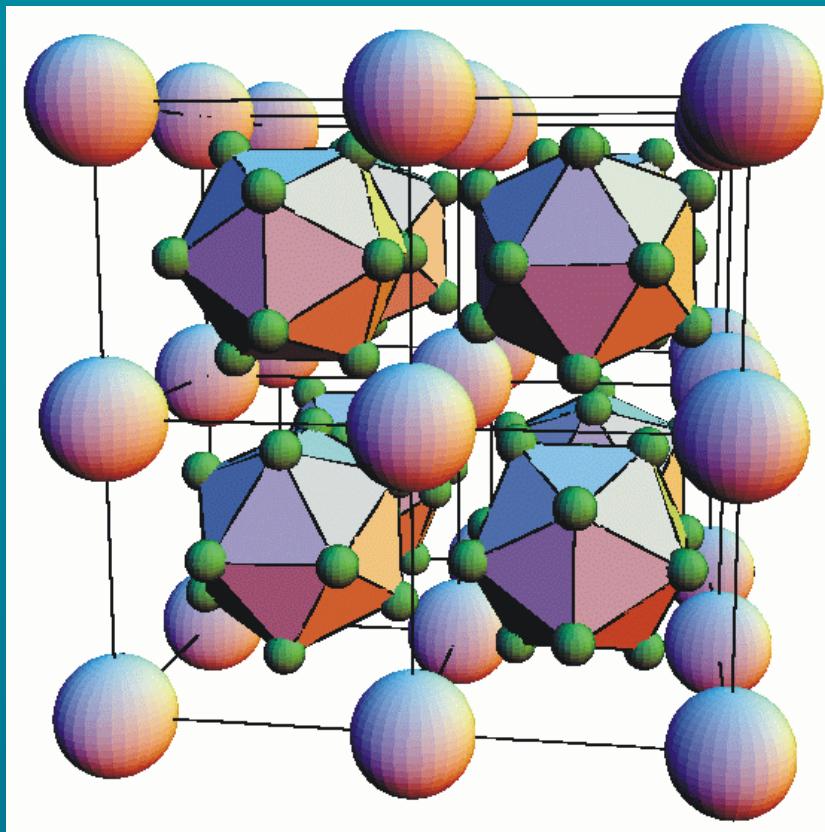


$$\rho \sim T^{1+\epsilon}$$

Subtlety: need to include disorder
A. Rosch, Phys Rev Lett **82**, 4280 (1999)

Are heavy fermion materials simply metals with small E_F ?

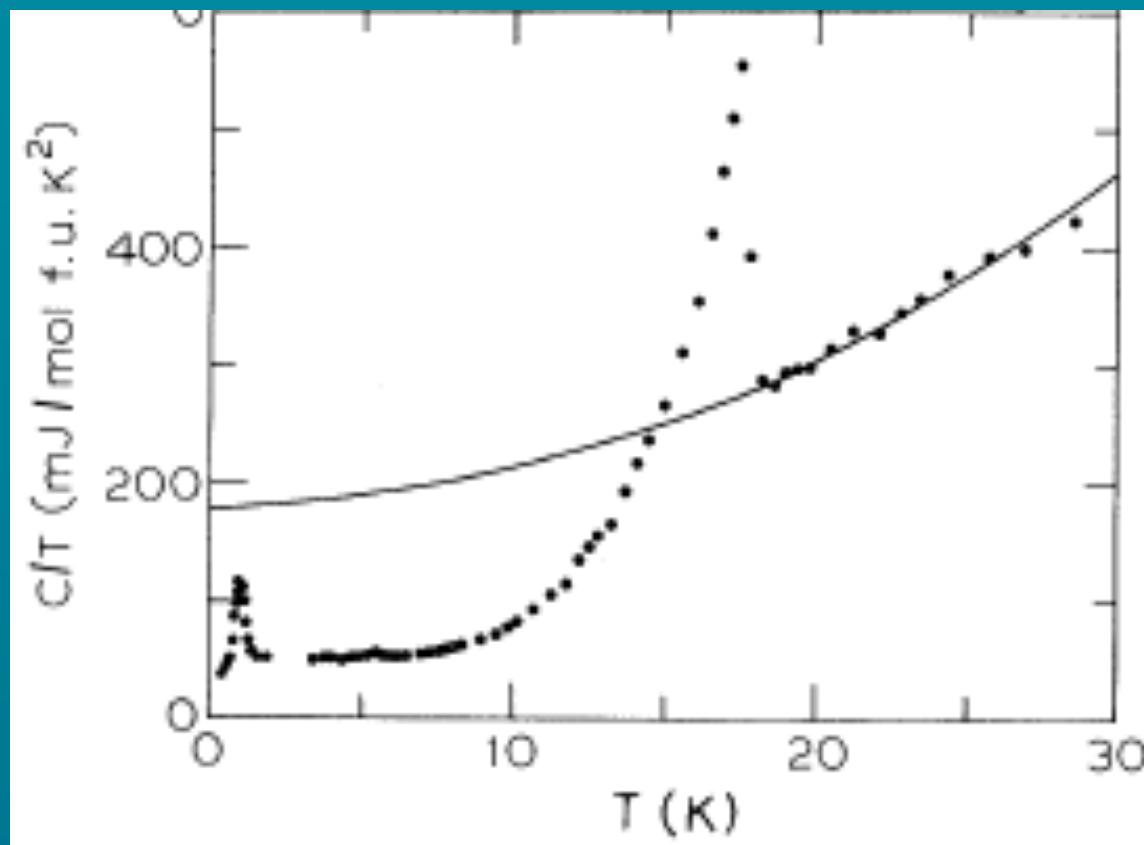
- Many outstanding puzzles – like UBe₁₃



Superconductivity without a Fermi surface?

Are heavy fermion materials simply metals with small E_F ?

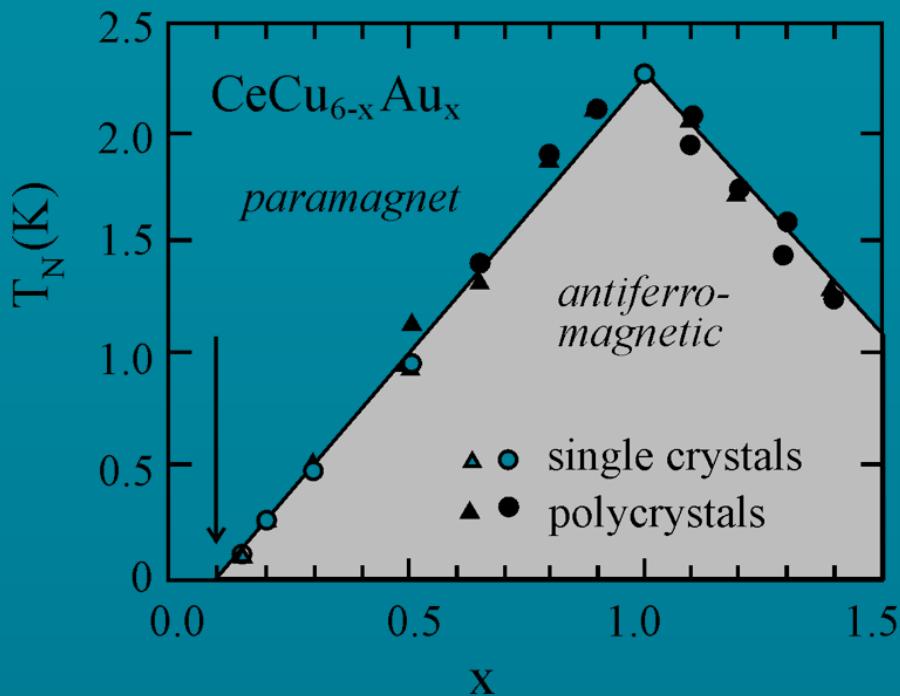
- Many outstanding puzzles – like the mysterious order in URu_2Si_2



URu_2Si_2 : T. T. M. Palstra, A. A. Menovsky, J. van den Berg, A. J. Dirkmaat, P. H. Kes, G. J. Nieuwenhuys and J. A. Mydosh Physical Review Letters **55**, 2727 (1985)

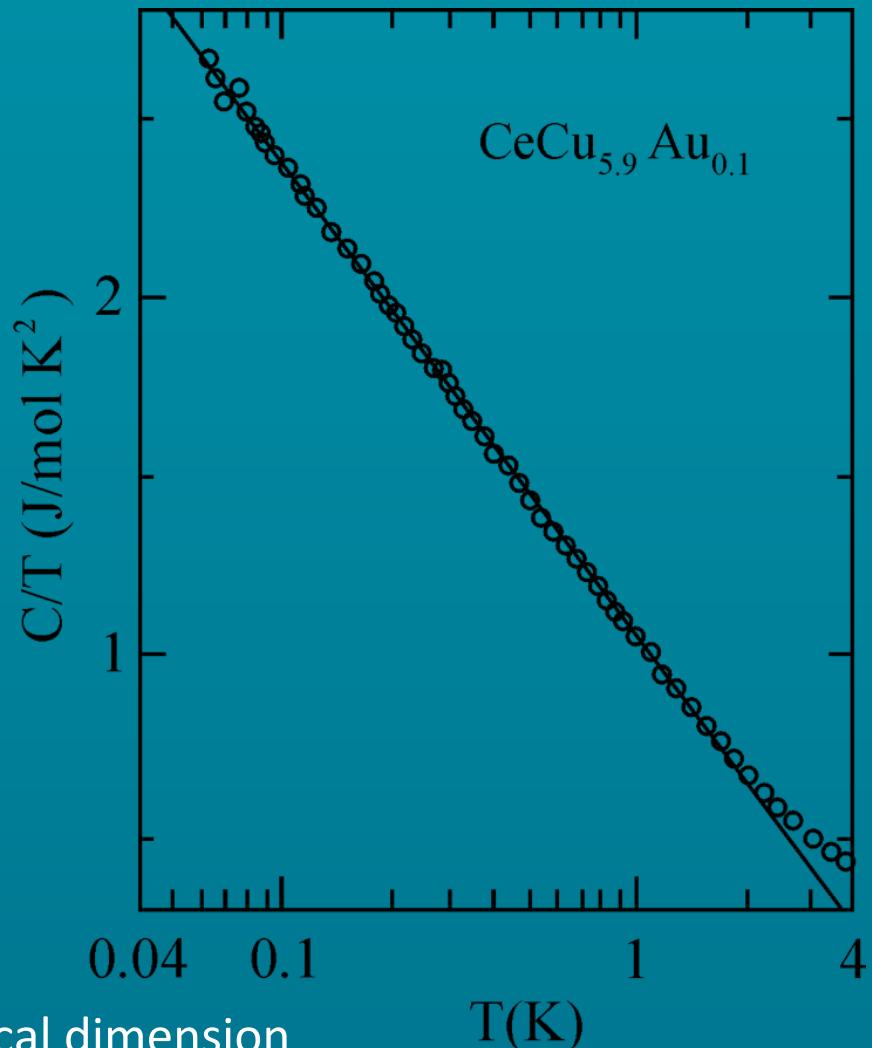
LGWH: Failure in heavy fermion AFM

Is this above or below the upper critical dimension?

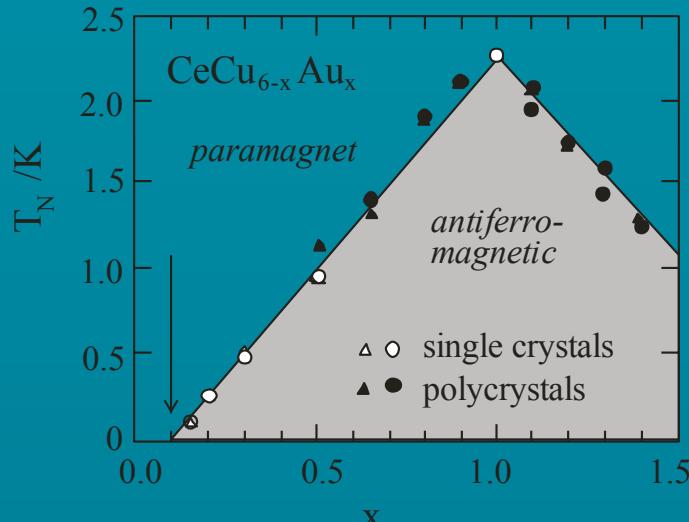


Control parameter is chemical doping:

- Fluctuations at $\mathbf{Q}=\text{AFM vector}$
- $D=3$ (perhaps 2), $z=2$ so above upper-critical dimension



... yet E/T scaling seen

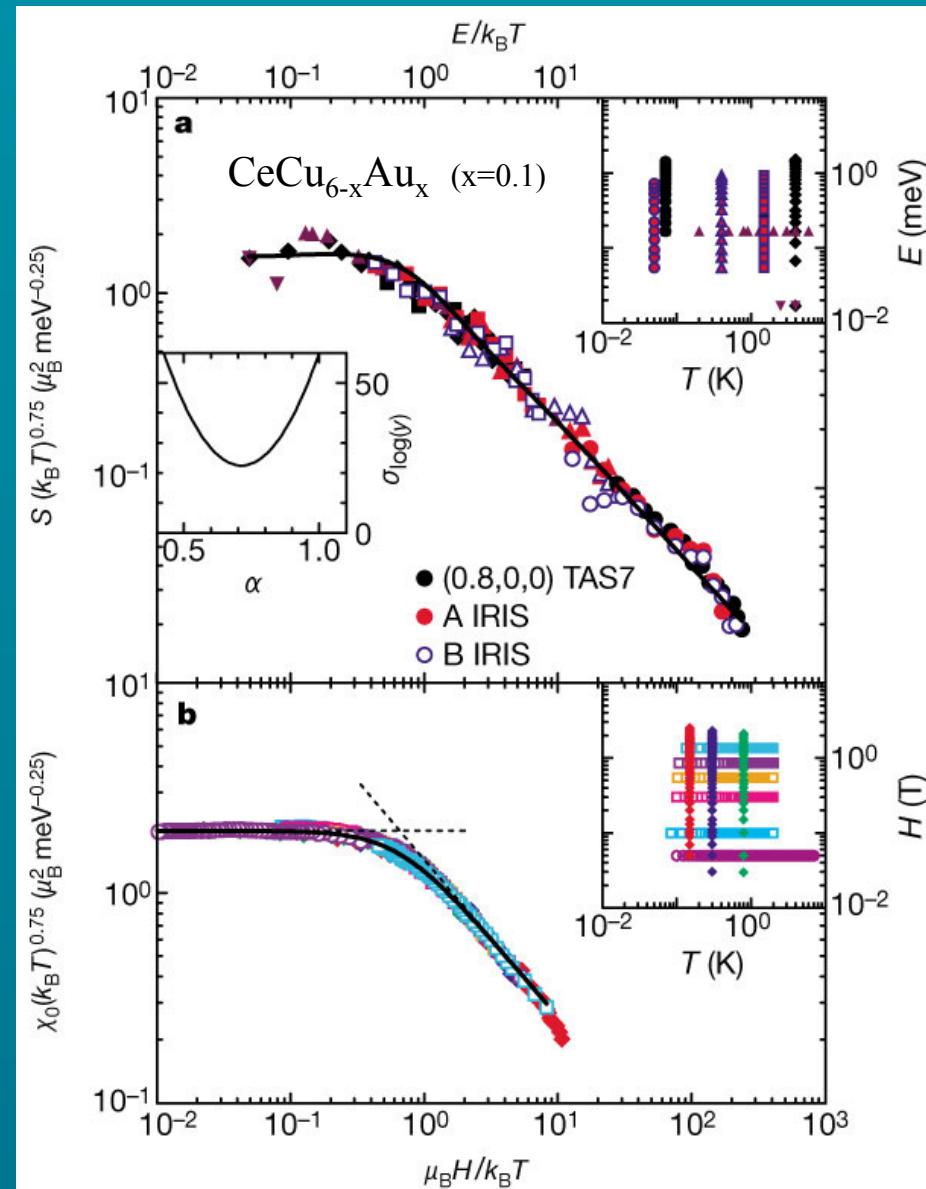


von Löhneysen et al (1994).

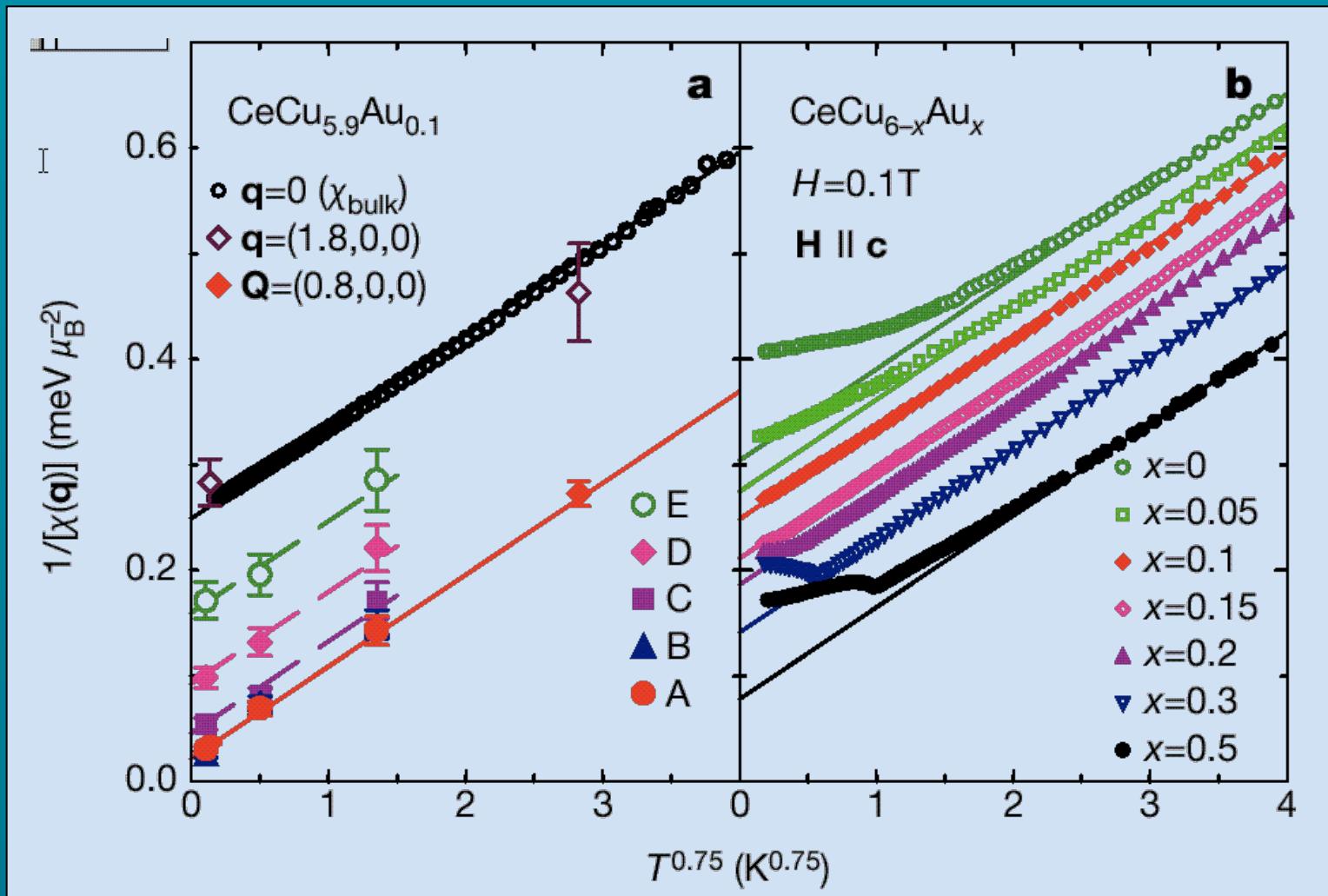
$$\chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right)$$

Physics below the upper critical dimension.

Schröder et al, Nature 407,351 (2000).



... and seems show criticality everywhere in k-space



$$\chi^{-1}(\mathbf{q}, E, T) = \left[\left(\left[T^2 + \left(\frac{g\mu_B}{k_B} \right)^2 H^2 \right]^{1/2} - iE/a \right)^\alpha + \theta(\mathbf{q})^\alpha \right] C^{-1}$$

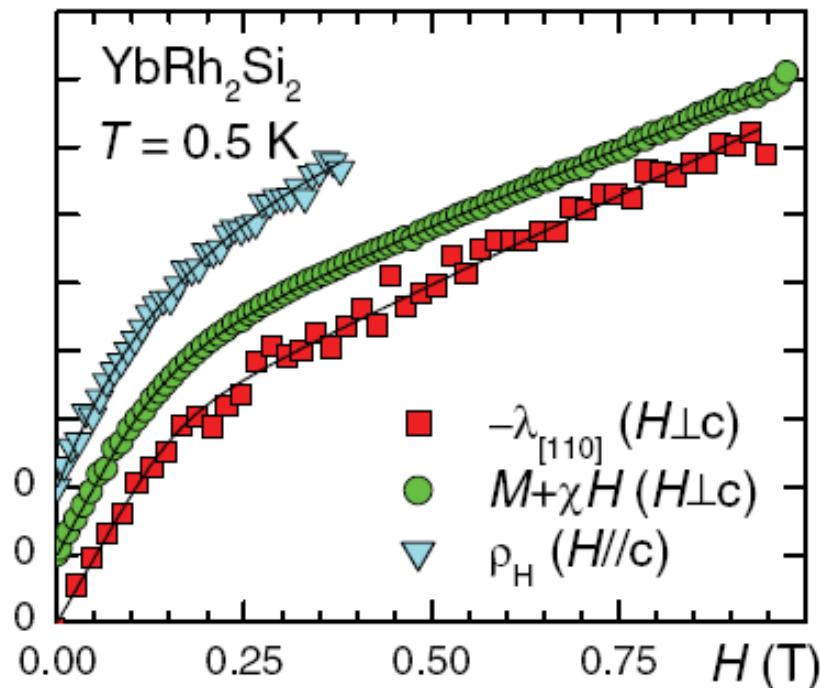
A. Schröder *et al.* Nature, 407, 351 (2000)

Evidence for new physics at a heavy fermion quantum critical point

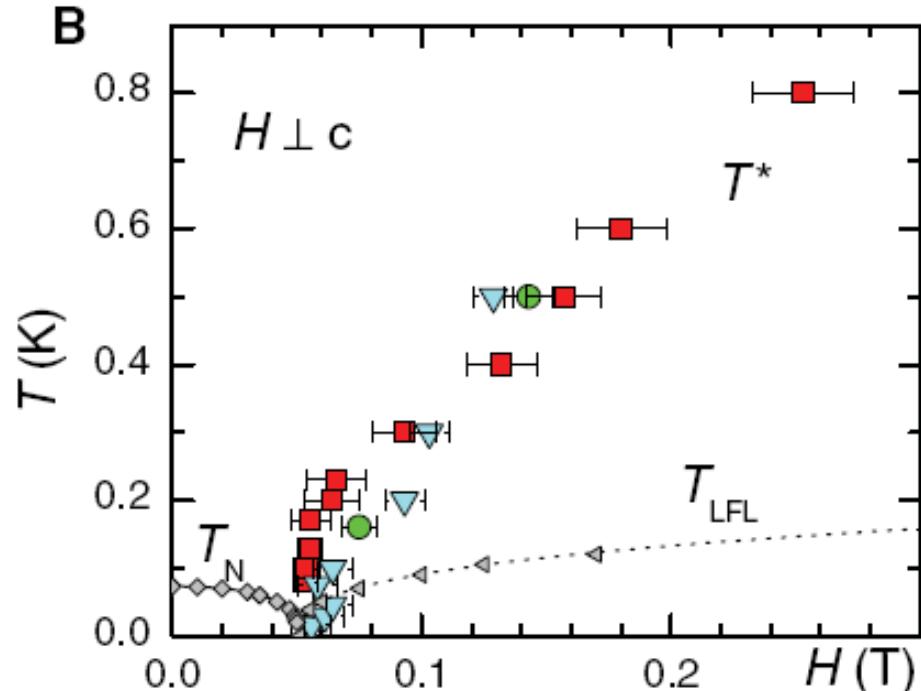
P. Gegenwart, T. Westerkamp, C. Krellner, Y. Tokiwa, S. Paschen, C. Geibel, F. Steglich, E. Abrahams, *Q. Sci. Science* **315**, #5814, 969-971 (2007).

A

arb. units



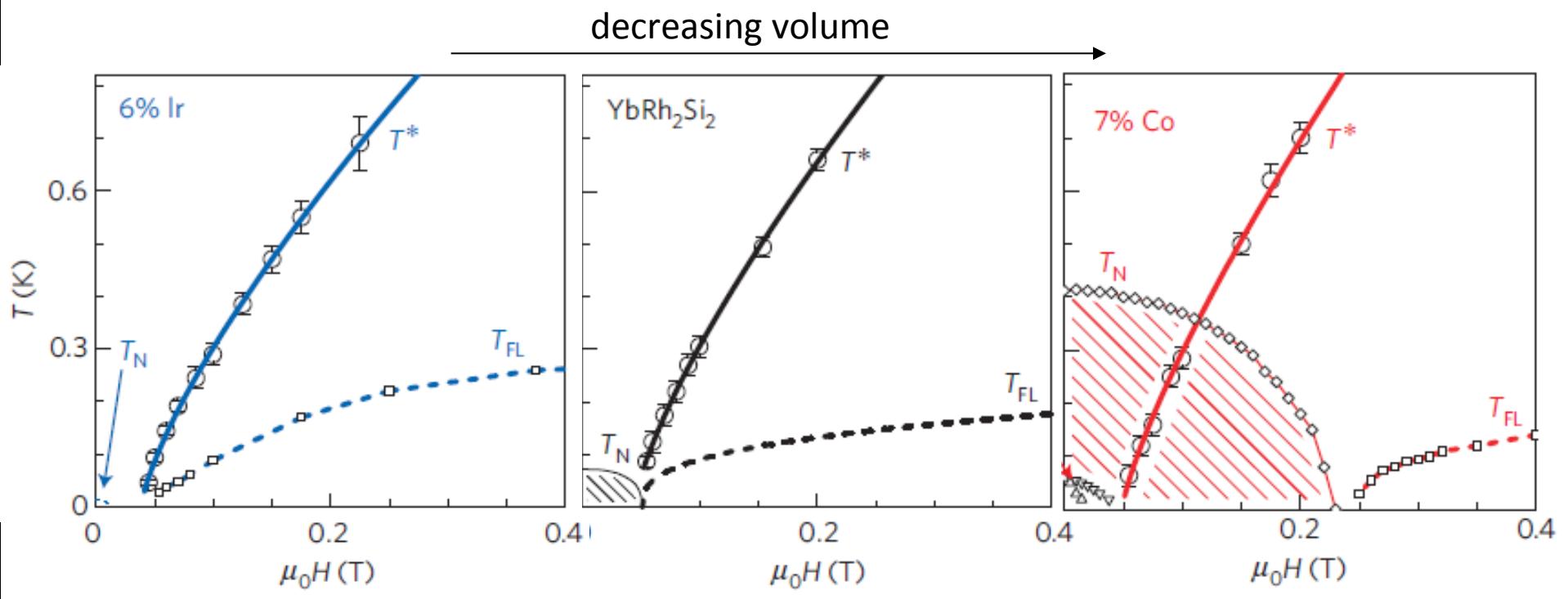
B



An additional energy scale appears to converge on the quantum phase transition:
Failure of the Landau assumption of a single order parameter controlling criticality

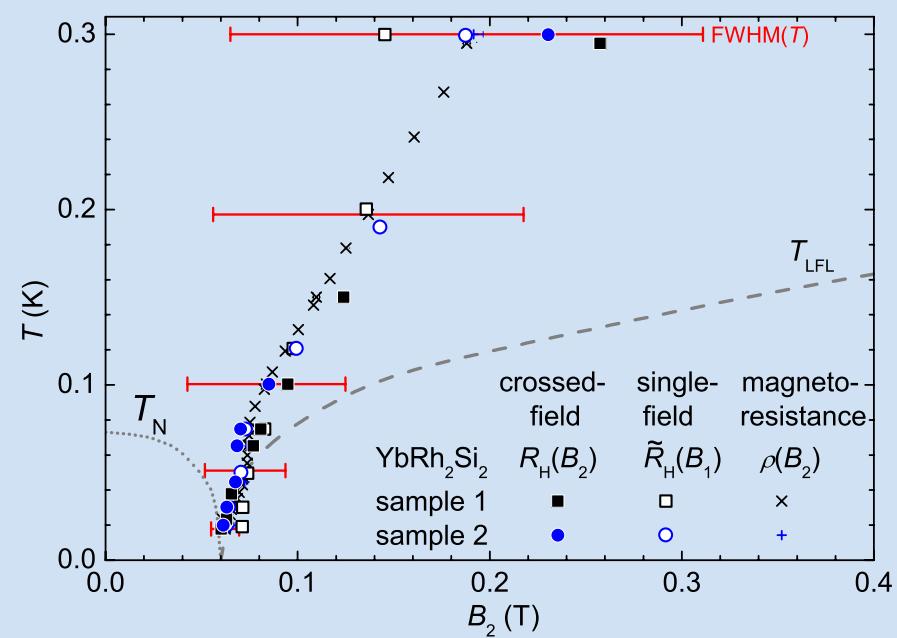
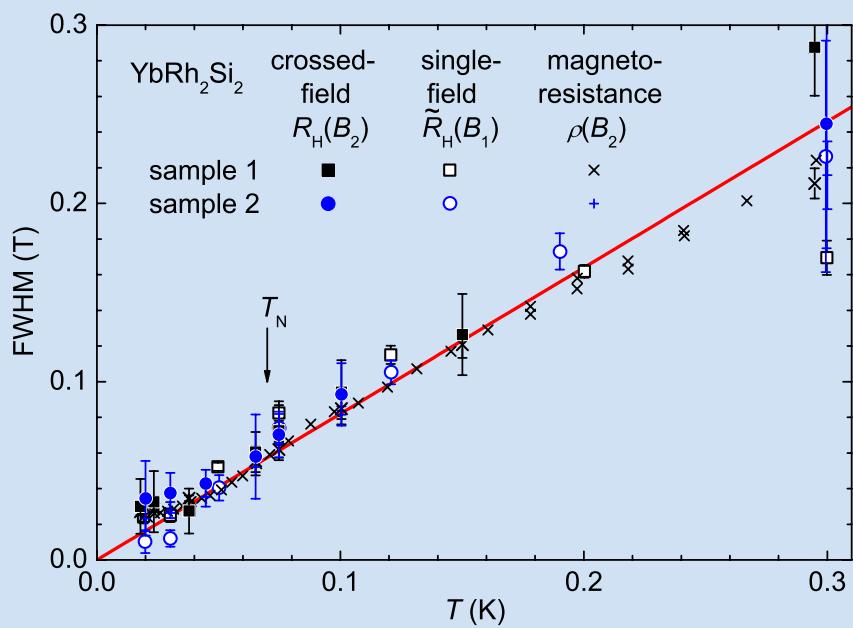
Phase diagram appears even richer

It looks like you can control the two scales independently

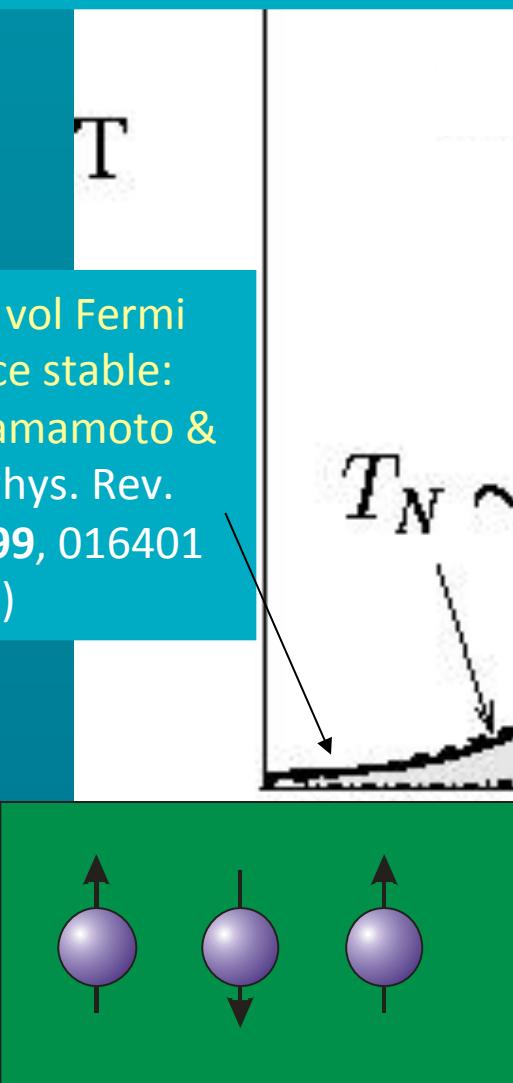


*S. Friedemann, T. Westerkamp, M. Brando, N. Oeschler, S. Wirth, P. Gegenwart,
C. Krellner, C. Geibel and F. Steglich
Nature Physics 5, #7, 465-469 (2009).*

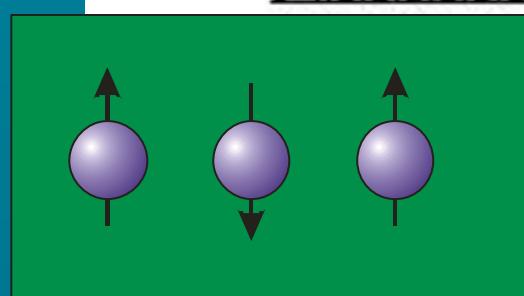
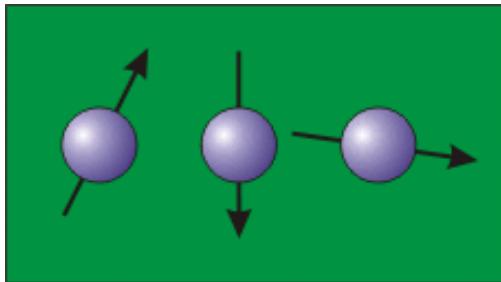
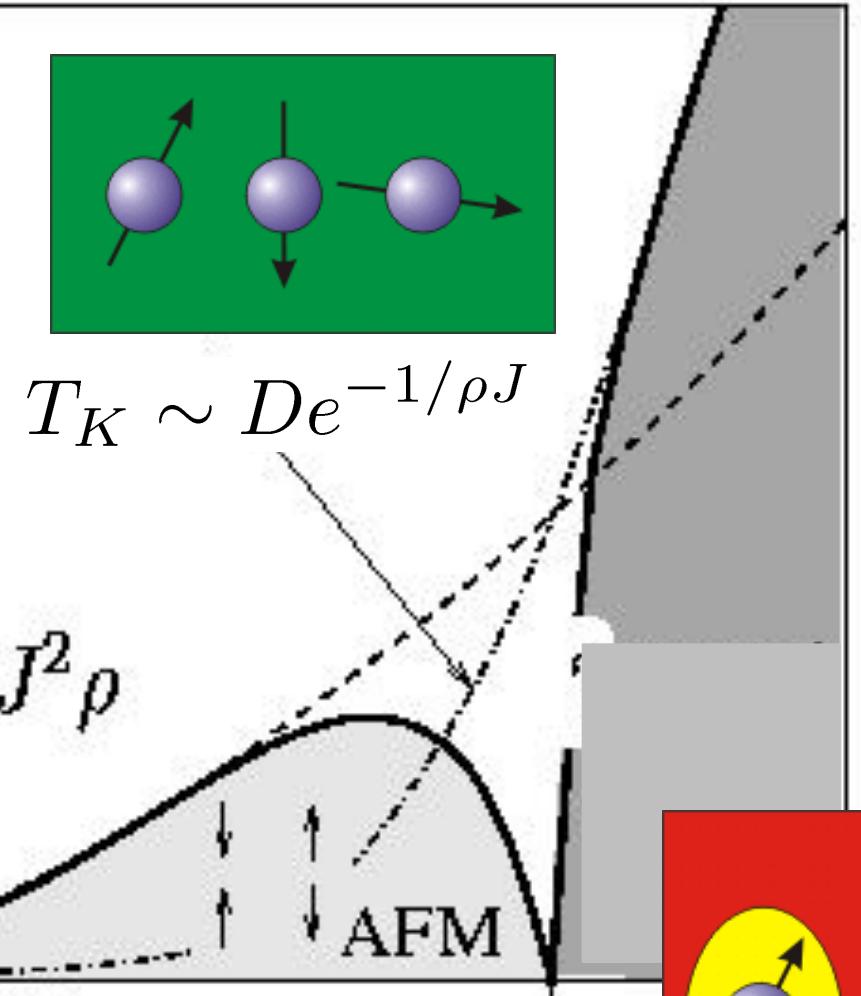
Moreover – evidence of E/T scaling



Sven Friedemann et al. PNAS, 107, 14547 (2010)

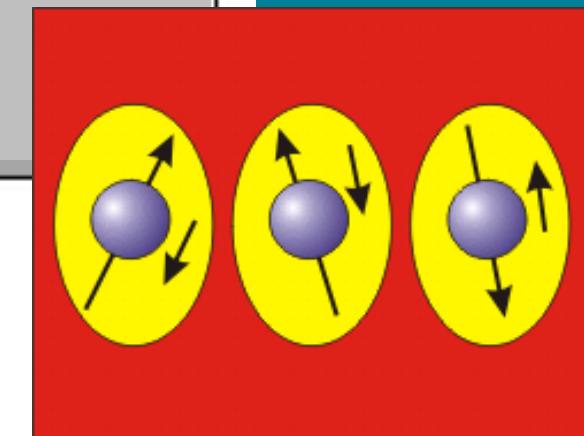


Small vol Fermi
surface stable:
S.J. Yamamoto &
Q.Si Phys. Rev.
Lett. **99**, 016401
(2007)



$T_K < T_{RKKY}$

$T_K > T_{RKKY}$



S. Doniach, 1978.

Using analogues to understand the physics

- Are the anomalies seen in quantum critical heavy fermion metals a consequence of Kondo physics OR a generic failure of our understanding of quantum criticality?
- Is there a “look – alike” for quantum criticality without the Kondo physics?
 - d -metals near a magnetic instability
 - AFM?
 - FM? (issues of non-analyticity)
- Is there a “look-alike” for the Kondo lattice which illustrates the same physics?
 - ${}^3\text{He}$ bilayers

Quantum criticality in metallic systems

- Paramagnon theory (1960s onward):
 - metals on the border of ferromagnetism.
 - Pd, Ni₃Al, Ni₃Ga, YNi₃, ZrZn₂
 - Puzzles: neither Stoner nor Heisenberg-like
 - Stoner: $B=a M + b M^3$ with $a \sim a_0 - \lambda(T/TF)^2$
 - T_c small, χ Curie-Weiss like, yet μ_{eff}/μ_0 large
 - Resolution: include spin-fluctuations
 - Berk, Schrieffer, Doniach, Engelsberg, Rice, Moriya, Yamada, Beal-Monod, Misawa, Lonzarich, Continentino ... and more
 - $B=(a+b\langle m \rangle^2)M + b M^3$
 - Modern incarnation: Hertz-Millis (RG) approach

$$S = \int \sum_{\omega} d^D q \left[\left((x - x_c) + q^2 + \frac{i\omega}{q^{z-2}} \right) |\psi|^2 + u|\psi|^4 + \dots \right]$$

LGWH: Landau-Ginzburg-Wilson-Hertz (...)

Itinerant antiferromagnetic quantum criticality ($Q \neq 0$)

Hilbert v. Löhneysen, Achim Rosch, Matthias Vojta, Peter Wölfle

Rev. Mod. Phys. **79**, 1015-1075 (2007).

Interplay between spin-fluctuations and the electron fluid

- LGWH: Upper critical dimension should formally be 2
- At $d=2$:
 - Long range interactions induced by fluid,
LGWH expansion breaks down (infinite number of
marginal operators)
 - Abanov and Chubukov, Phys Rev Lett 93, 255702 (2004)
- At $d=3$:
 - Breakdown of Fermi liquid at hot spots – need to treat
fully self-consistently
- At $Q=2k_F$:
 - $d=2$ First order:
 - Ioffe, Millis Phys. Rev. B 51, 16151 (1995).

Ferromagnetic quantum critical points ($Q=0$)

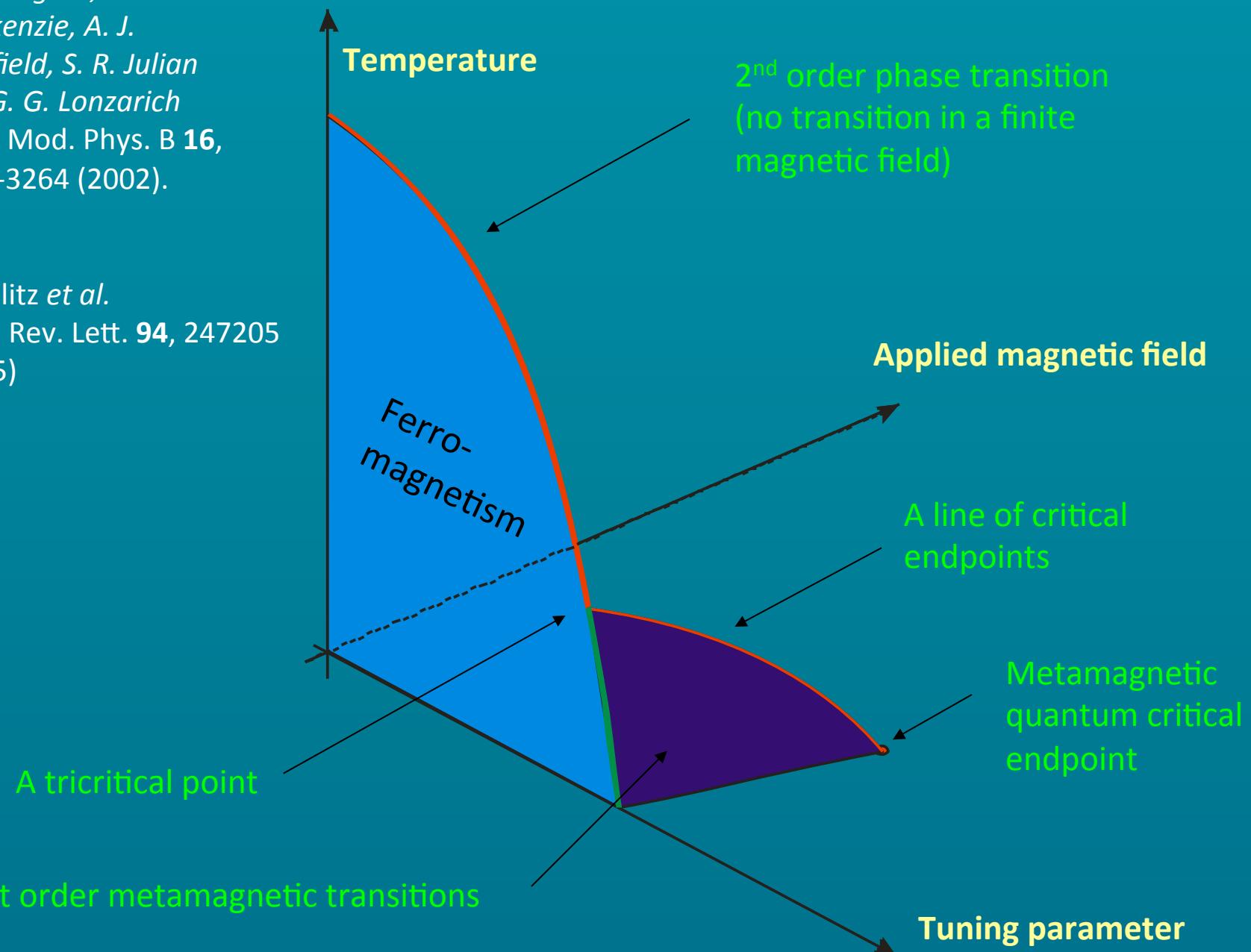
- Coupling to soft modes: particle-hole excitations in the spin channel:
 - $F \rightarrow F_{\text{conventional}} + w m^4 \ln (m^2/m_0^2 + T^2/T_0^2)$
 - D. Belitz, T.R. Kirkpatrick, Jorg Röllbühler, PRL **94**, 247205 (2005) and references therein.
Reminiscent of other fluctuation induced first order transitions.
 - Non-analytic action in q : *e.g.* $d=3 \rightarrow q^2 \ln q$
 $d=2 \rightarrow -|q|^{3/2} m_q^{-2}$ at QCP. $|q|$ near QCP.
 - A. V. Chubukov, C. Pépin, J. Rech, PRL **92**, 147003 (2004).
 - D. Belitz, T. R. Kirkpatrick, and T. Vojta, Phys. Rev. B **55**, 9452 (1997).
 - Over counting? No – observe corrections in Fermi liquid theory.
 - G.Y. Chitov and A. J. Millis, PRB **64**, 054414 (2001).
- Special pleading: Negative curvature of the density of states

Conclusion: generically driven first order, or order parameter must “curl up”.
But you can find a true LGWH quantum critical point at finite magnetic field.

Generic phase diagram

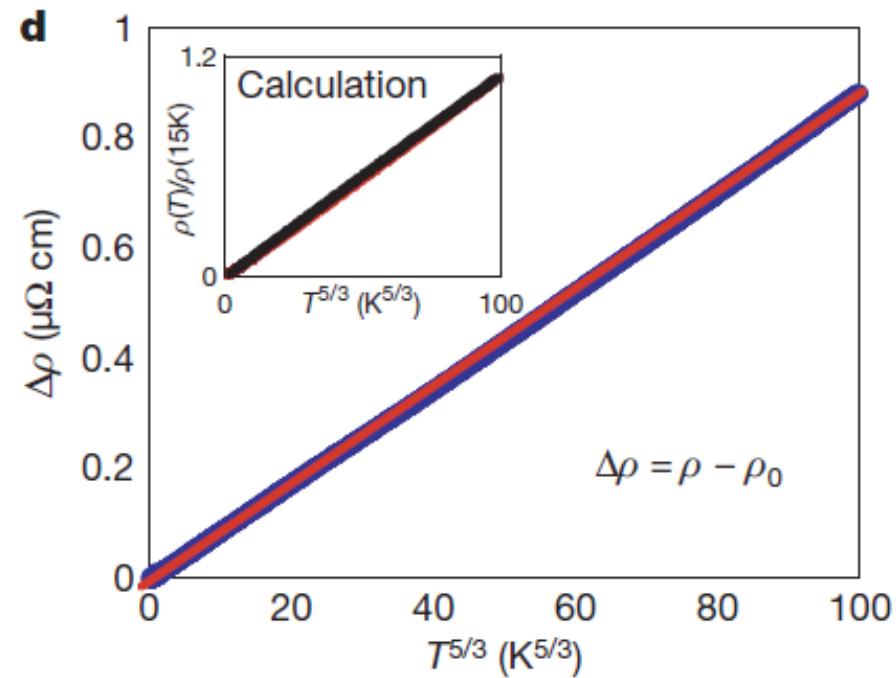
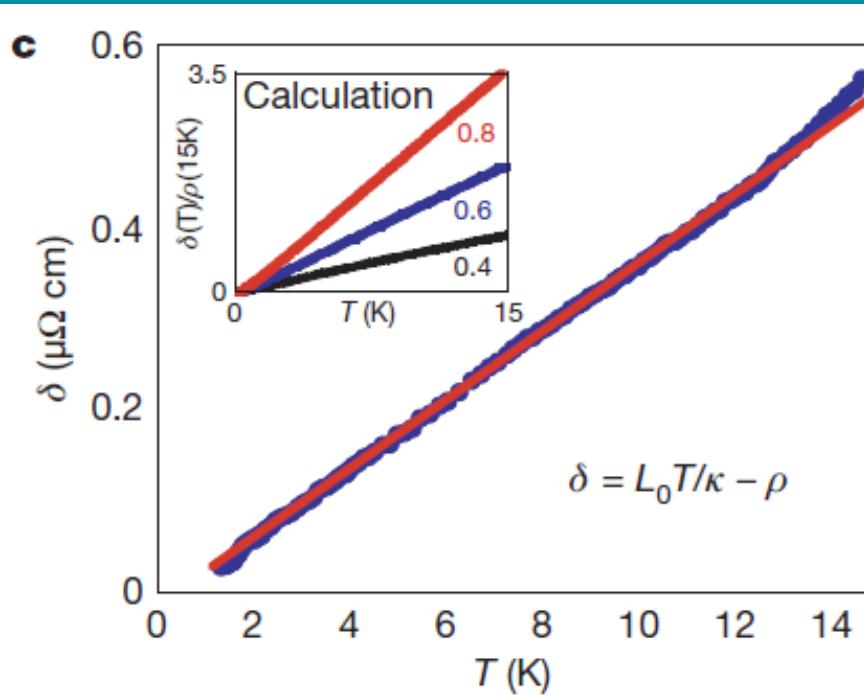
S. A. Grigera, A. P.
Mackenzie, A. J.
Schofield, S. R. Julian
and G. G. Lonzarich
Int. J. Mod. Phys. B **16**,
3258-3264 (2002).

D. Belitz *et al.*
Phys. Rev. Lett. **94**, 247205
(2005)



LGWH: successes – ZrZn₂ a clean itinerant FM (>1K)

A Marginal Fermi Liquid

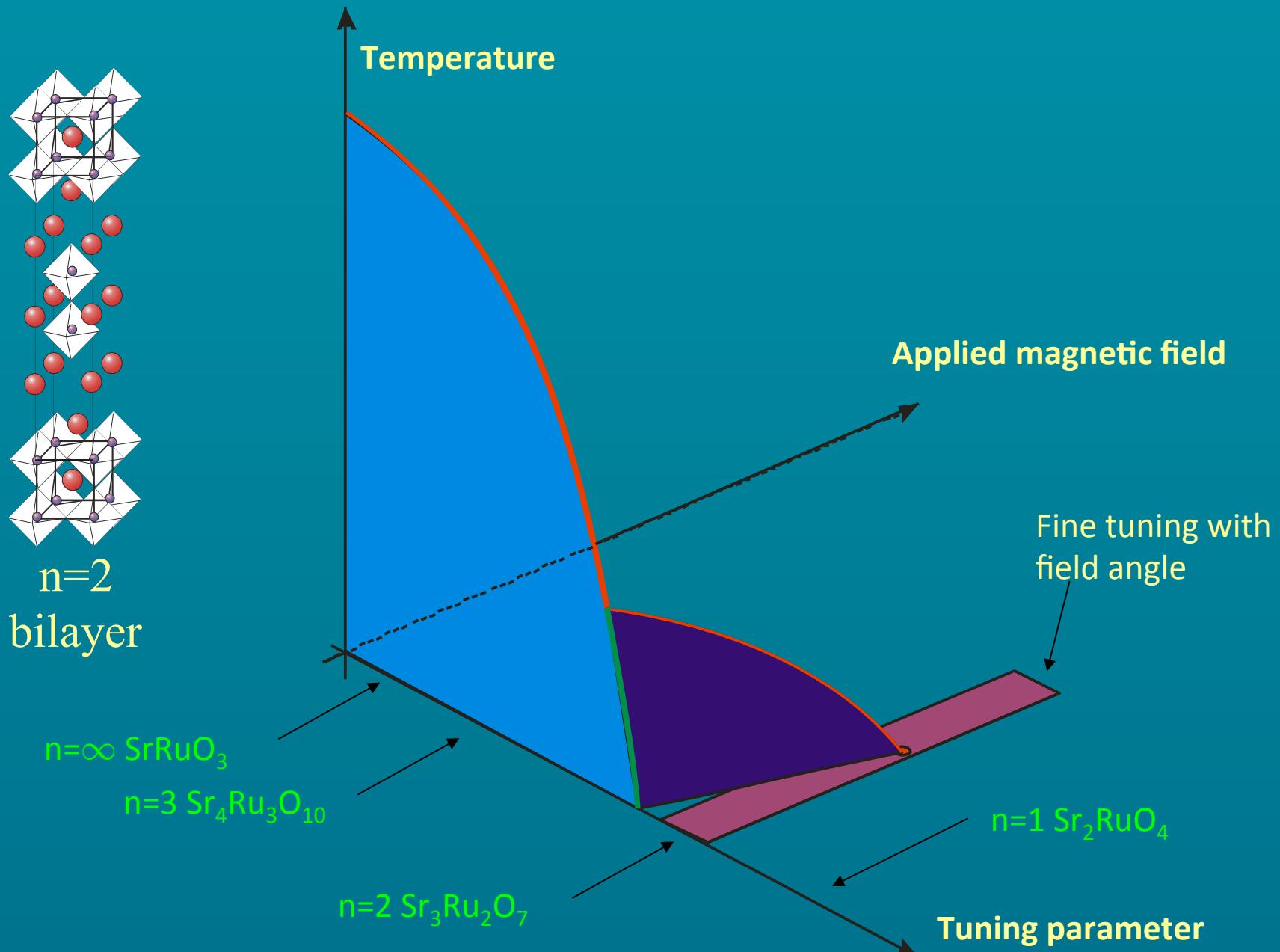


R. P. Smith, M. Sutherland, G. G. Lonzarich, S. S. Saxena, N. Kimura, S. Takashima, M. Nohara and H. Takagi
 Nature **455**, #7217, 1220-1223 (2008).

Exposing physics normally too weak to see: e.g. Michael Reizer (1989)

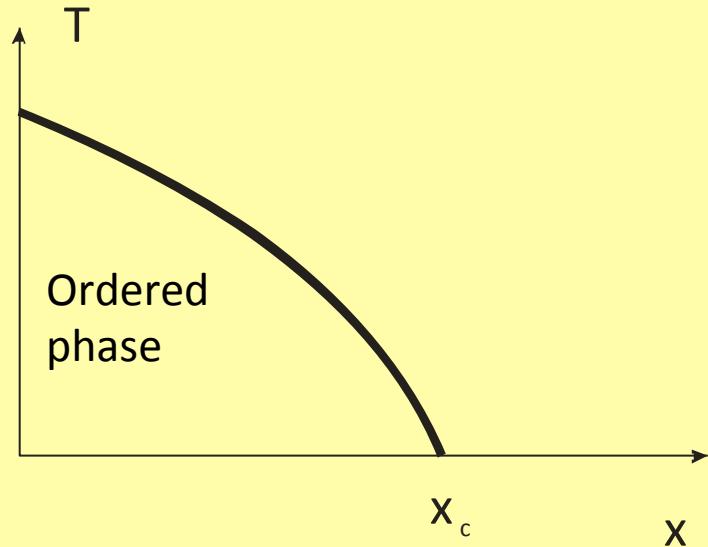
See: Non-Fermi liquids A. J. Schofield Contemp. Phys. **40**, #2, 95-115 (1999).

The Ruddlesden-Popper Series: $\text{Sr}_{1+n}\text{Ru}_n\text{O}_{1+3n}$



Generating quantum criticality

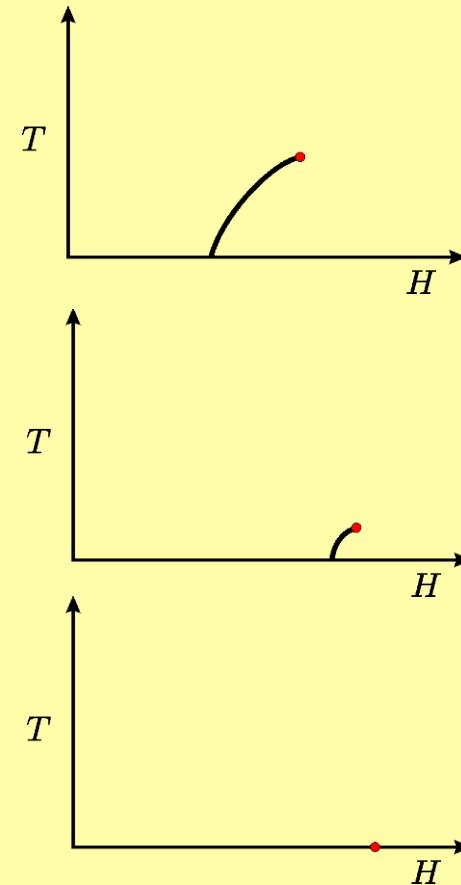
Continuous phase transition driven to $T=0$



Example: **CePd₂Si₂** Antiferromagnetism tuned by pressure.

[S. R. Julian *et. al.* J. Phys. C (1996)]

Critical end-point driven to $T=0$



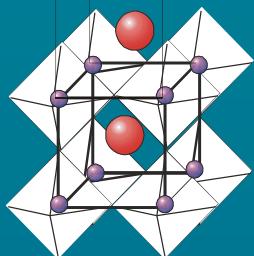
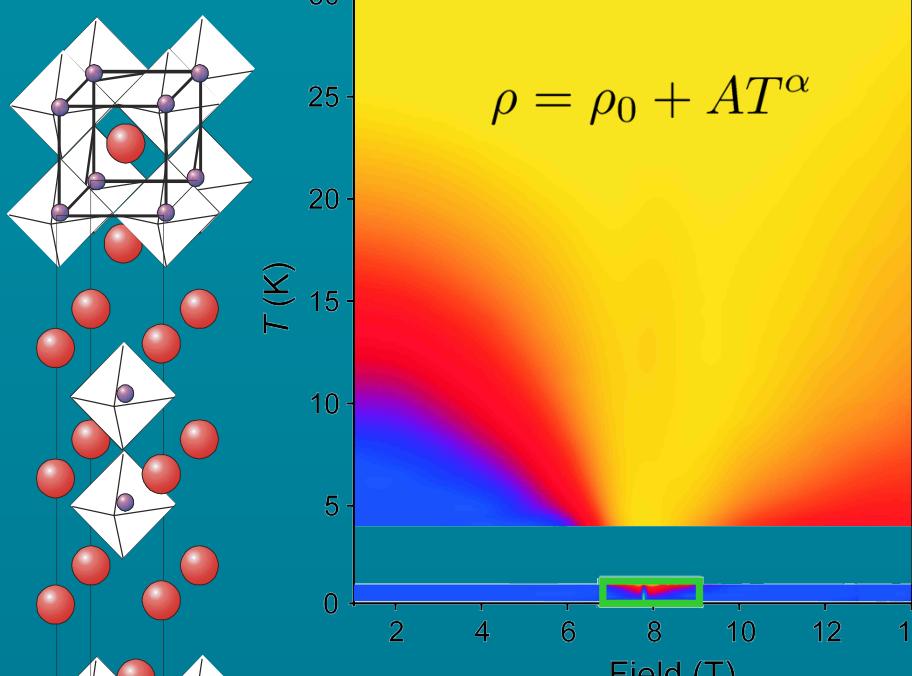
Example: **Sr₃Ru₂O₇** Metamagnetism tuned by magnetic field angle

[S.A. Grigera *et. al.*, Science (2001)]

The metamagnetic quantum critical end-point (QCEP)

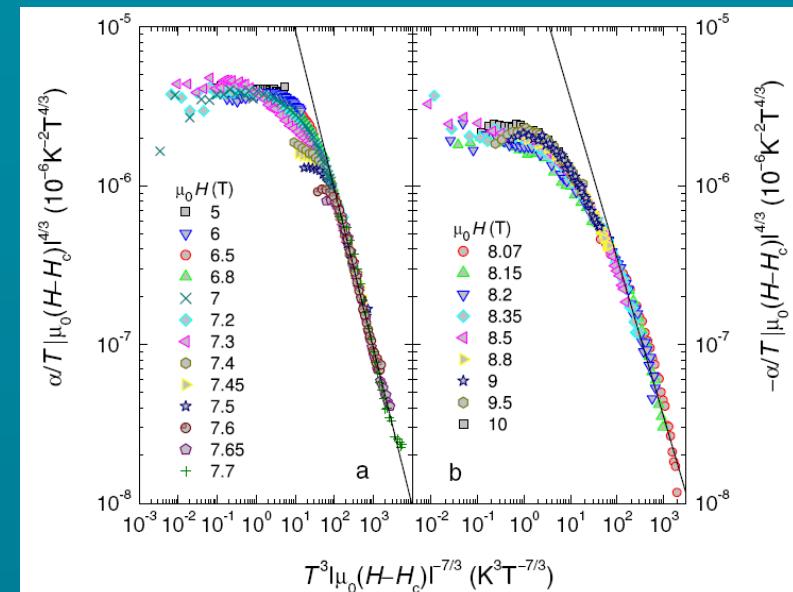
Theory of the metamagnetic quantum critical endpoint:

A.J. Millis, A. J. Schofield, G.G. Lonzarich and S.A. Grigera, Phys. Rev. Lett. **88**, 217204 (2002)



S.A.Grigera, R.S.Perry, A.J.Schofield,
M.Chiao, S.R.Julian, G.G.Lonzarich,
S.I.Ikeda, Y.Maeno, A.J.Millis,
A.P.Mackenzie,

Science, **294**, 329 (2001).



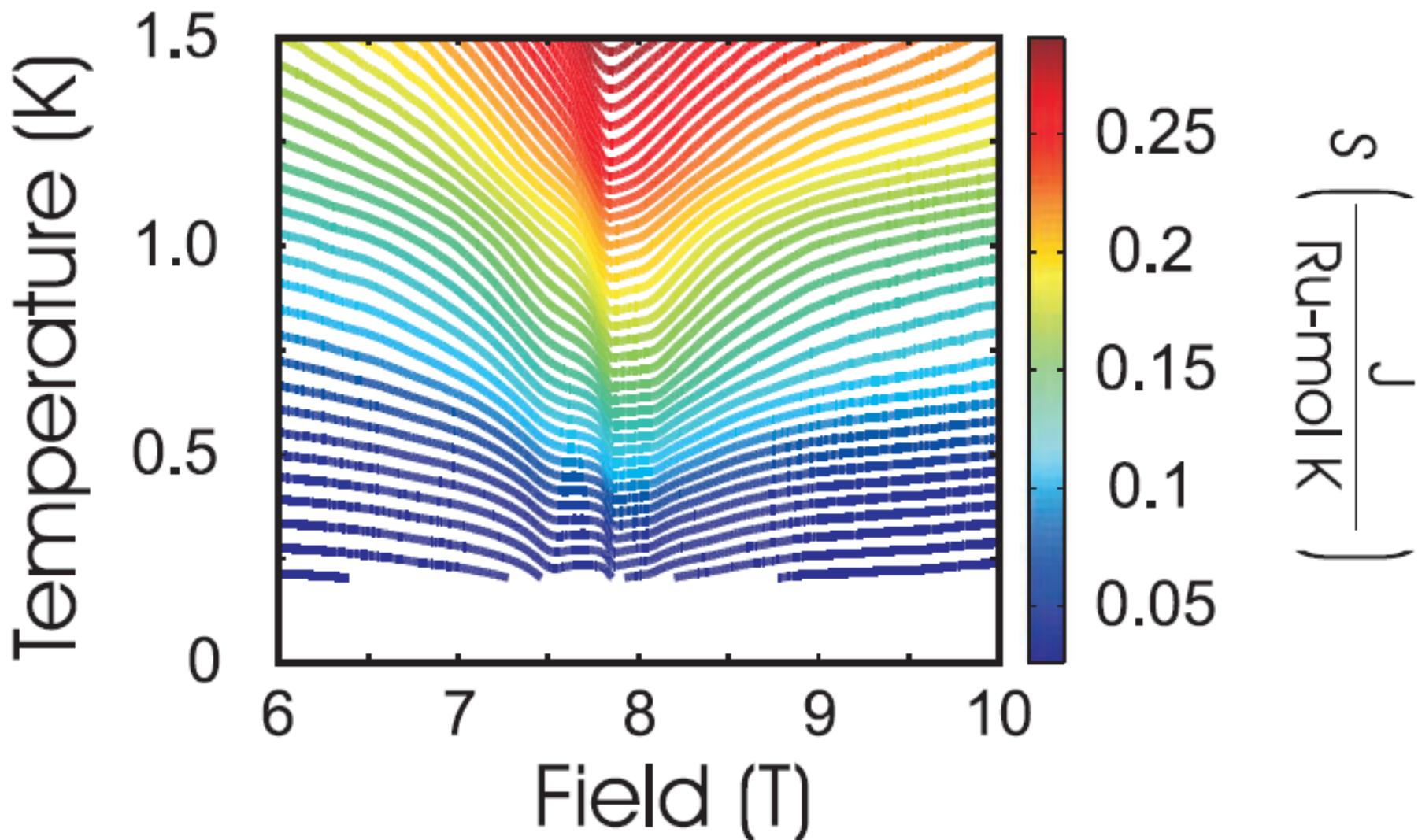
P. Gegenwart, F. Weickert, M. Garst, R.S. Perry and Y. Maeno,

Phys. Rev. Lett. **96**, 136402 (2006).

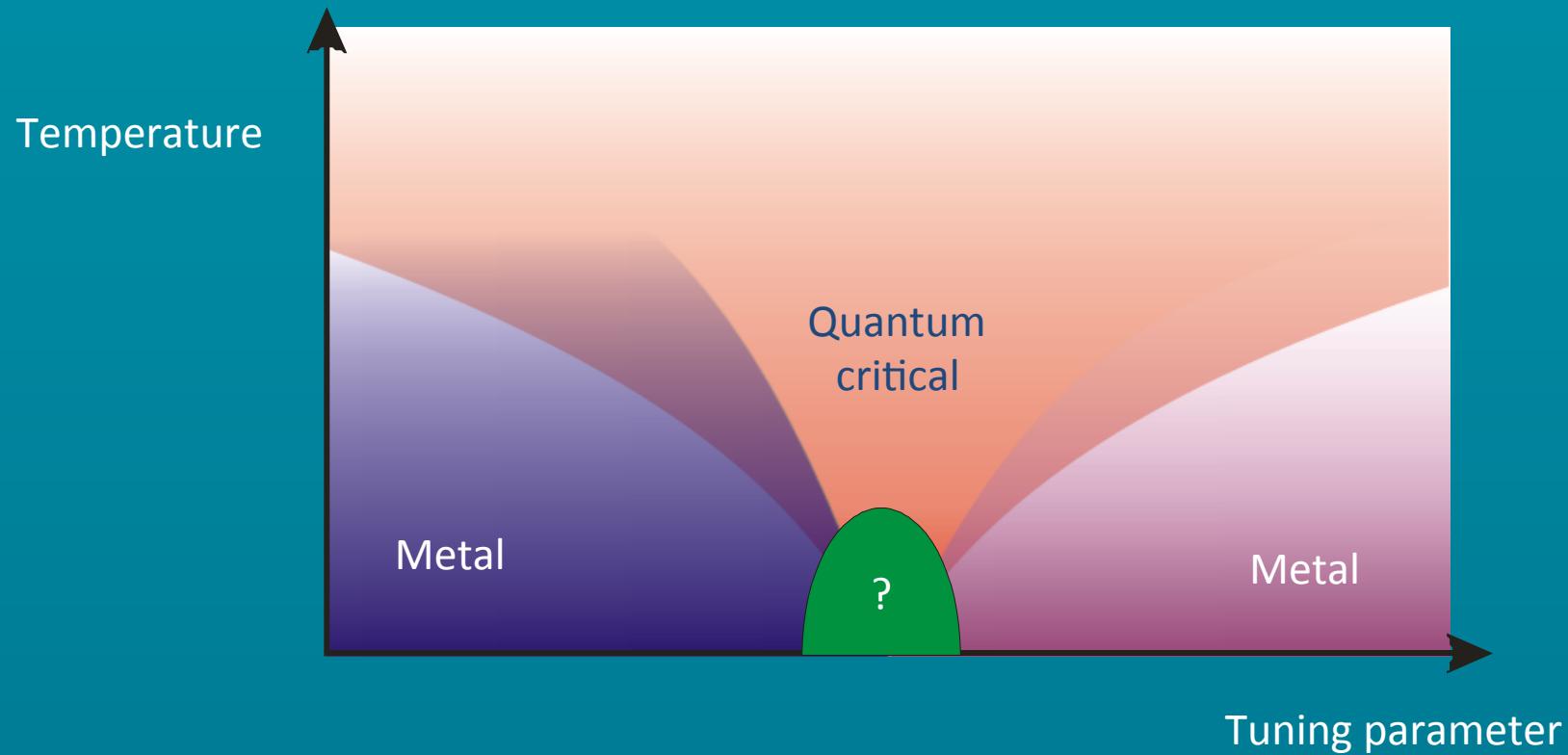
QCEP scenario consistent with thermal conductivity: F. Ronning, et al. PRL 97, 067005 (2006)

Quantum criticality – a playground for new phases

Quantum criticality concentrates the entropy: devices???

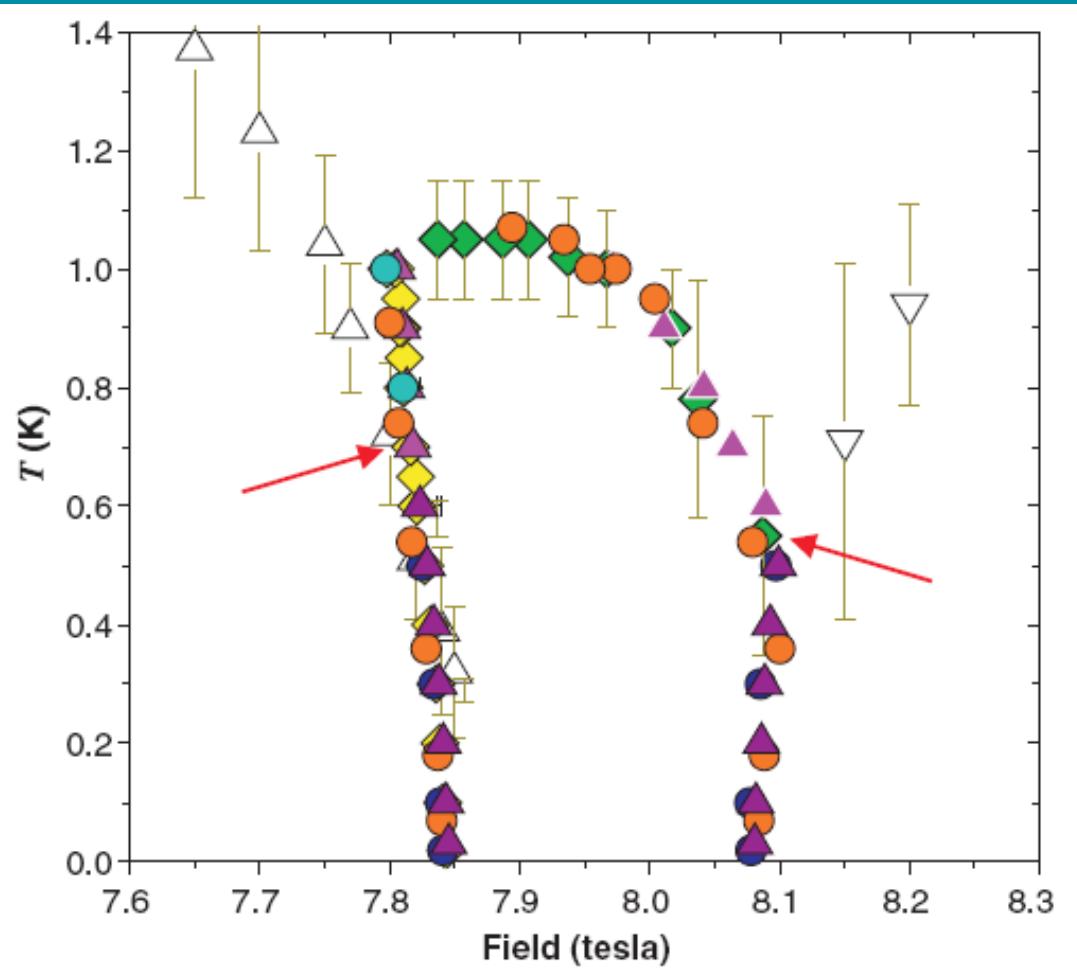


Quantum criticality driving new order in the vicinity of the QCP



Often superconductivity...but what if superconductivity cannot form?

Quantum “dark matter”



“Disorder sensitive phase formation linked to metamagnetic quantum criticality”:

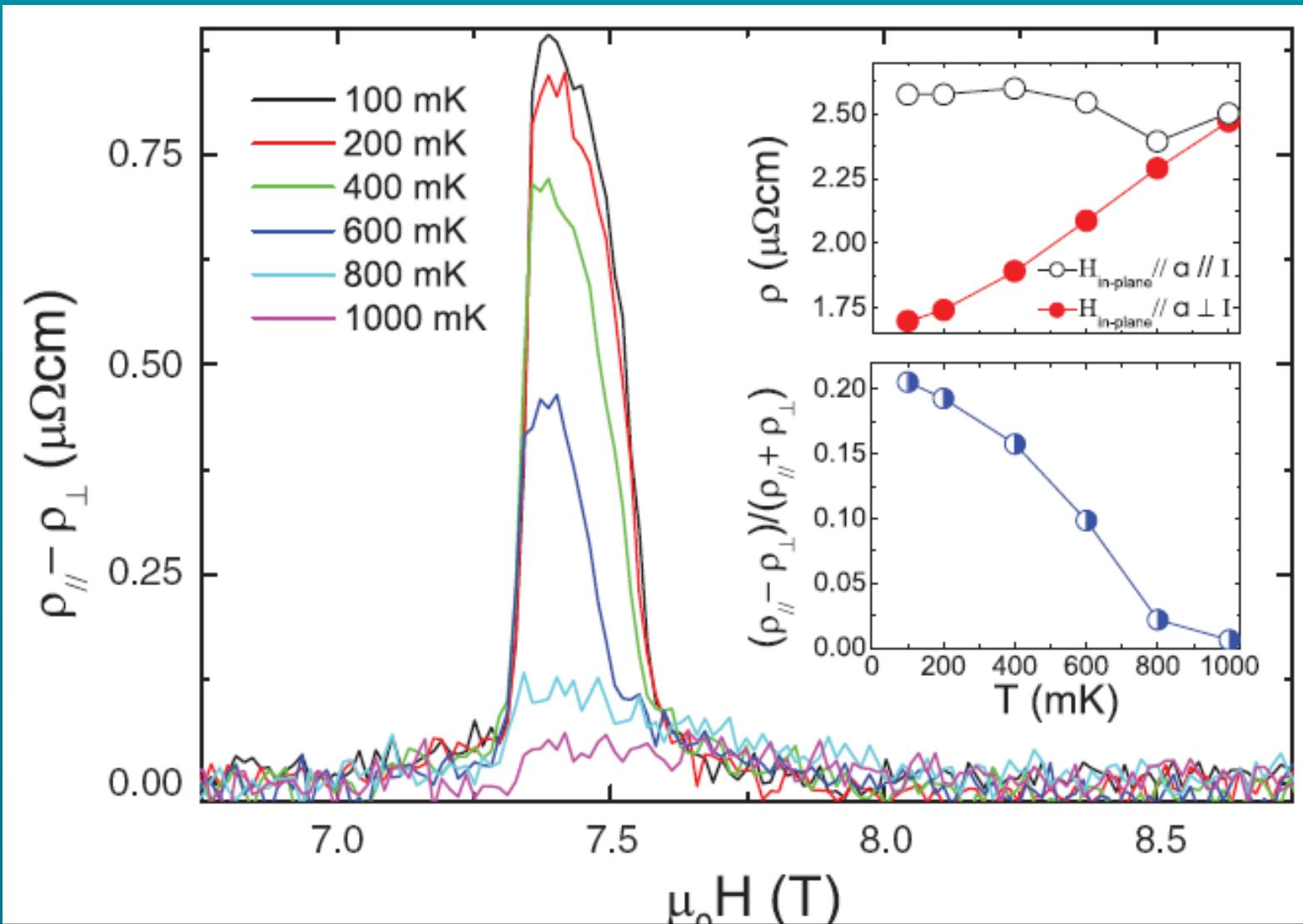
S. A. Grigera, P. Gegenwart, R. A. Borzi, F. Weickert, A. J. Schofield, R. S. Perry, T. Tayama, T. Sakakibara, Y. Maeno, A. G. Green, A. P. Mackenzie, *Science* **306**, 1154 (2004)

Metal-to-Metal transition seen in susceptibility, dc magnetization, resistivity, magnetostriction.

- URu_2Si_2 , $\text{Sr}_3\text{Ru}_2\text{O}_7$, Cuprates, ...

Condensed dark matter: matter that has a thermodynamic effect but whose order parameter is transparent to current probes.

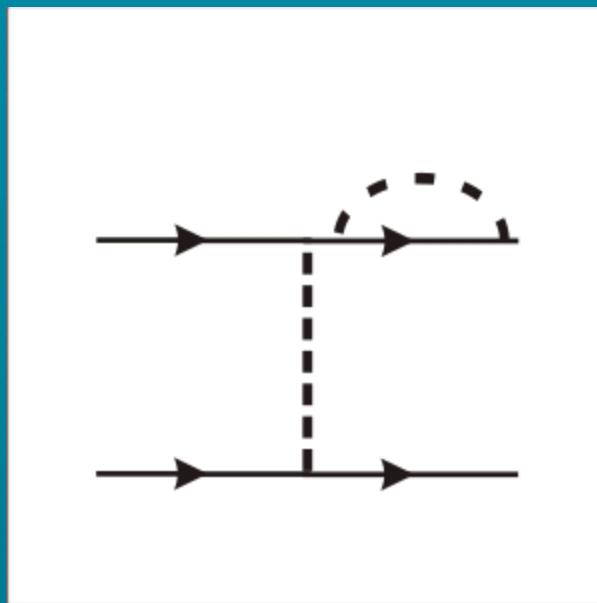
Resistivity anisotropy appears...
as if magnetic field aligns domains



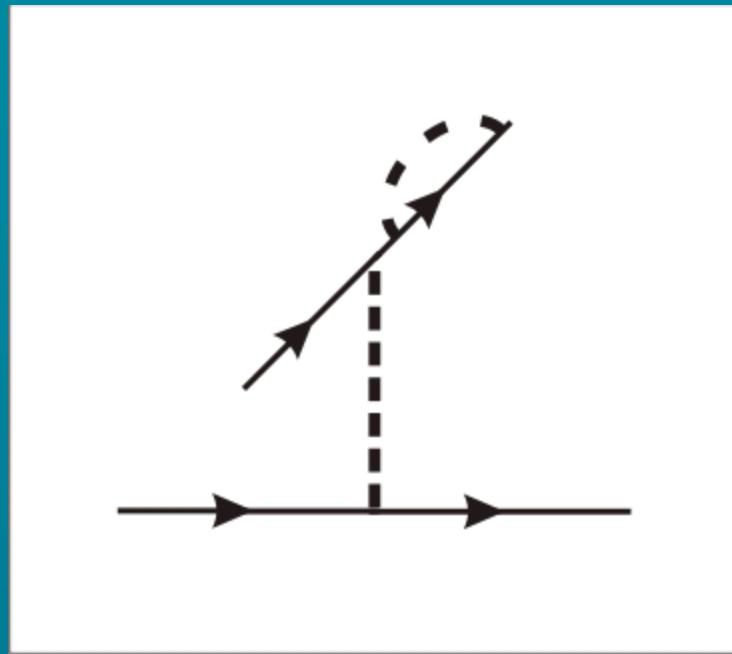
R.A. Borzi, S. A. Grigera, J. Farrell, R.S.Parry, S. J. S. Lister, S. L. Lee, D. A. Tennant,
Y. Maeno, A. P. Mackenzie, Science, **315**, 214 (2007).



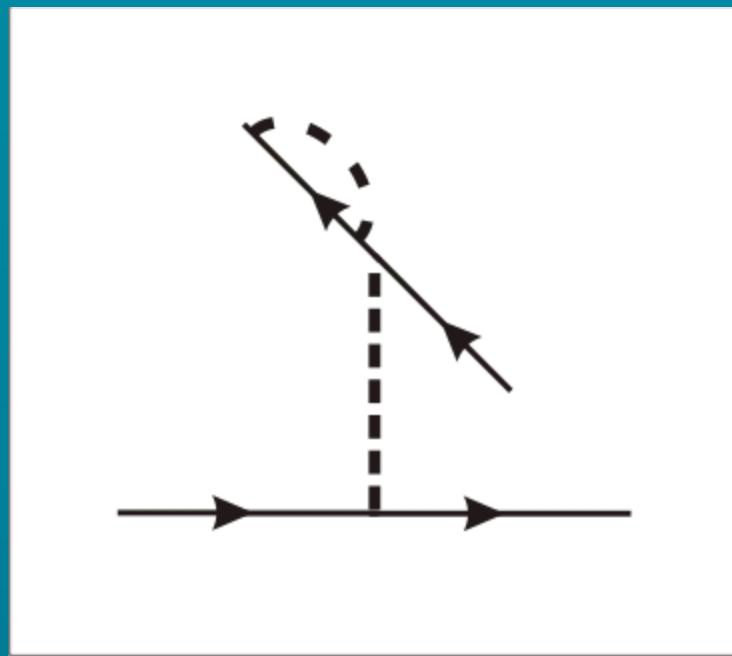
From particle-particle pairing to particle-hole pairing



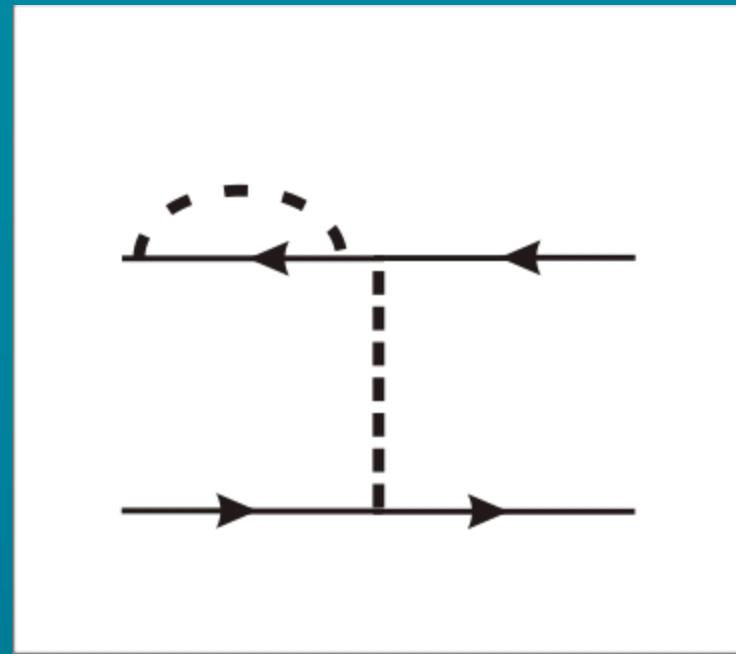
From particle-particle pairing to particle-hole pairing



From particle-particle pairing to particle-hole pairing



From particle-particle pairing to particle-hole pairing



Dark order states as magnetic analogues of unconventional superconductors

A. J. Schofield phys. stat. sol. (b) **247**, #3, 563-569 (2010).

Superconductors: part-part

Conventional: *s*-wave

$$\Delta = \sum_{k,k',\sigma} V_{k,k'} \langle c_{k'\sigma}^\dagger c_{-k'\bar{\sigma}}^\dagger \rangle$$

Unconventional: *p*-wave, *d*-wave,...

$$\Delta(k) = \sum_{k,k',\sigma} V_{k,k'} \langle c_{k'\sigma}^\dagger c_{-k'\bar{\sigma}}^\dagger \rangle$$

Inhomogeneous: FFLO

$$\Delta(q) = \sum_{k,k',\sigma} V_{k,k'} \langle c_{k'+q/2,\sigma}^\dagger c_{-k'+q/2,\bar{\sigma}}^\dagger \rangle$$

Mixed State

Magnets: part-hole

Conventional: Stoner ferromagnetism

$$M = \sum_{k,\sigma,\sigma'} g_{\sigma,\sigma'} \langle c_{k\sigma}^\dagger c_{k\sigma'} \rangle$$

“Pomeranchuk”: *p*-wave, *d*-wave,...

$$M(k) = \sum_{k,\sigma,\sigma'} g_{k,k';\sigma,\sigma'} \langle c_{k'\sigma}^\dagger c_{k'\sigma'} \rangle$$

Inhomogeneous: “spirals”, density waves

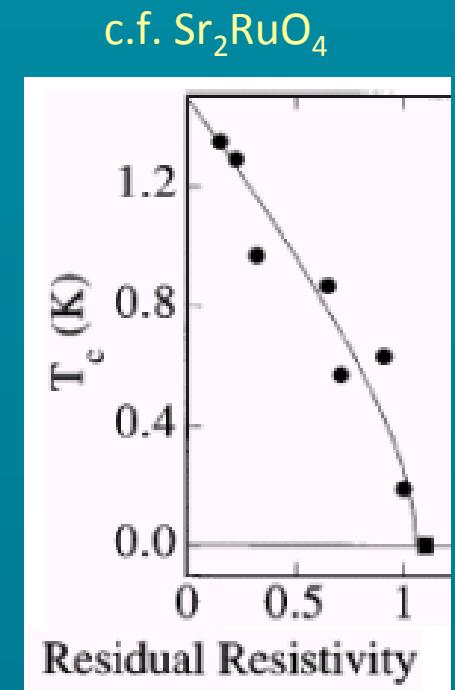
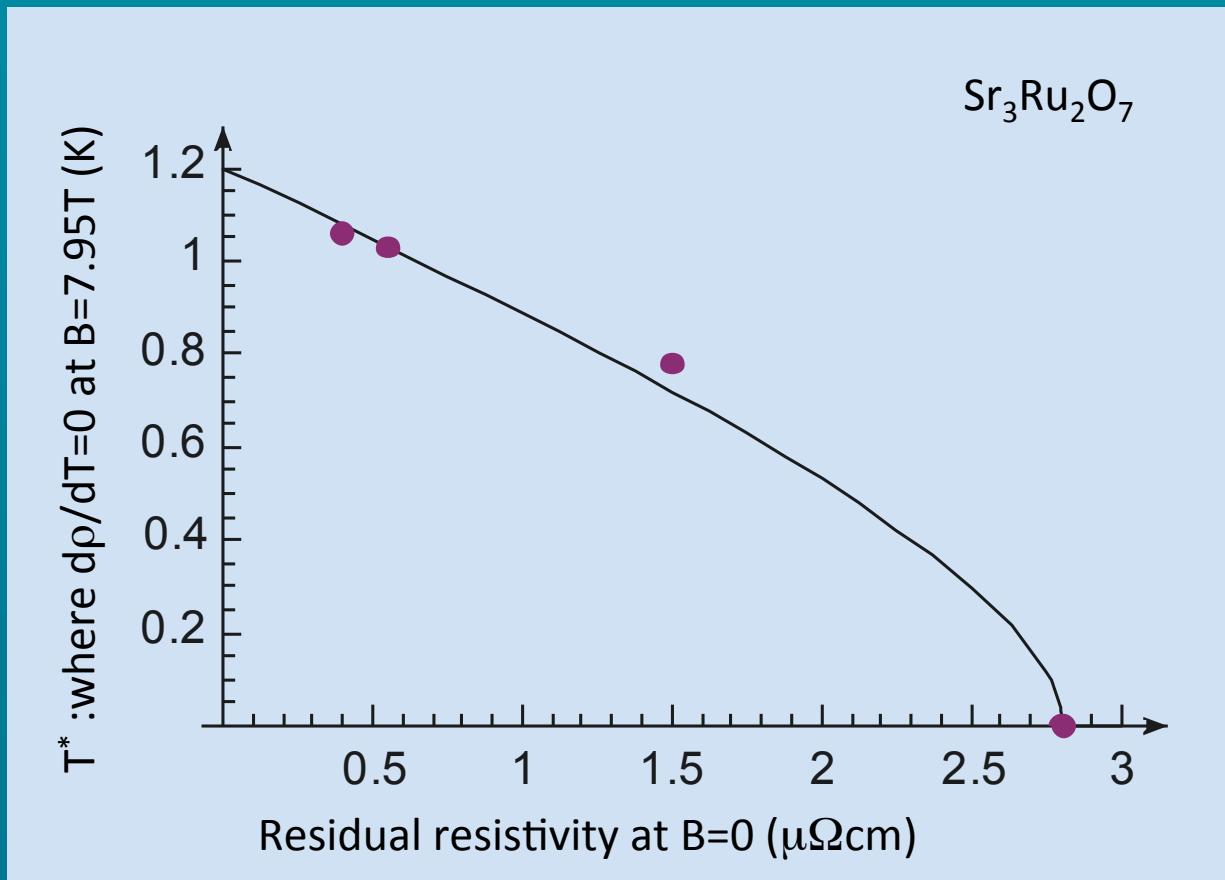
$$M(q) = \sum_{k,\sigma} g_{k,k';\sigma,\sigma'} \langle c_{k'+q/2,\sigma}^\dagger c_{k'-q/2,\sigma'} \rangle$$

Spin textured state



Predictions: magnetic analogues of unconventional superconductors should show strong disorder dependence

A. F. Ho and A. J. Schofield EPL **84**, #2, 27007 (2008).



A. P. Mackenzie *et al.*
Phys. Rev. Lett. **80**,
161 (1998)

- No systematic studies. Existing data from S. A. Grigera

Can we make Kondo lattice look-alike?

- Evidence from d-metals of some resonance with the Landau-Ginzburg-Wilson-Hertz view of quantum criticality
- Can we create quantum criticality in a Kondo lattice “look-alike”?

Kondo breakdown in a look-alike?

- Bilayers of ^3He :

Michael Neumann, Ján Nyéki, Brian Cowan and John Saunders
Science **317**, #5843, 1356-1359 (2007).

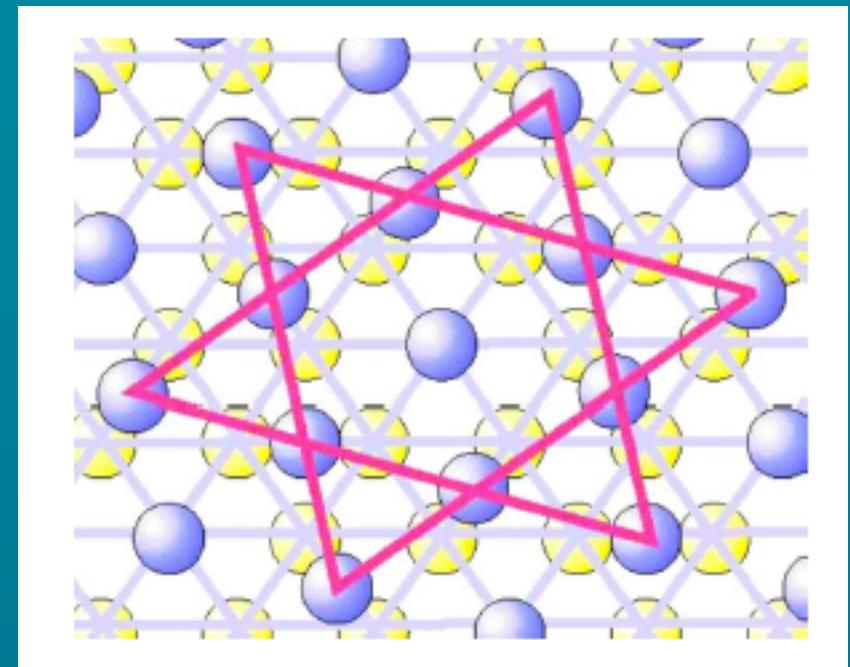
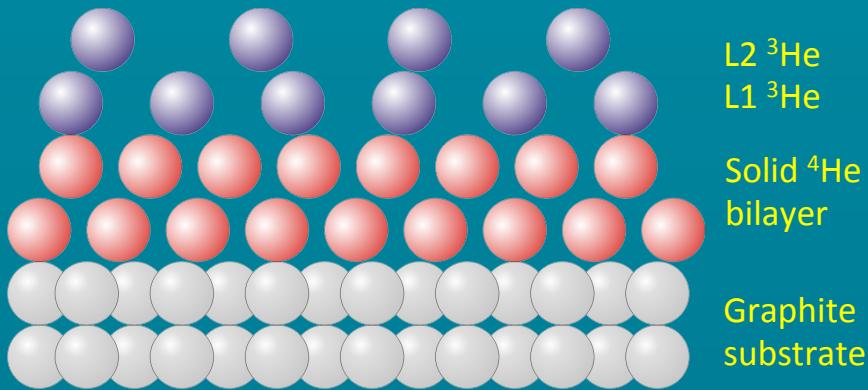
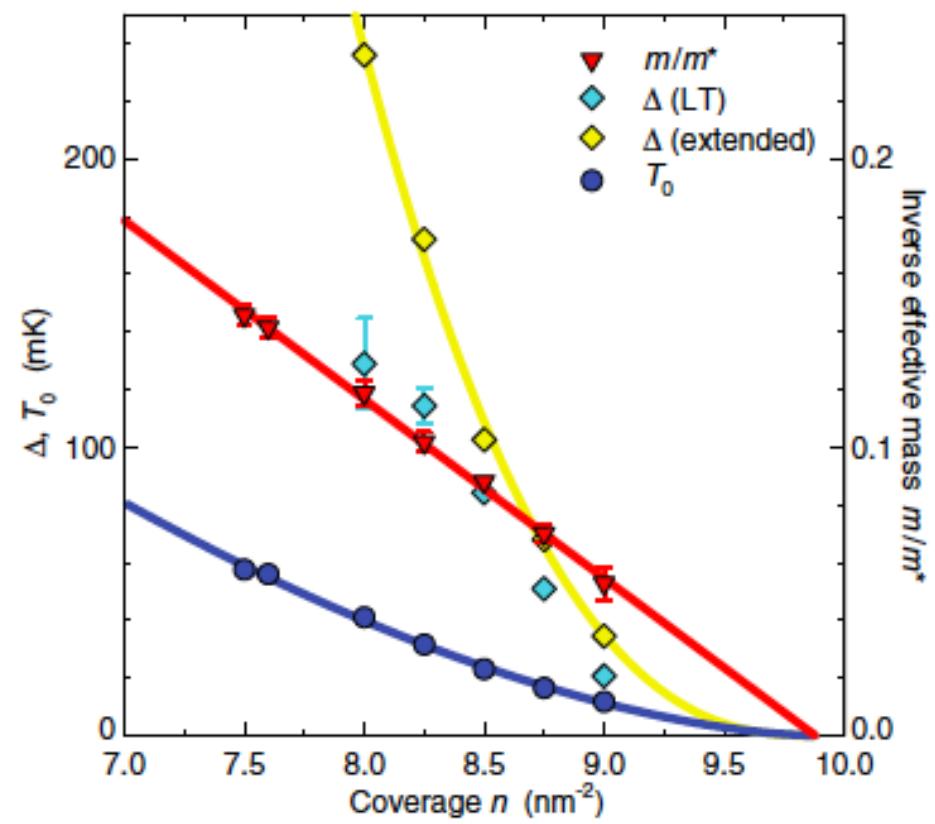
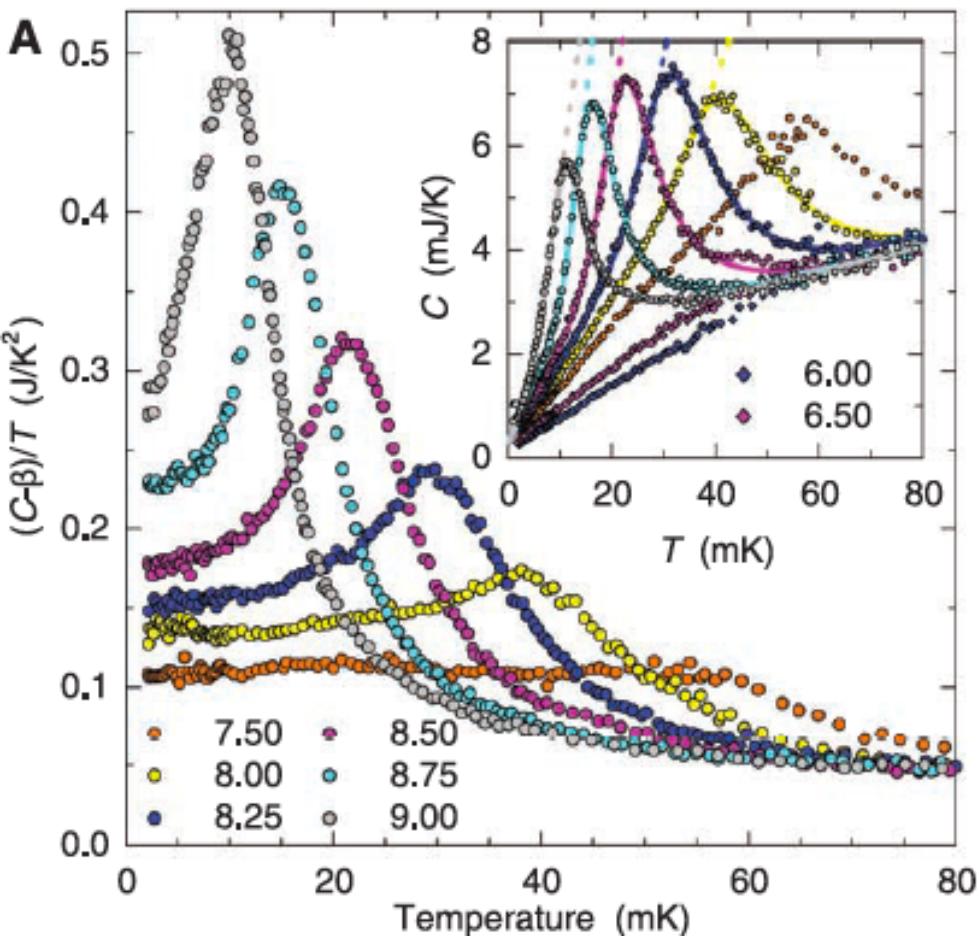


Figure from A. Benlagra and C. Pepin,
PRB, 79, 045112 (2009)

Kondo breakdown in a look-alike?

- Bilayers of ^3He :

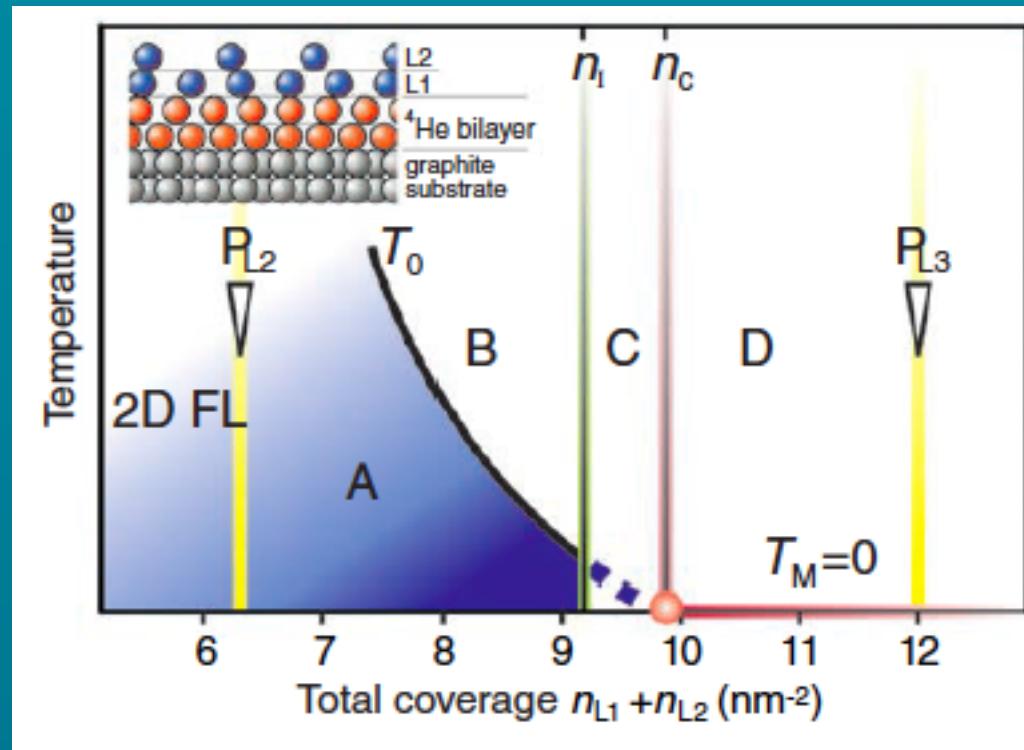
Michael Neumann, Ján Nyéki, Brian Cowan and John Saunders
Science **317**, #5843, 1356-1359 (2007).



Kondo breakdown in a look-alike?

- Bilayers of ^3He :

Michael Neumann, Ján Nyéki, Brian Cowan and John Saunders
Science **317**, #5843, 1356-1359 (2007).



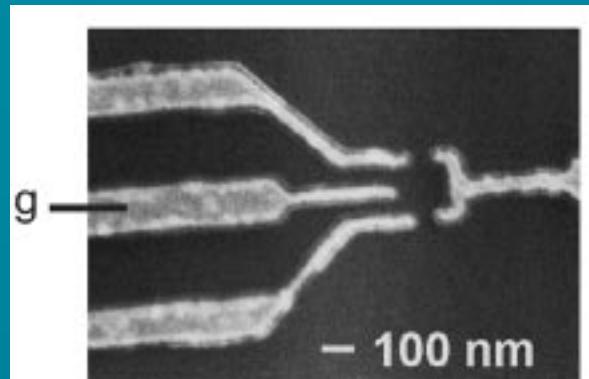
Evidence of quantum criticality in a Kondo like system: does it resemble heavy fermion anomalous quantum critical points?

Future Trends and Perspectives

- Developments in materials:
 - Epitaxial growth [c.f. *H. Shishido et al.* *Science* **327**, #5968, 980-983 (2010).]
- Developments in look-alikes:
 - Quantum critical d-metals, oxides, organics...
 - Realizing an Anderson lattice in a semiconductor
 - Realizing an Anderson lattice in a cold atomic gas
- Developments in experiments:
 - ARPES, tunnelling, high field facilities
 - How could we use the new field of quantum metrology?

Realizing an Anderson lattice in semiconductors

- Inspiration 1: Kondo dots



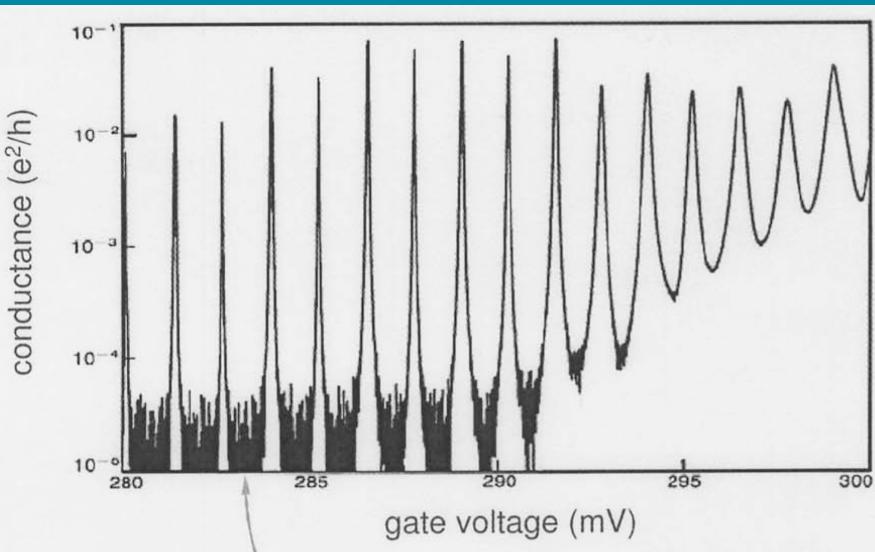
*Andrei Kogan, Sami Amasha, M. A.
Kastner
Science 304, #5675, 1293-1295 (2004).*



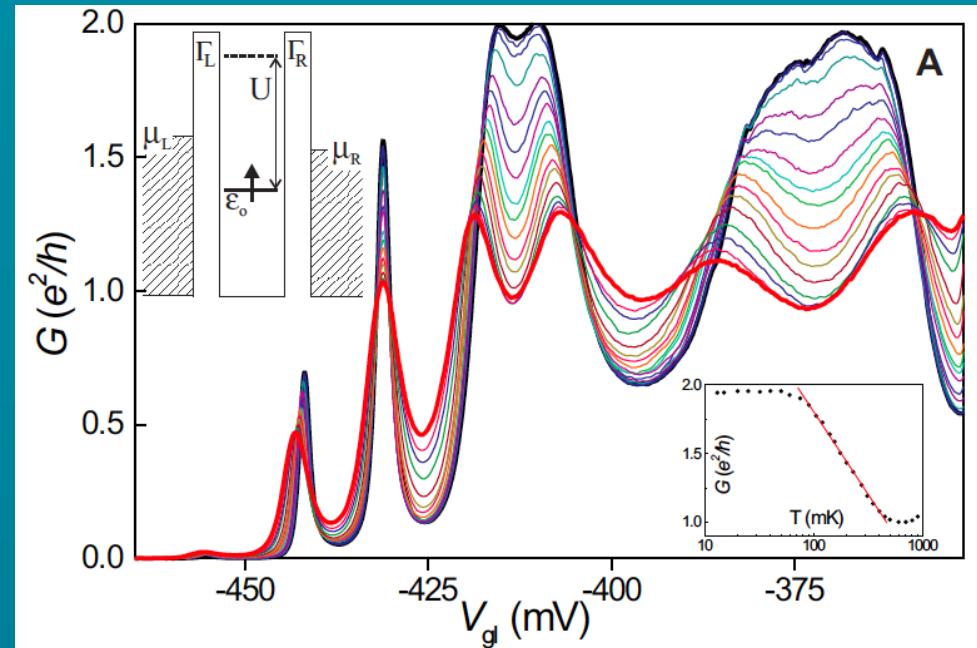
(c) Luis Diest - Ohio University

Realizing an Anderson lattice in semiconductors

- Inspiration 1: Kondo dots



M. Kastner (1993)



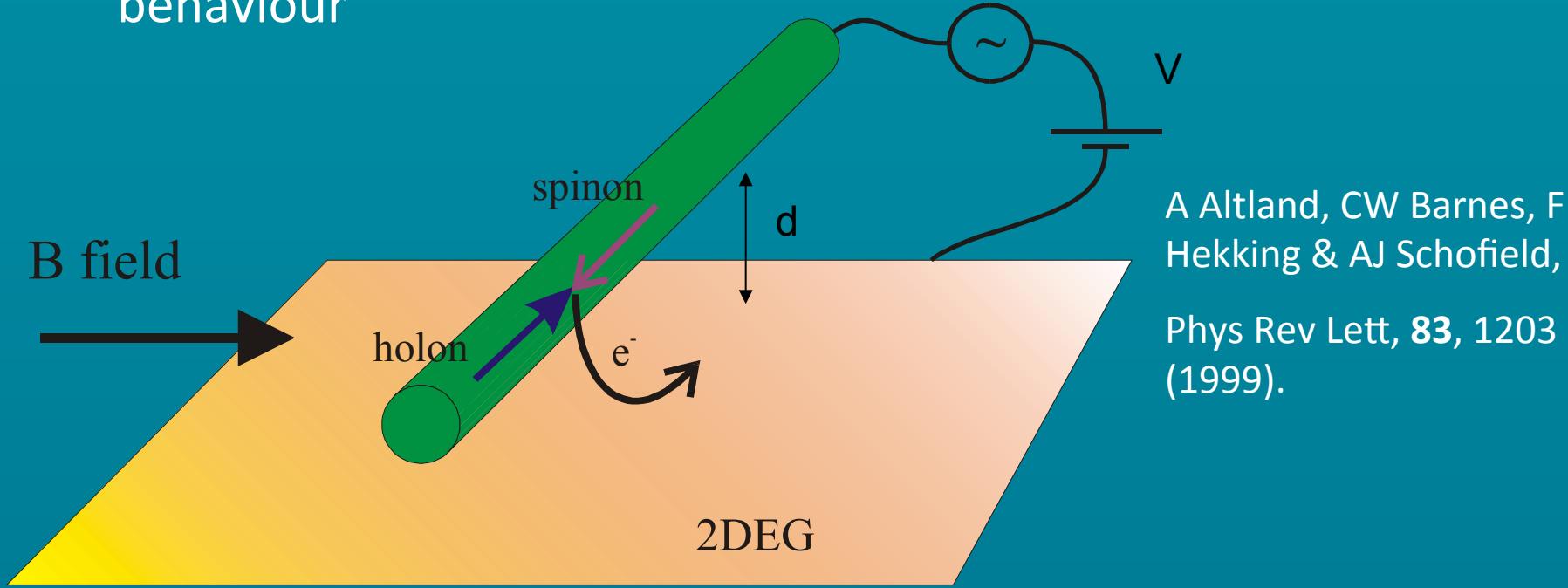
Van der Wiel et al. Science 289, 2105 (2004)

For a review, see M. Pustilnik and L. I. Glazman: [cond-mat/0501007](https://arxiv.org/abs/cond-mat/0501007).

Could you make a lattice of Coulomb blockaded dots?

Realizing an Anderson lattice in semiconductors

- Inspiration 2: Bilayer 2DEGs used to probe Luttinger liquid behaviour

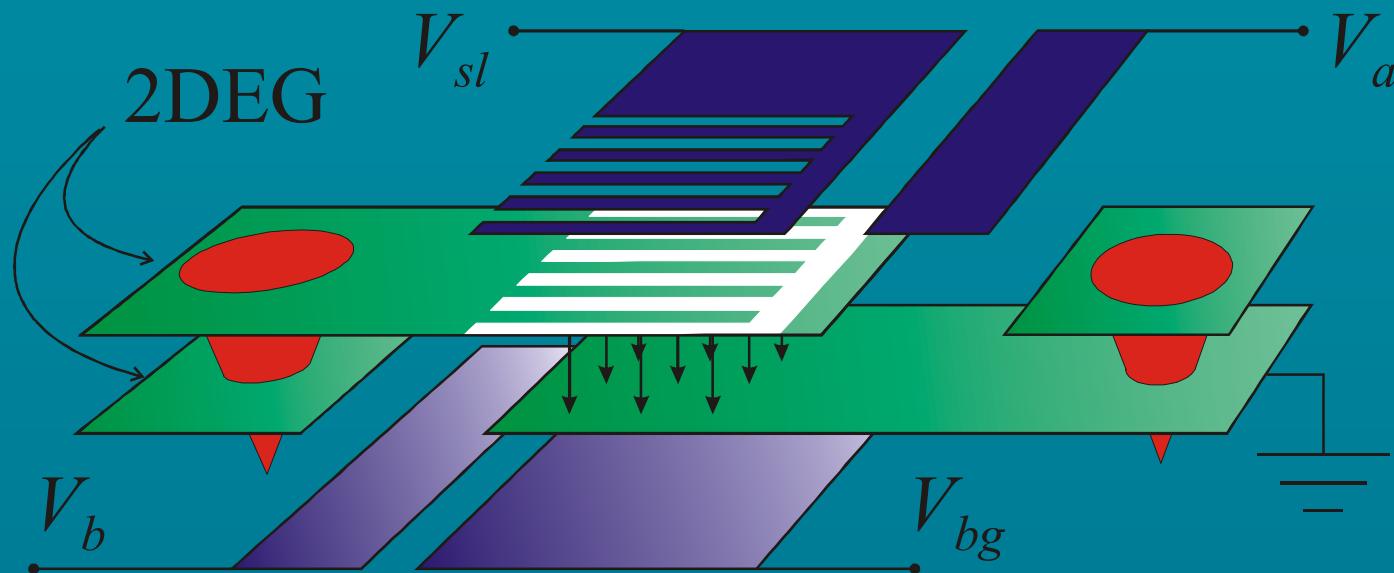


A Altland, CW Barnes, F Hekking & AJ Schofield,
Phys Rev Lett, **83**, 1203
(1999).

- Tunnelling between a 2D system (known probe) and a 1D wire (unknown subject).
- Smooth interface: momentum conserved along wire
- Transverse magnetic field tunes relative momentum

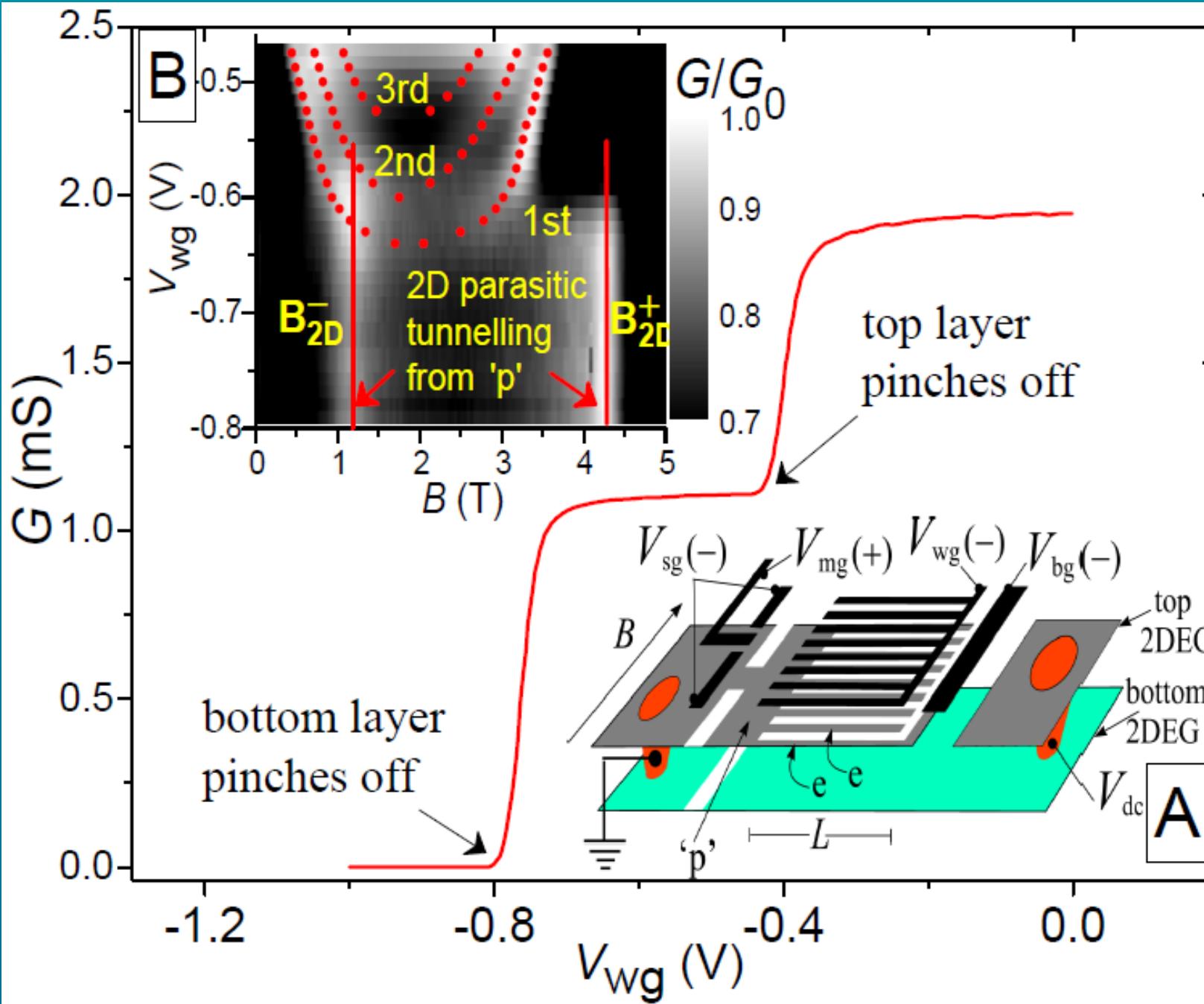
Realizing an Anderson lattice in semiconductors

- Inspiration 2: Bilayer 2DEGs used to probe Luttinger liquid behaviour

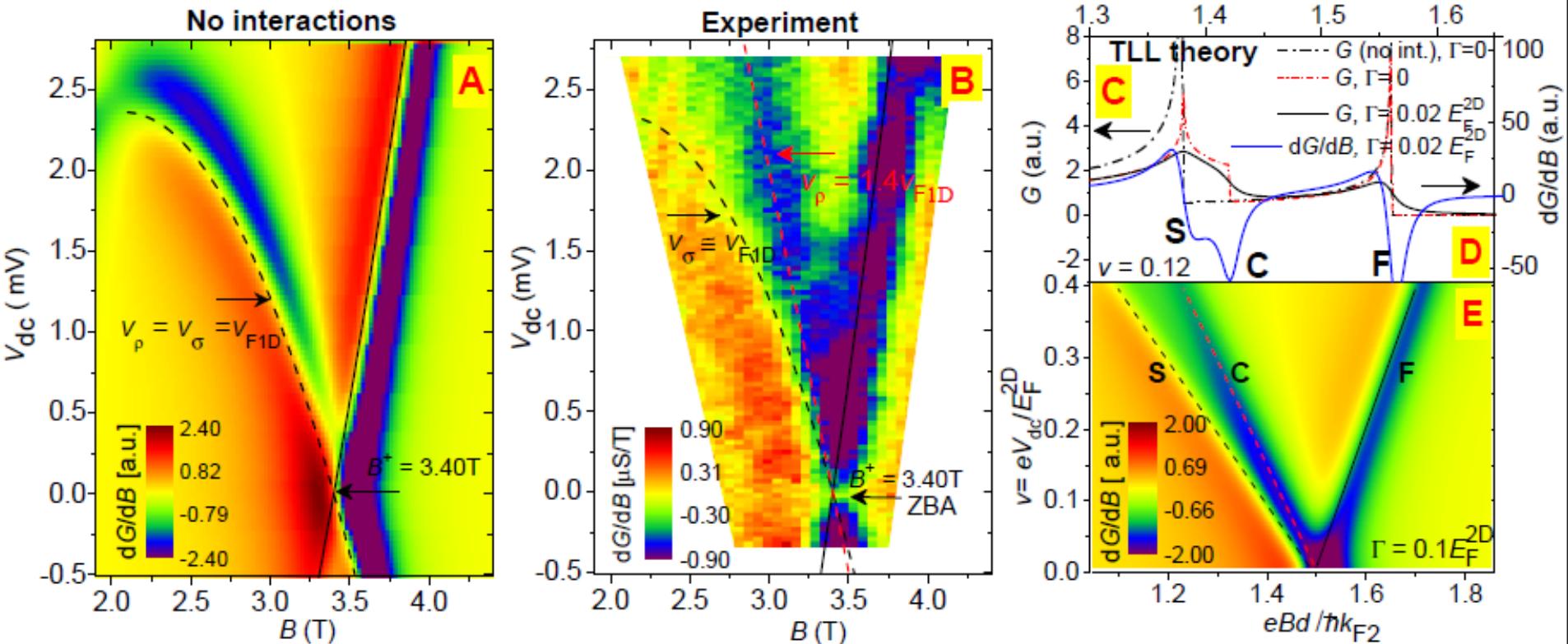


Y. Jompol, C. J. B. Ford, J. P. Griffiths, I. Farrer, G. A. C. Jones, D. Anderson, D. A. Ritchie, T. W. Silk and A. J. Schofield
Science **325**, #5940, 597-601 (2009).

Achieving a single channel wire:



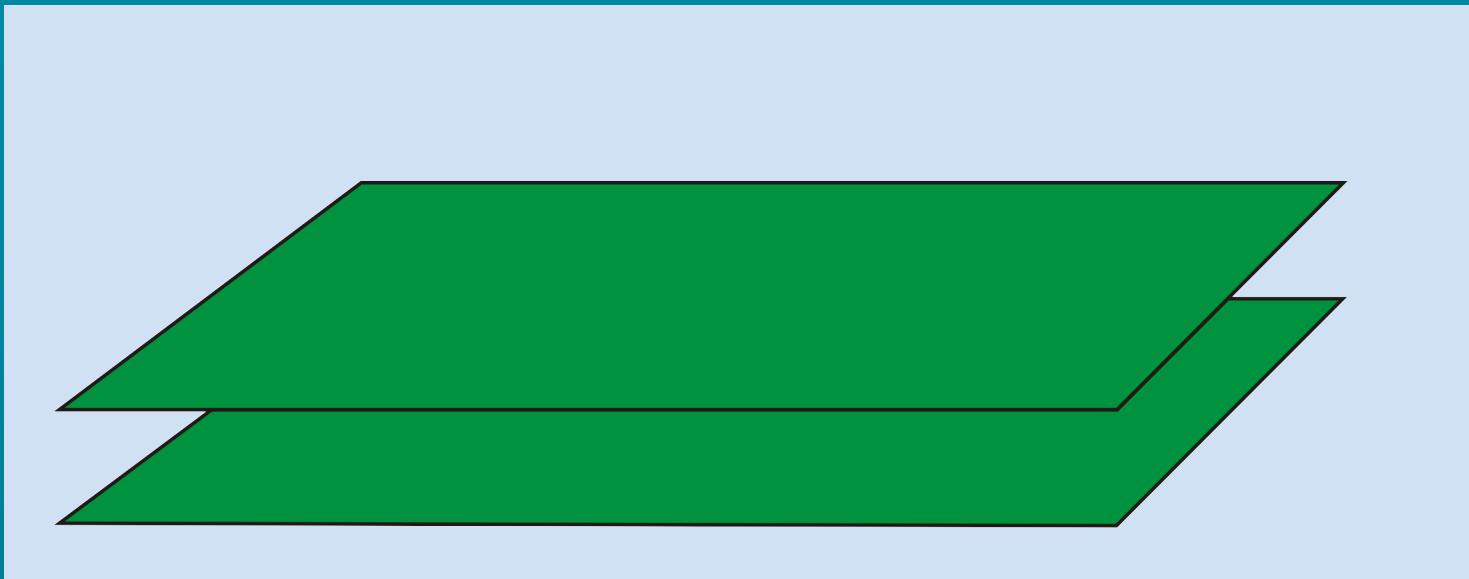
Finally – dispersing spinons and holons



- See spinon and holon beyond the universal region.

Y. Jompol, C. J. B. Ford, J. P. Griffiths, I. Farrer, G. A. C. Jones, D. Anderson, D. A. Ritchie, T. W. Silk and A. J. Schofield
Science 325, #5940, 597-601 (2009).

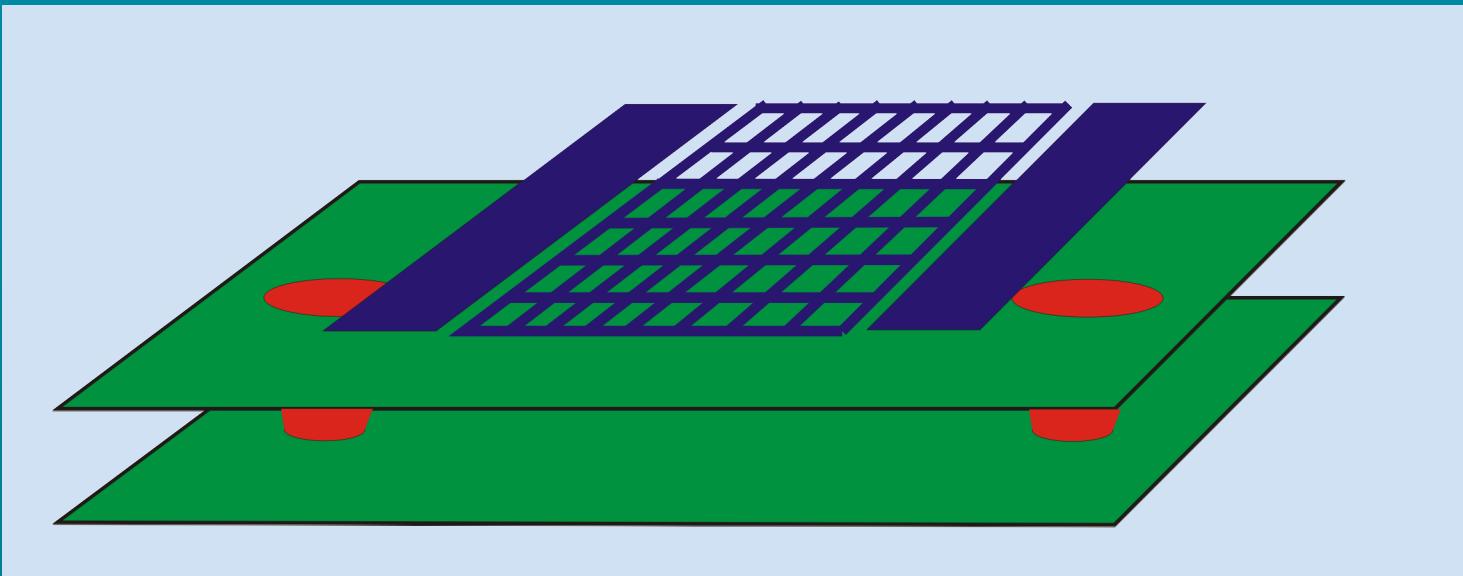
Realizing an Anderson lattice in a bilayer system



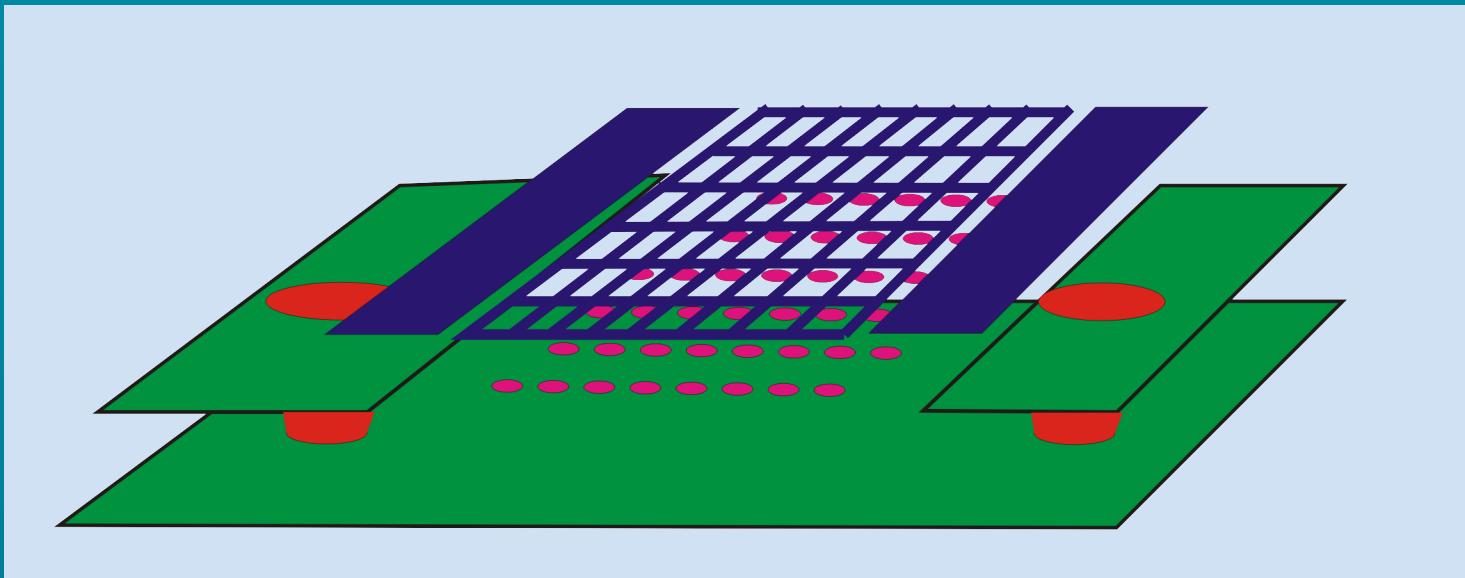
Realizing an Anderson lattice in a bilayer system



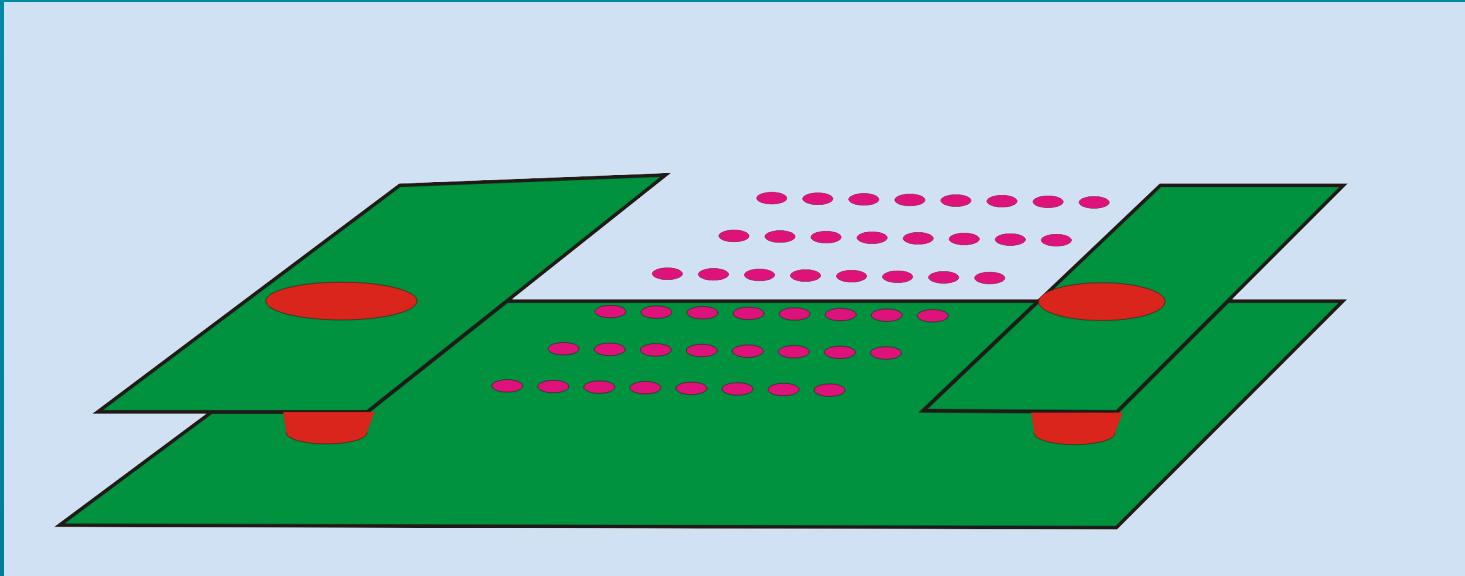
Realizing an Anderson lattice in a bilayer system



Realizing an Anderson lattice in a bilayer system



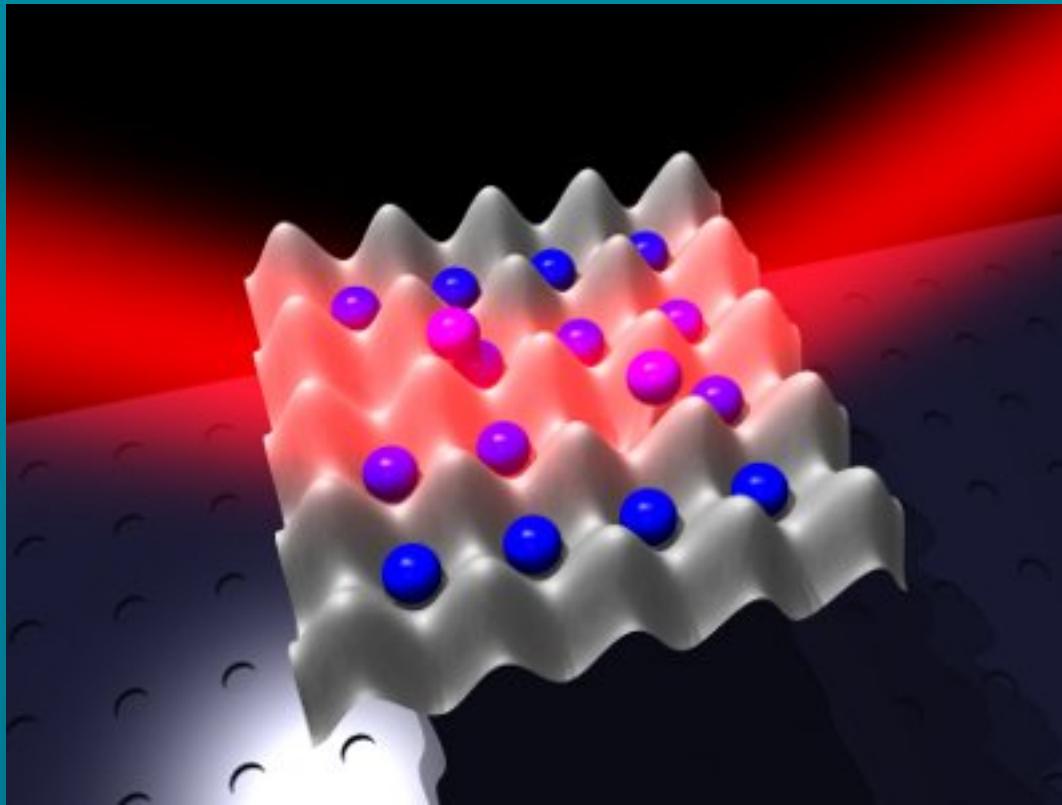
Realizing an Anderson lattice in a bilayer system



Would be a very dilute Anderson/Kondo lattice – can we improve this?

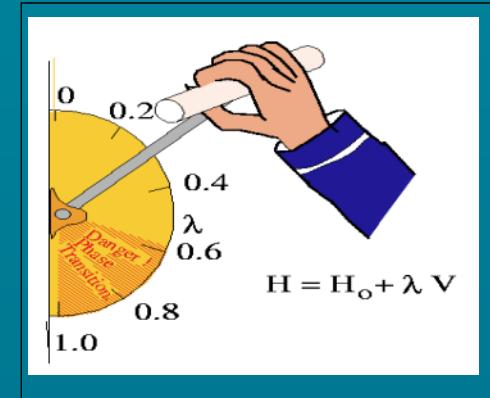
Cold quantum gases as quantum simulators

- Remarkable developments in cold quantum gases



- Ultimate control of the Hamiltonian – lattice shape, interactions etc.

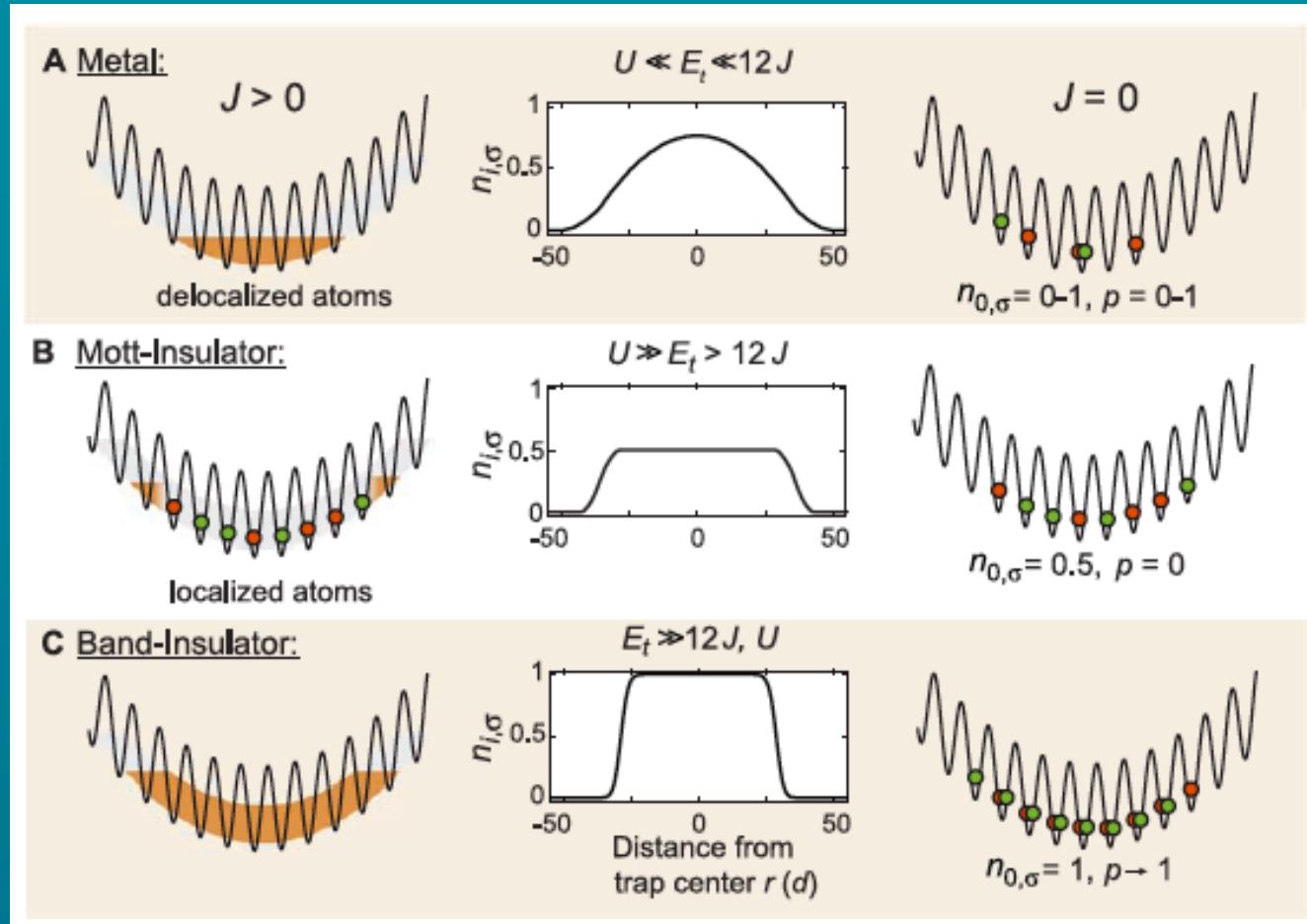
The hand of Landau...



...becomes the hand of the experimentalist
(changes are adiabatic).

Realizing an Anderson Lattice in an optical lattice

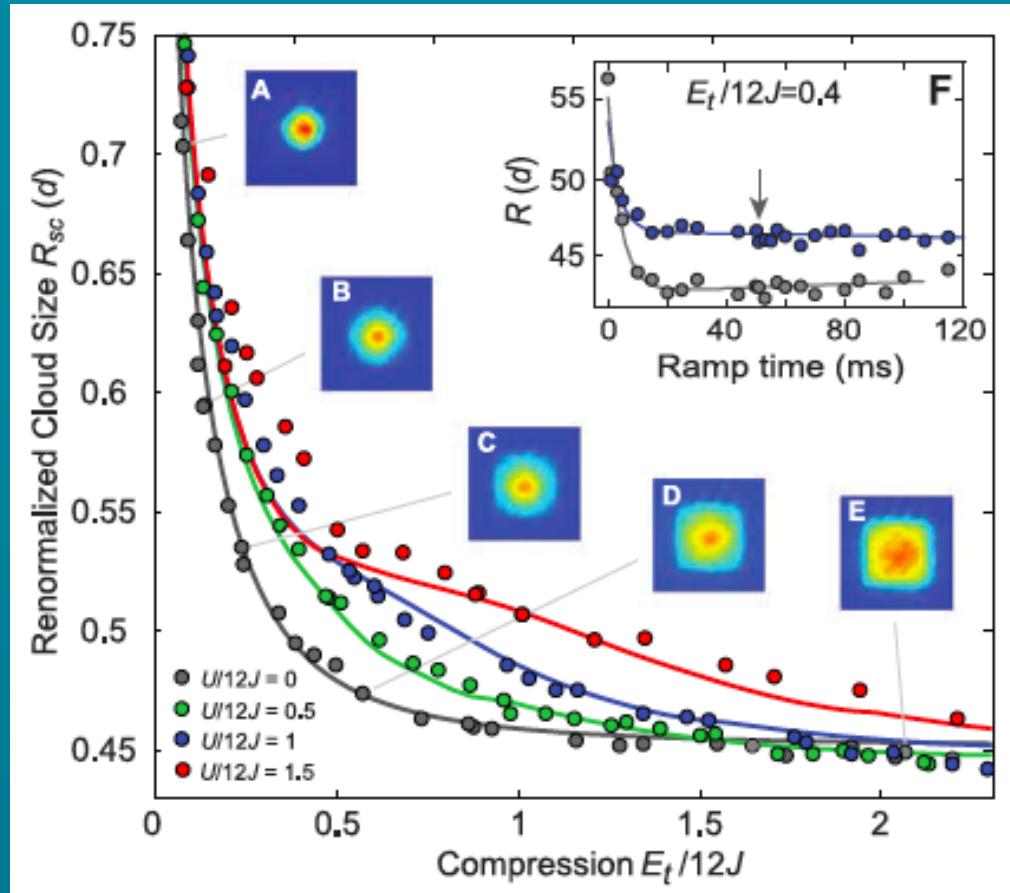
- Inspiration: the (Fermi) Hubbard model in cold atomic gases



U. Schneider, L. Hackermüller, S. Will, Th. Best, I. Bloch, T. A. Costi, R. W. Helmes, D. Rasch and A. Rosch Science 322, #5907, 1520-1525 (2008).

Realizing an Anderson Lattice in an optical lattice

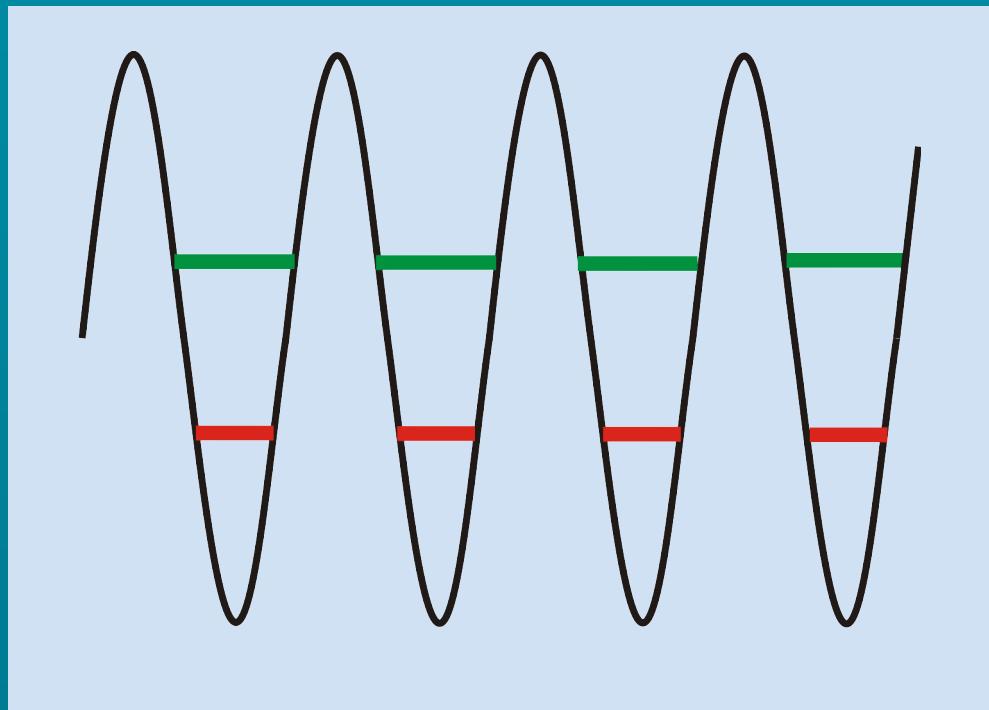
- Inspiration: the (Fermi) Hubbard model in cold atomic gases



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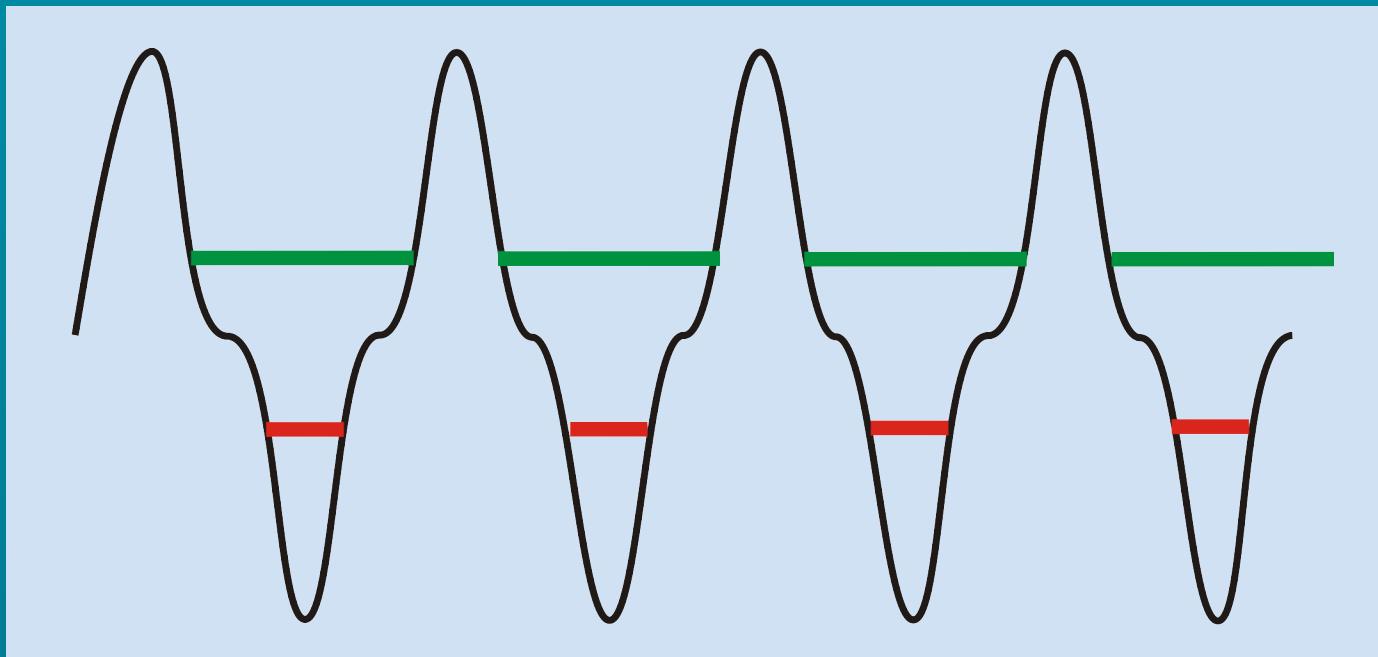
Realizing an Anderson Lattice in an optical lattice

- Idea 1: a tuned optical lattice



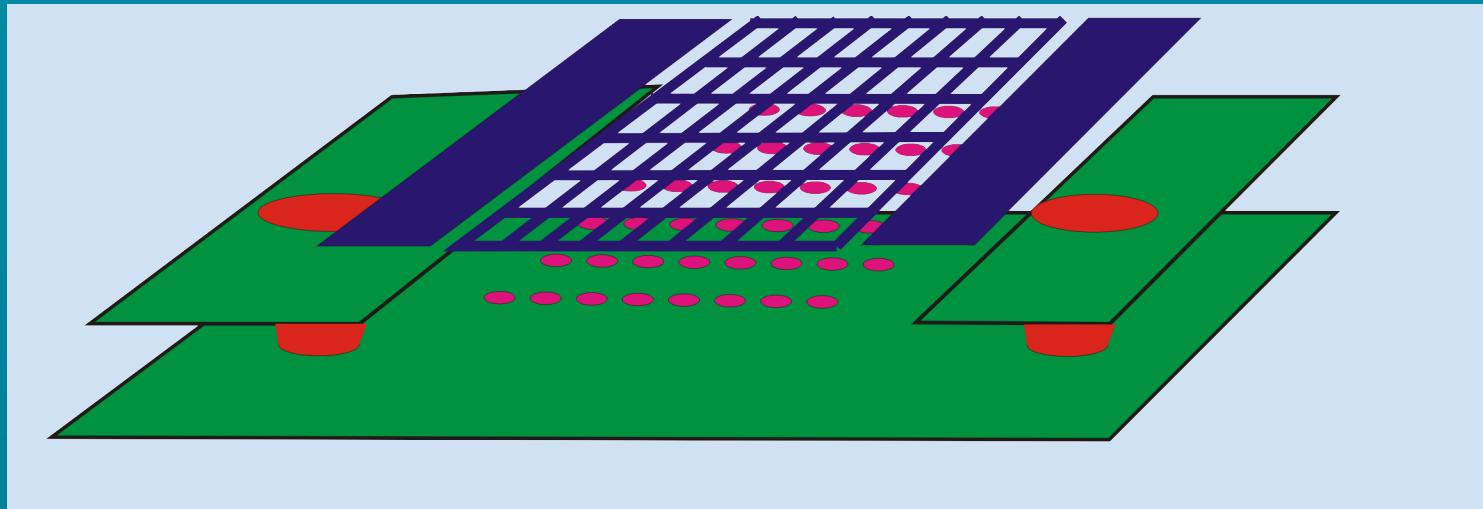
Realizing an Anderson Lattice in an optical lattice

- Idea 1: a shaped optical lattice (multi-chroic lasers)



Realizing an Anderson Lattice in an optical lattice

- Idea 2: A bilayer – with K. Bongs



Like the semiconductor bilayer but created with standing waves of light and an imaging system on the top layer.

Developments in experiments

- Can we make use of high precision optical frequencies?
 - Hansch and Hall – Nobel prize in 2005

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PHYSICAL REVIEW LETTERS

11 SEPTEMBER 2000

Optical Frequency Synthesizer for Precision Spectroscopy

R. Holzwarth, Th. Udem, and T. W. Hänsch

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany

J. C. Knight, W. J. Wadsworth, and P. St. J. Russell

Optoelectronics Group, Department of Physics, University of Bath, Claverton Down, Bath BA2 7AY, United Kingdom

(Received 4 May 2000)

We have used the frequency comb generated by a femtosecond mode-locked laser and broadened to more than an optical octave in a photonic crystal fiber to realize a frequency chain that links a 10 MHz radio frequency reference phase-coherently in one step to the optical region. By comparison with a similar frequency chain we set an upper limit for the uncertainty of this new approach to 5.1×10^{-16} . This opens the door for measurement and synthesis of virtually any optical frequency and is ready to revolutionize frequency metrology.

Summary and conclusions

- All the world of strong correlation physics seems to be contained in the Heavy Fermions
- Many open puzzles still remain
- Can some be solved by understanding heavy fermion look-alikes?
- Future trends:
 - Dilute Kondo lattices in semiconductors
 - Kondo lattices in cold atoms
 - New experiments

