

重费米子物理与重费米子超导基础

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中国科学院物理研究所



<http://hf.iphy.ac.cn>



2019年凝聚态理论前沿暑期讲习班 @青岛大学

中科院物理所重费米子研究组EX9简介



杨义峰 (百人计划、优青、青年拔尖)
强关联理论与计算

○ 强关联微观理论

- 重费米子微观理论、非平衡理论
- 非常规超导理论

○ 强关联数值计算

- 强关联计算: NRG / DMRG / DQMC 等
- 强关联材料计算: DFT + DMFT

课题组网页

<http://hf.iphy.ac.cn>

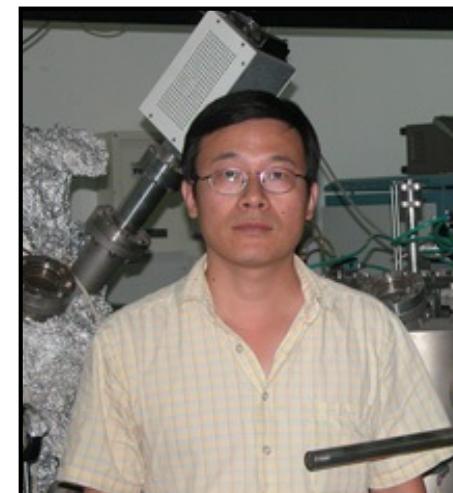


孙培杰 (百人计划)
极低温热电输运



EX9组部分学生合影

欢迎合作



曹立新 (百人计划)
超导和稀磁半导体薄膜生长与表征

第一部分

重费米子物理基础与历史简介

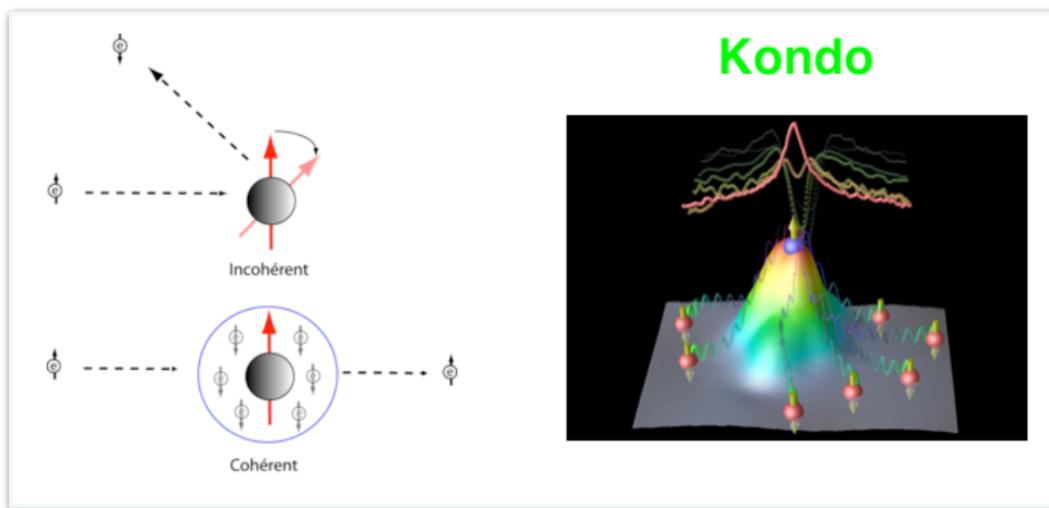
- 近藤效应简介
- 重费米子物理简介
- 重费米子物理的核心问题

第二部分

重费米子超导的发展与机理研究

- 重费米子超导与竞争序
- 重费米子超导机理与前沿问题

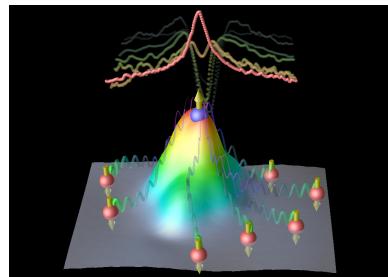
一、近藤效应简介



参考文献

A. C. Hewson, The Kondo problem to heavy fermions

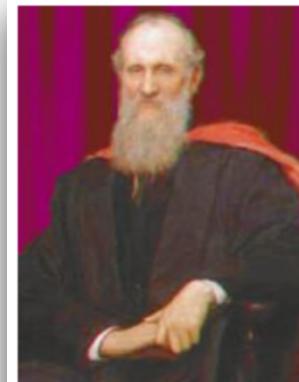
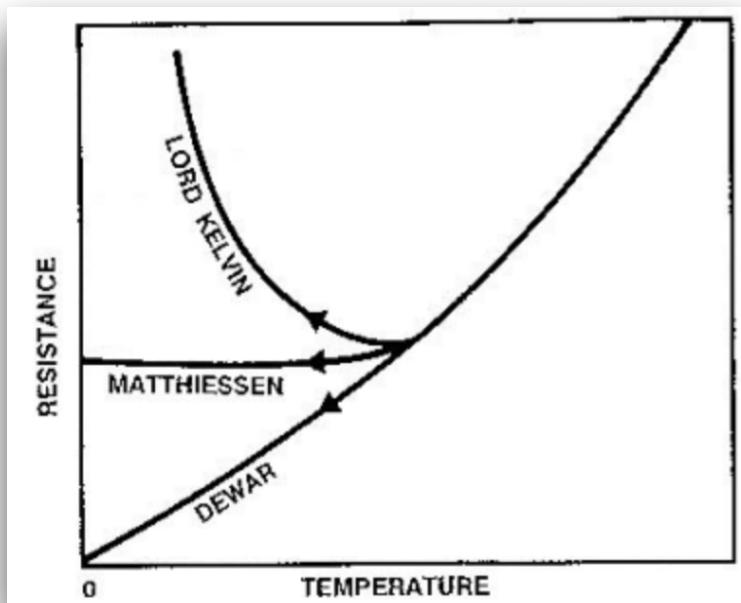
低温电阻行为



Historic development of Kondo

✓ (19th Century) Electron behavior @ $T \rightarrow 0$

InT	Kondo/2D WL
T	Non-Fermi liquid
T^2	Fermi liquid
T^5	Phonon scattering
$e^{-\Delta/T}$	Insulator/Semiconductor
$e^{T^{-1/n}}$	VRH (Semiconductor)
...	...

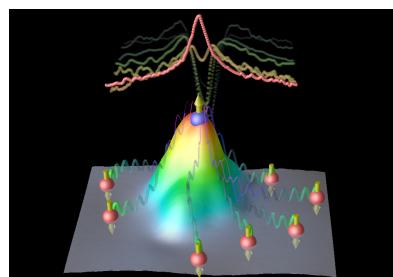


Kelvin



Dewar

低温电阻行为



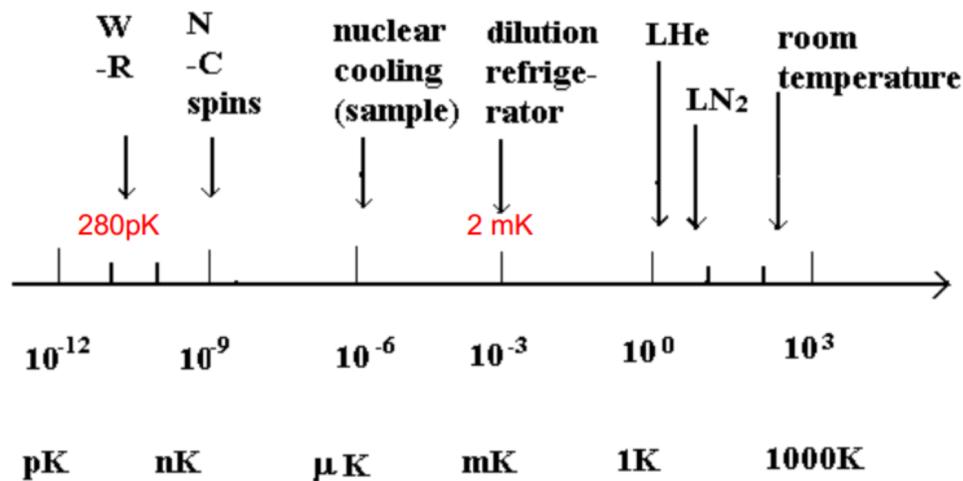
● Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement

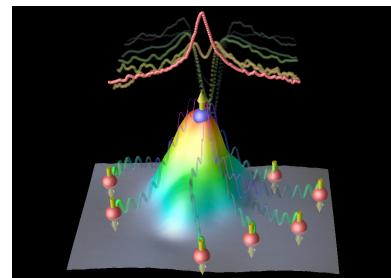
☺1895: 空气被液化 -192°C (81K)

☺1898: 氢气被液化 -253°C (20K)

☺1908: 氦气被液化 4.25K



超导的发现



● Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911 Superconductivity in Hg

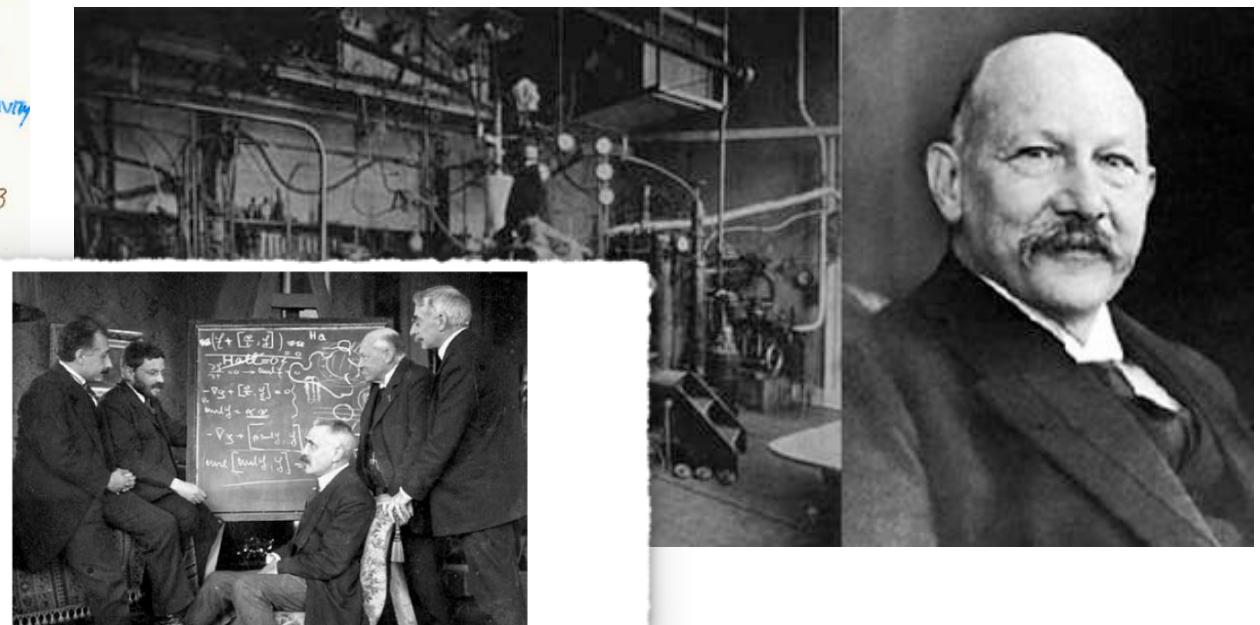
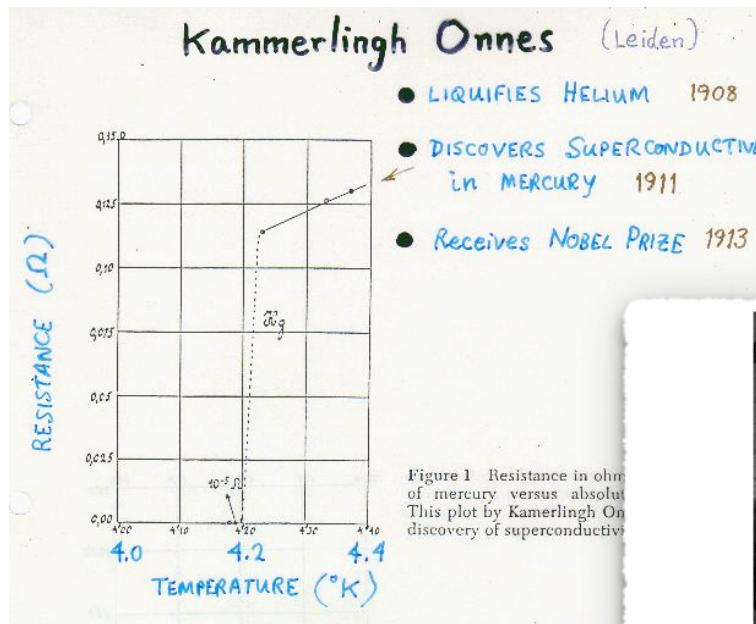
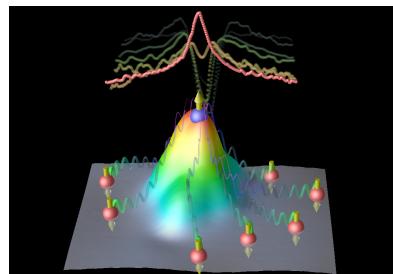


Fig. 1. Albert Einstein, Paul Ehrenfest, Paul Langevin, Heike Kamerlingh Onnes, and Pierre Weiss discussing superconductivity during the "Magnet-Woche" in Leiden in November 1920 (Photo: AIP)

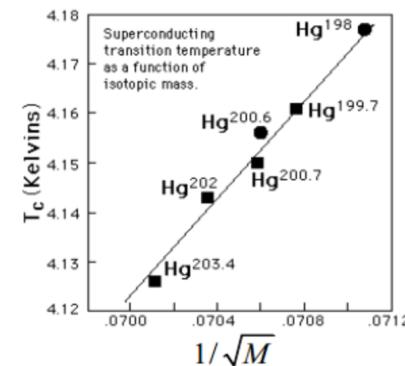
超导理论的建立



Historic development of Kondo

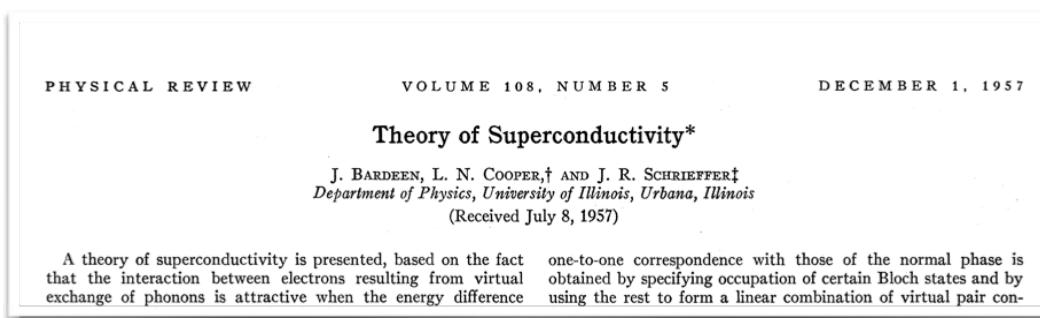
- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911 Superconductivity in Hg
- ✓ 1957 BCS Theory of Superconductivity

Smoking gun: Isotope effect

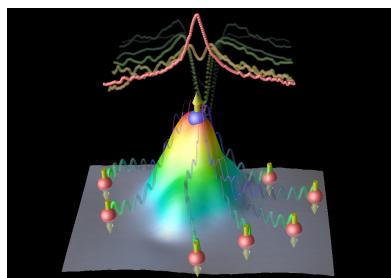


E. Maxwell, Phys. Rev. (1950)

C.A. Reynolds et al., Phys. Rev. 78 (1950).

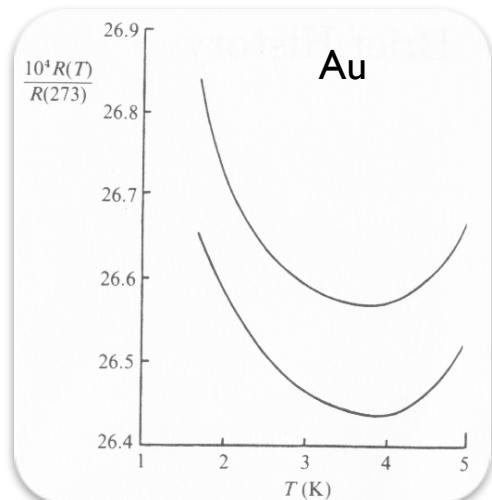


电阻极小值问题



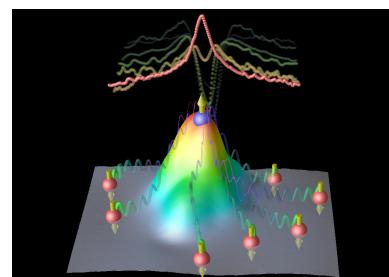
● Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity $\text{EXP} \rightarrow \text{THEO}$
- ✓ 1934 Resistivity minima



de Haas, de Boer and van den Berg, 1934

电子极小值问题



Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity EXP→THEO
- ✓ 1934 Resistivity minima

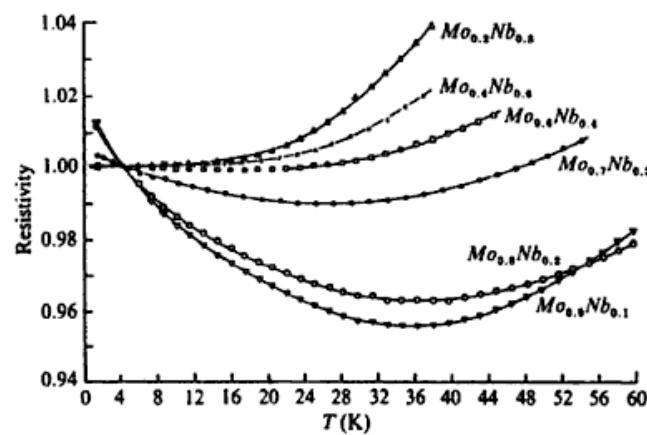


Figure 2.6 Resistance minima for Fe in a series of Mo-Nb alloys (from Sarachik et al., 1964). Compare the depths of the minima with the corresponding moments in figure 1.8.

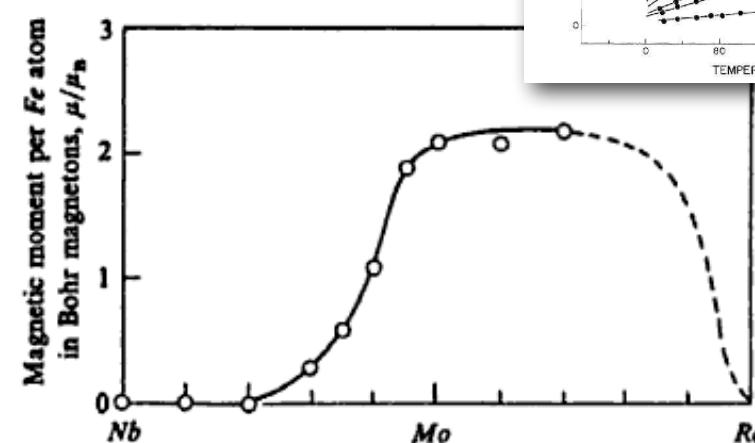


Figure 1.8. The magnetic moment in μ_B of Fe in various Mo-Nb and Mo-Re alloys as a function of alloy composition (Clogston et al., 1962).

Smoking gun: Resistance minima **Magnetic impurity**

问题的解决

Journal of the Physical Society of Japan
Vol. 74, No. 1, January, 2005, pp. 4–7
©2005 The Physical Society of Japan

SPECIAL TOPICS

Kondo Effect — 40 Years after the Discovery



Kondo Lattices and the Mott Metal–Insulator Transition

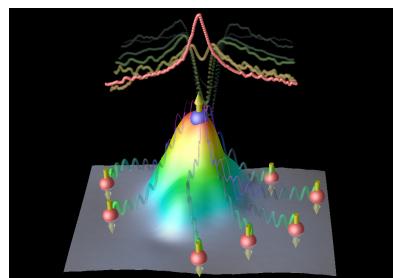
Ph. NOZIÈRES

*Laboratoire d'Etude des Propriétés Electroniques des Solides, Centre National de la Recherche Scientifique,
BP166, 38042 Grenoble Cedex 9, France*

(Received August 29, 2004)

In the Summer of 1964, I visited Urbana in order to work with my thesis advisor David Pines. The first person I met when entering the building was John Bardeen, just out of the seminar room. He was beaming when he told me “the long standing puzzle of the resistance minimum in metals is gone: we just heard a young Japanese theorist who has a beautiful explanation that is obviously the good one”. The Kondo effect was born—and John was right! But it was only the

近藤问题



Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

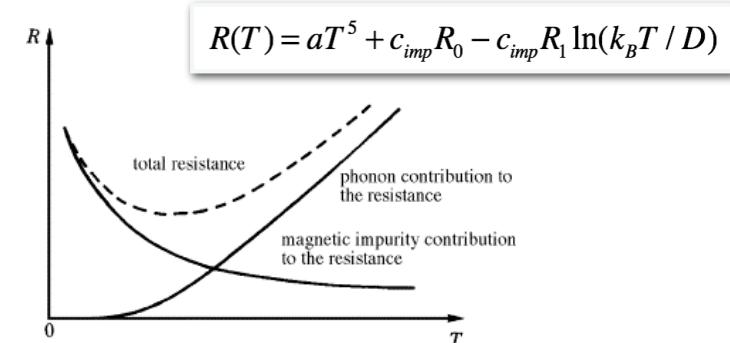
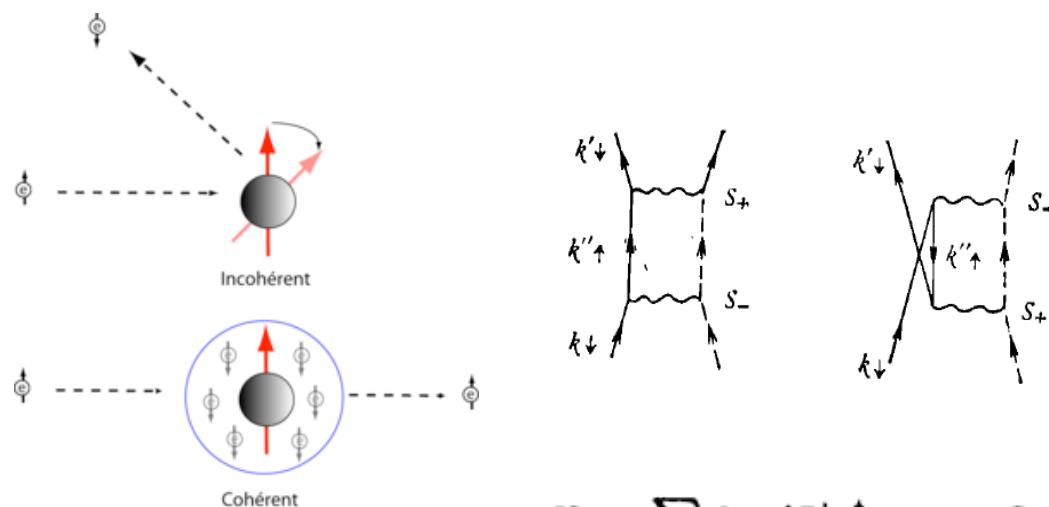
Electro-technical Laboratory
Nagatacho, Chiyodaku, Tokyo

(Received March 19, 1964)



Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity $\text{EXP} \rightarrow \text{THEO}$
- ✓ 1934 Resistivity minima
- ✓ 1964 Kondo effect (logarithmic resistivity)



$$H_{sd} = \sum_{\mathbf{k}, \mathbf{k}'} J_{\mathbf{k}, \mathbf{k}'} (S^+ c_{\mathbf{k}, \downarrow}^\dagger c_{\mathbf{k}' \uparrow} + S^- c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}' \downarrow} + S_z (c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}', \uparrow} - c_{\mathbf{k}, \downarrow}^\dagger c_{\mathbf{k}', \downarrow}))$$



A poor man's derivation of scaling laws for the Kondo problem

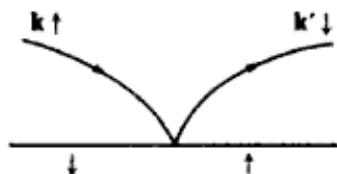
P. W. ANDERSON

Cavendish Laboratory,[†] Cambridge, England
and

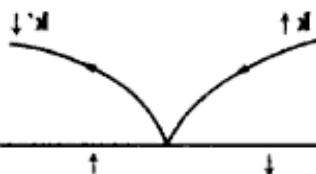
Bell Telephone Laboratories, Murray Hill, New Jersey, USA
MS. received 28th April 1970

Abstract. The scaling laws derived by a complicated space-time approach for the Kondo problem in previous work are rederived by a 'cutoff renormalization' technique used previously in the theory of superconductivity.

spin-flip scattering



spin-flip scattering



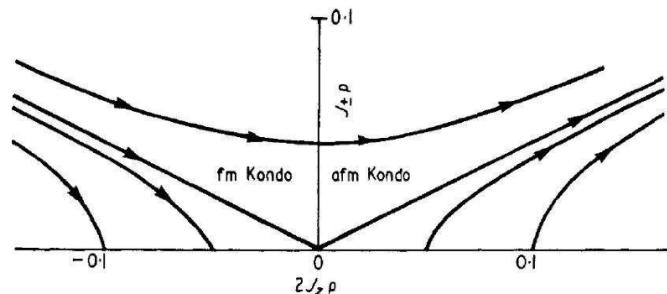
=

$$\sum_{k'} \frac{1}{\epsilon_{k'} - \mu} = \rho_0 \int_T^D \frac{d\epsilon}{\epsilon} = \rho_0 \ln \frac{D}{T}$$

logarithmic divergence

$$H_{sd} = \sum_{\mathbf{k}, \mathbf{k}'} J_{\mathbf{k}, \mathbf{k}'} (S^+ c_{\mathbf{k}, \downarrow}^\dagger c_{\mathbf{k}', \uparrow} + S^- c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}', \downarrow} + S_z (c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}', \uparrow} - c_{\mathbf{k}, \downarrow}^\dagger c_{\mathbf{k}', \downarrow}))$$

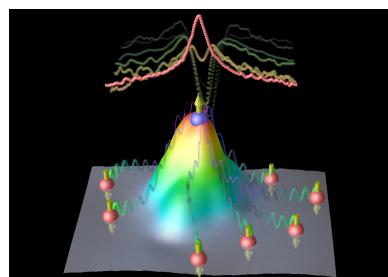
Poor Man's Scaling



$$k_B T_K = D e^{-\frac{1}{2Jz\rho}}$$

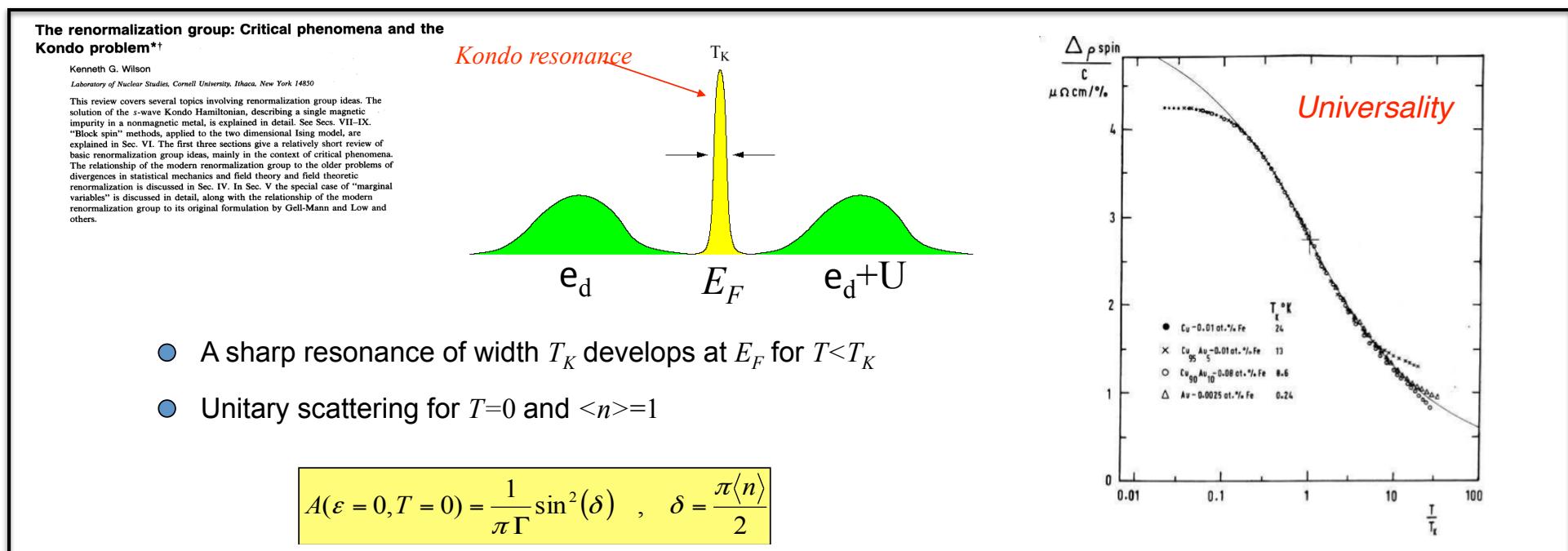
Rich man's scaling
Ken Wilson
Numerical Renormalization Group
Nobel Prize 1982 for
critical phenomena using renormalization group

近藤问题的解决

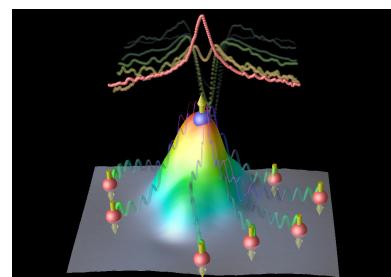


● Historic development of Kondo

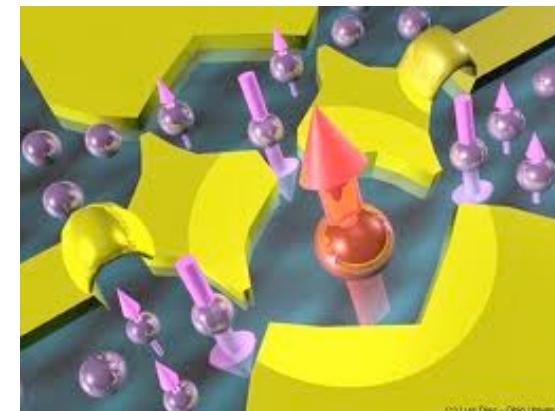
- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity $\text{EXP} \rightarrow \text{THEO}$
- ✓ 1934-1964 Kondo effect $\text{EXP} \rightarrow \text{THEO}$
- ✓ Many approaches: EOM, NRG, QMC ...



近藤问题的解决



Kondo cloud



Historic development of Kondo

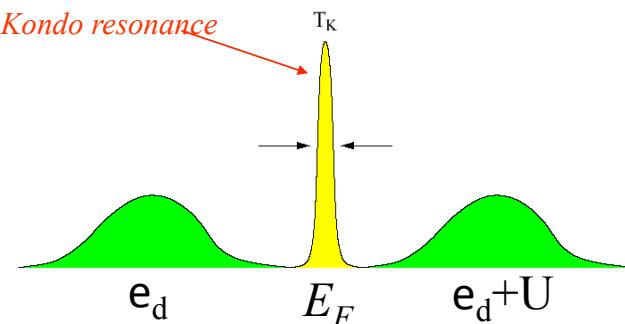
- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
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The renormalization group: Critical phenomena and the Kondo problem^{*†}

Kenneth G. Wilson

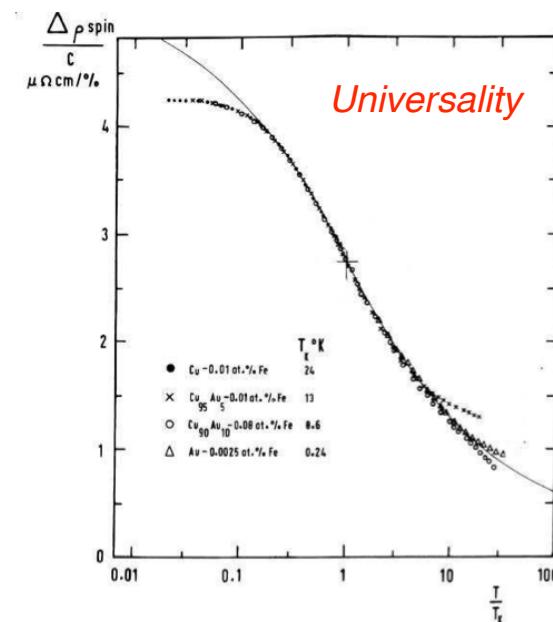
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

This review covers several topics involving renormalization group ideas. The solution of the s-wave Kondo Hamiltonian, describing a single magnetic impurity in a nonmagnetic metal, is explained in detail. See Secs. VII-IX. "Block spin" methods, applied to the two dimensional Ising model, are explained in Sec. VI. The first three sections give a relatively short review of basic renormalization group ideas, mainly in the context of critical phenomena. The relationship of the modern renormalization group to the older problems of divergences in statistical mechanics and field theory and field theoretic renormalization is discussed in Sec. IV. In Sec. V the special case of "marginal variables" is discussed in detail, along with the relationship of the modern renormalization group to its original formulation by Gell-Mann and Low and others.

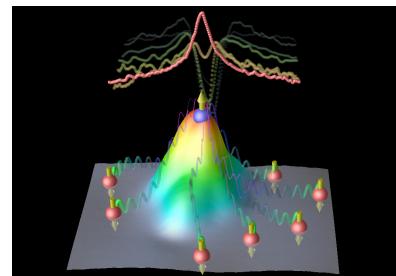


- A sharp resonance of width T_K develops at E_F for $T < T_K$
- Unitary scattering for $T=0$ and $\langle n \rangle = 1$

$$A(\varepsilon = 0, T = 0) = \frac{1}{\pi \Gamma} \sin^2(\delta) \quad , \quad \delta = \frac{\pi \langle n \rangle}{2}$$



近藤问题的解决



mean-field approach

● Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity $EXP \rightarrow THEO$
- ✓ 1934-1964 Kondo effect $EXP \rightarrow THEO$
- ✓ Many approaches: EOM, NRG, QMC ...

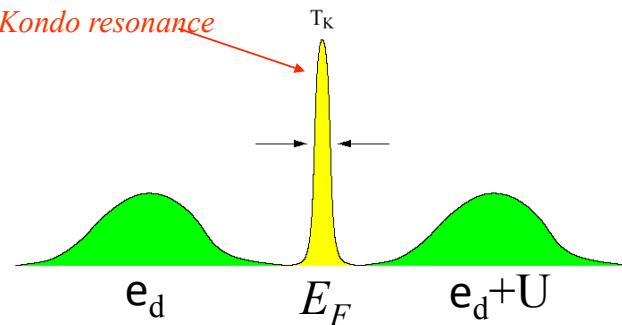
辅助理解：Anderson模型的图像

The renormalization group: Critical phenomena and the Kondo problem^{*†}

Kenneth G. Wilson

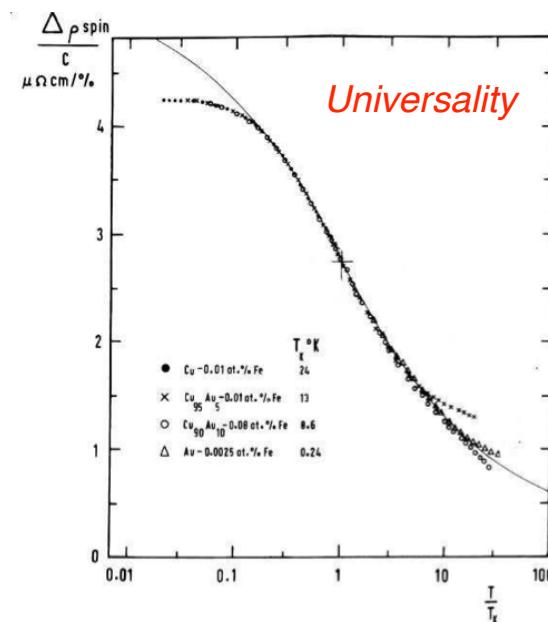
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

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- A sharp resonance of width T_K develops at E_F for $T < T_K$
- Unitary scattering for $T=0$ and $\langle n \rangle = 1$

$$A(\varepsilon = 0, T = 0) = \frac{1}{\pi \Gamma} \sin^2(\delta) \quad , \quad \delta = \frac{\pi \langle n \rangle}{2}$$



Nanotechnology has rekindled interest in the Kondo effect, one of the most widely studied phenomena in condensed-matter physics

Revival of the Kondo effect

Leo Kouwenhoven and Leonid Glazman

WHY would anyone still want to study a physical phenomenon that was discovered in the 1930s, explained in the 1960s and has been the subject of numerous reviews since the 1970s? Although the Kondo effect is a well known and widely studied phenomenon in condensed-matter physics, it continues to capture the imagination of experimentalists and theorists alike.

The effect arises from the interactions between a single magnetic atom, such as cobalt, and the many electrons in an otherwise non-magnetic metal. Such an impurity typically has an intrinsic angular momentum or "spin" that interacts with the electrons. As a result, the mathematical description of the system is a difficult many-body problem.

However, the Kondo problem is well defined, making it an attractive testing



The theory that describes the scattering of electrons from a localized magnetic impurity was initiated by the work of Jun Kondo in 1964

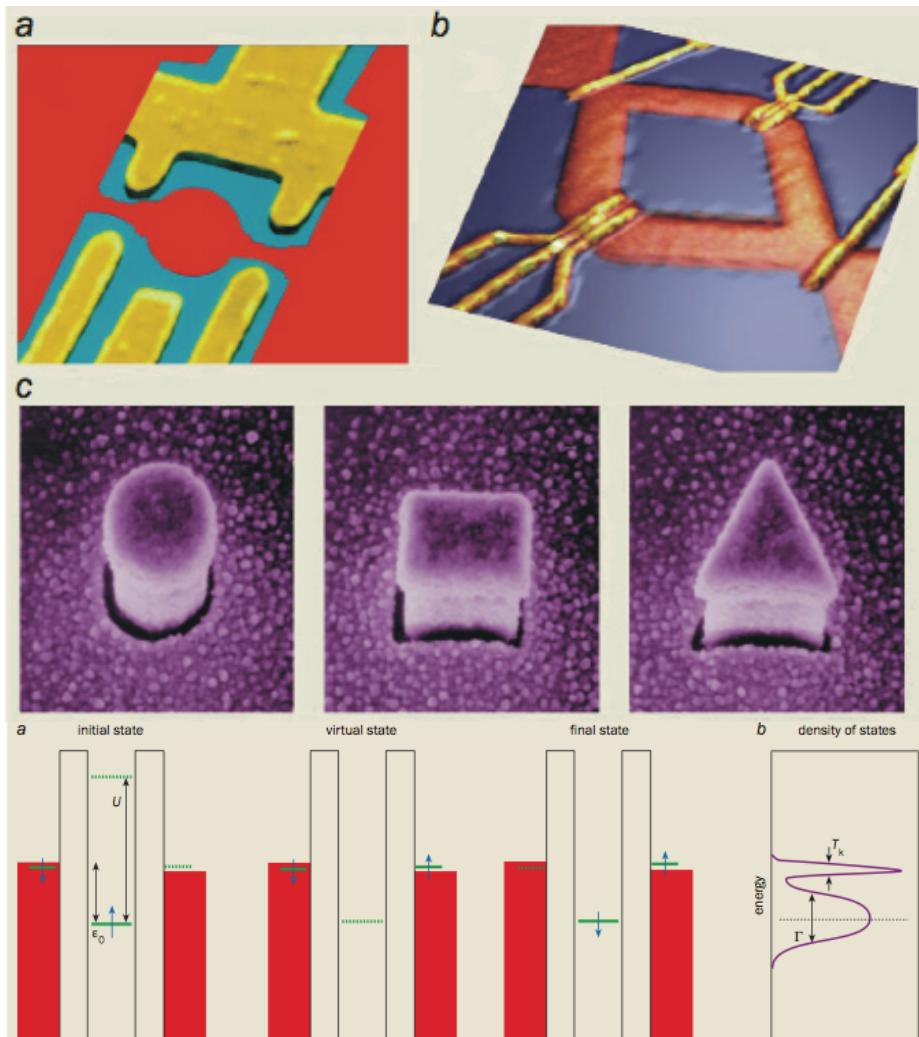
behave as a single entity. Indeed, superconductivity is a prime example of a many-electron phenomenon.

Other metals, like copper and gold, remain conducting and have a constant finite resistance, even at the lowest accessible temperatures. The value of the low-temperature resistance depends on the number of defects in the material. Adding defects increases the value of this "saturation resistance" but the character of the temperature dependence remains the same.

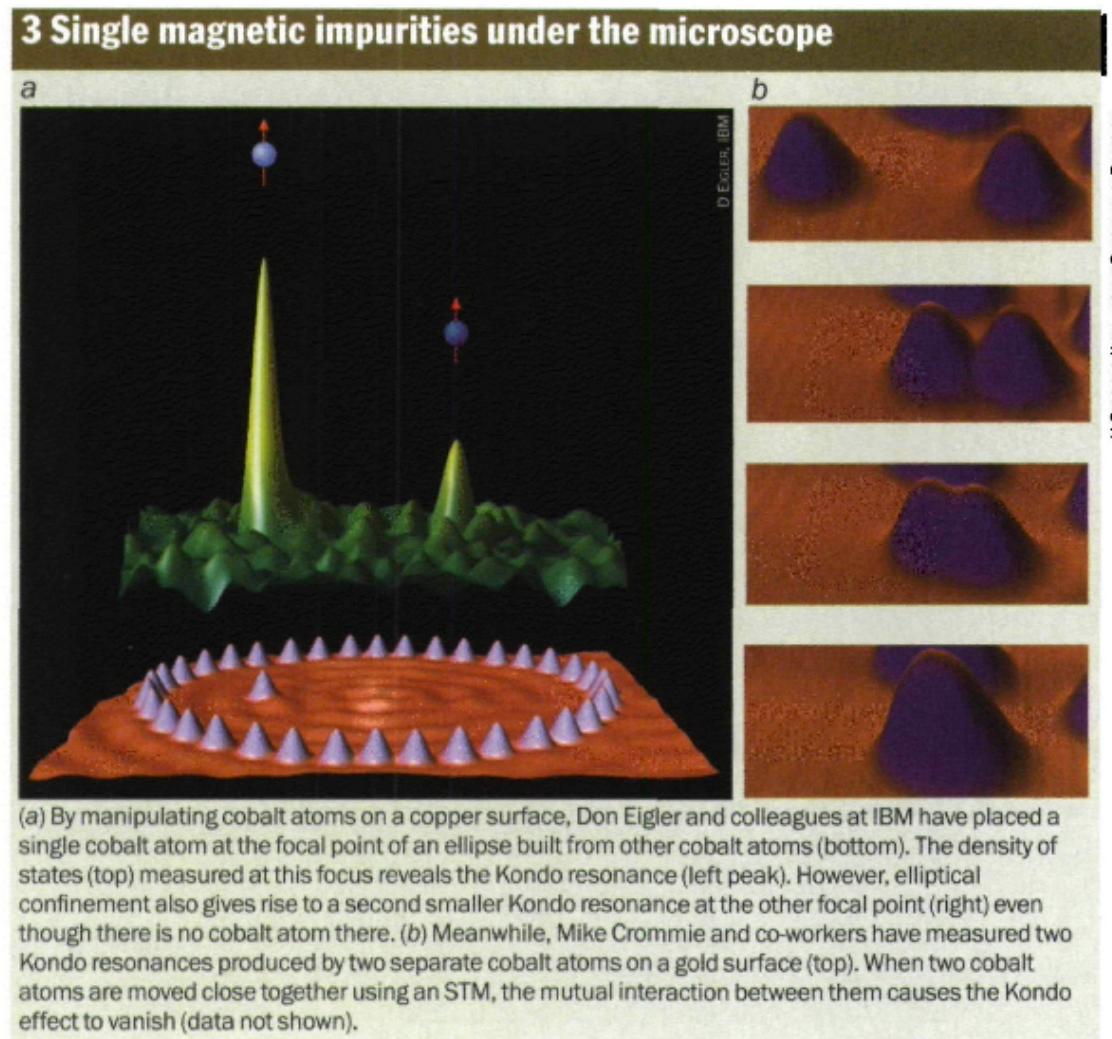
However, this behaviour changes dramatically when magnetic atoms, such as cobalt, are added. Rather than saturating, the electrical resistance increases as the temperature is lowered further. Although this behaviour does not involve a phase transition, the so-called Kondo temperature – roughly speaking the tem-

量子点系统中的近藤效应

Quantum dots

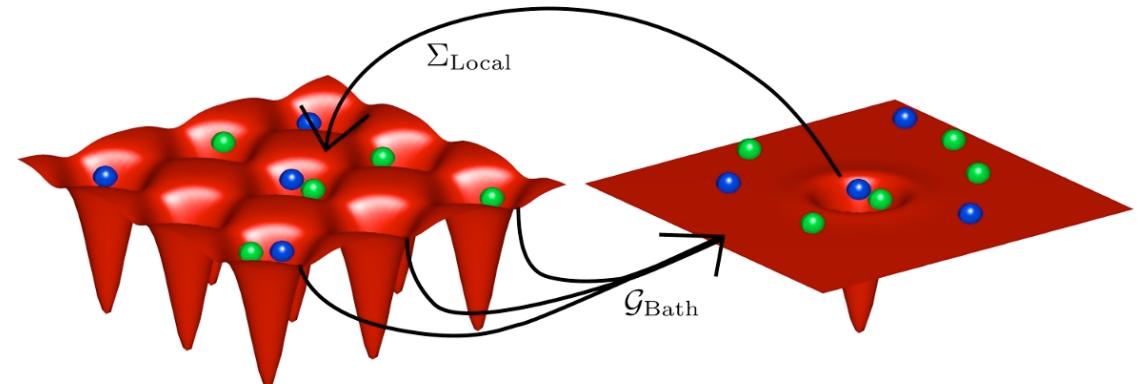


STM操纵原子



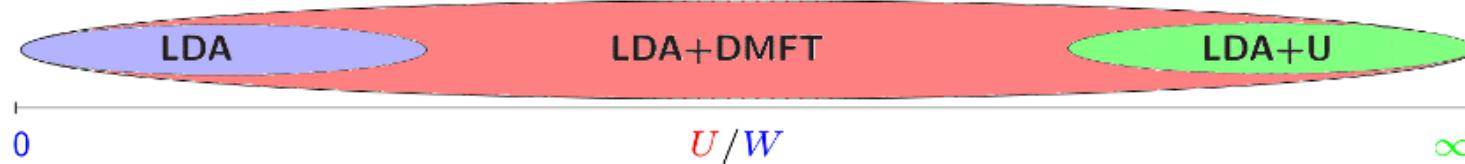
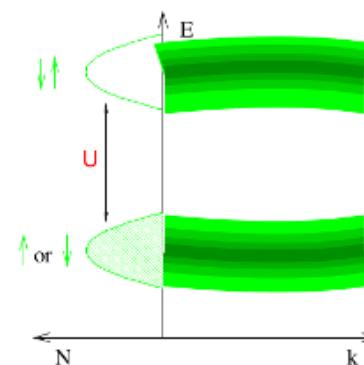
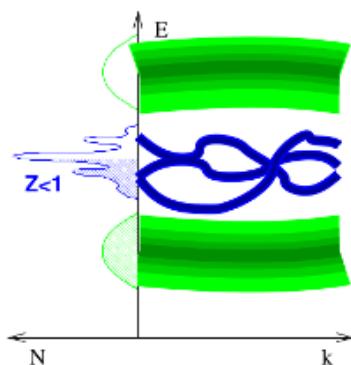
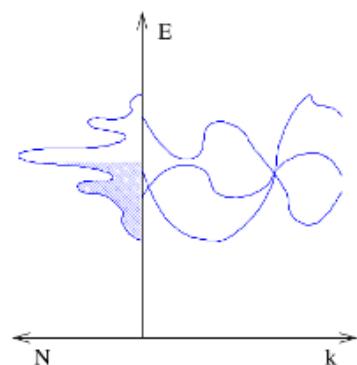
有趣的物理往往都发生在局域和巡游的边界：高温超导、庞磁阻等等

Coulomb X Hybridization

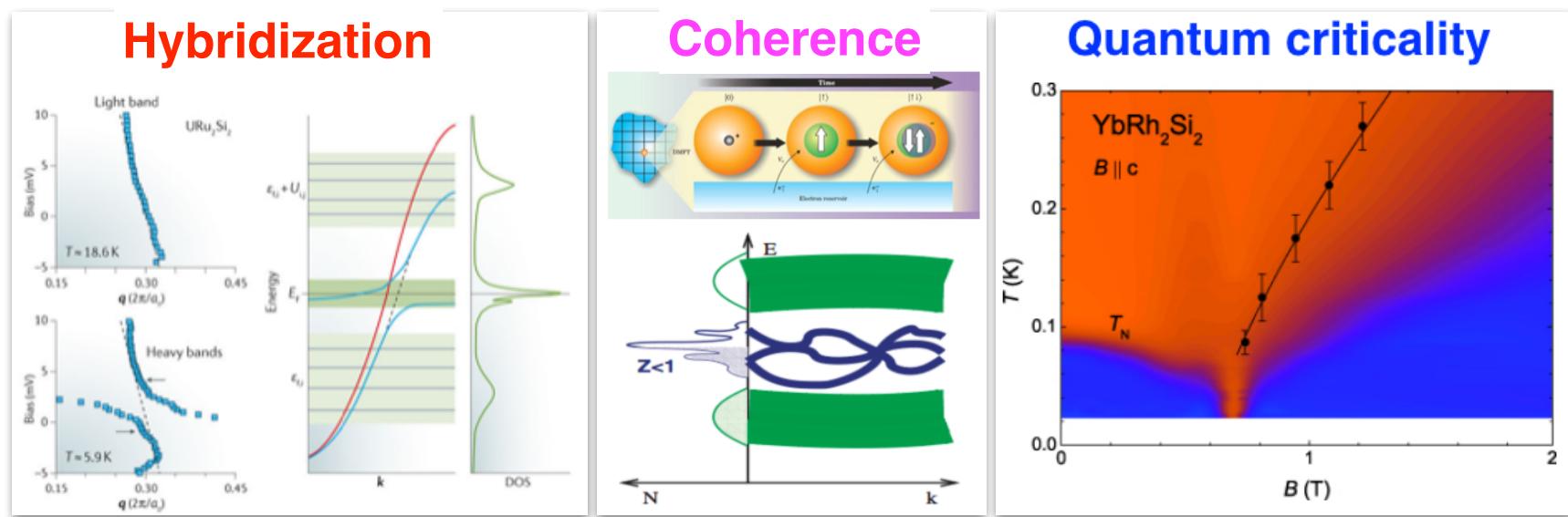


localization vs itinerancy

Dynamical mean-field theory



二、重费米子物理简介



参考文献

P. Coleman, Heavy Fermions: Electron at the Edge of Magnetism



SPIN EXCHANGE IN SUPERCONDUCTORS

B. T. Matthias, H. Suhl, and E. Corenzwit

Bell Telephone Laboratories,

Murray Hill, New Jersey

(Received July 15, 1958)

B. T. Matthias

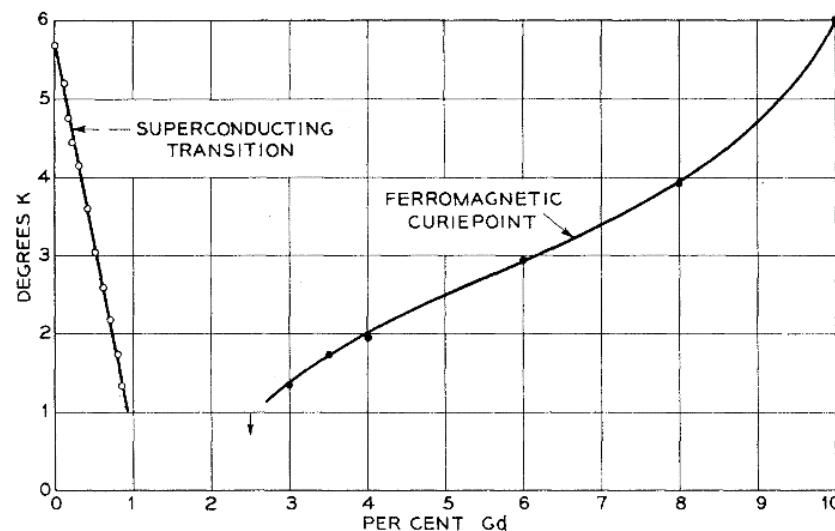


FIG. 3. Ferromagnetic and superconducting transition temperatures of solid solutions of gadolinium in lanthanum.

(B. T. Matthias, et al, PRL 1, 92 (1958))

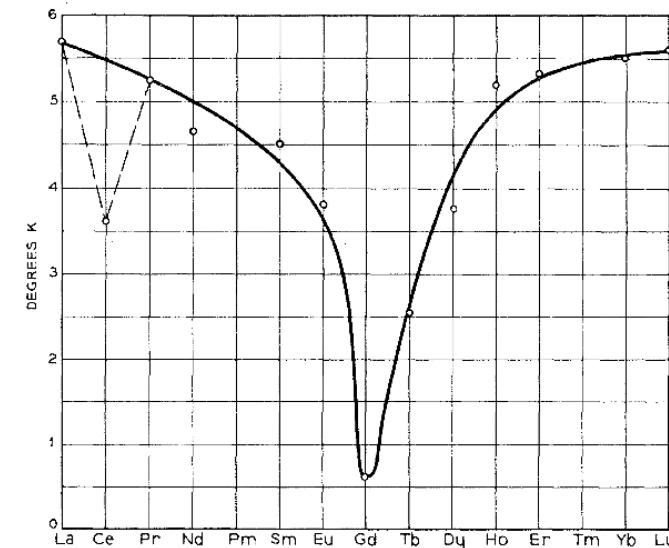
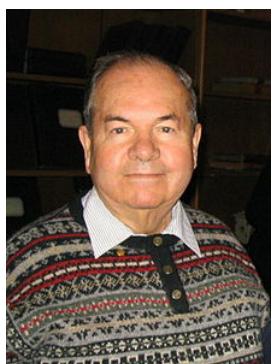


FIG. 2. Superconducting transition temperatures of 1 at % rare earth solid solutions in lanthanum.

57	178.91	58	140.12	59	140.91	60	144.24	61	(145)	62	150.36	63	151.96	64	157.26	65	158.93	66	162.50	67	164.93	68	167.26	69	168.93	70	173.04	71	174.57
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu															
Lanthanum	Cerium	Praseodymium	Neodymium	Promethium	Samarium	Europium	Gadolinium	Terbium	Dysprosium	Holmium	Erbium	Thulium	Ytterbium	Lutetium															
5d ¹⁰ 6s ²	4f ¹⁵ d ^{6s²}	4f ⁹ 6s ²	4f ¹⁰ 6s ²	4f ¹¹ 6s ²	4f ¹² 6s ²	4f ¹³ 6s ²	4f ¹⁵ 6s ²	4f ¹⁵ 6s ²	4f ¹⁶ 6s ²	5d ¹⁰ 6s ²																			

- **Magnetism and (conventional) SC are mutually exclusive.**
- **Anomaly depression in Ce .**

Why?



A. A. Abrikosov



L. P. Gor'kov

SOVIET PHYSICS JETP

VOLUME 12, NUMBER 6

JUNE, 1961

*CONTRIBUTION TO THE THEORY OF SUPERCONDUCTING ALLOYS WITH
PARAMAGNETIC IMPURITIES*

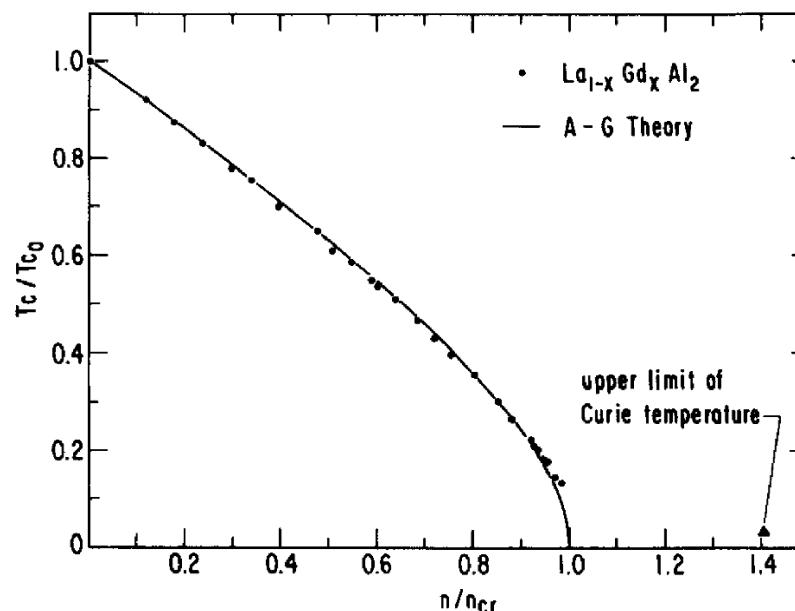
A. A. ABRIKOSOV and L. P. GOR'KOV

Institute for Physics Problems, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 25, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 1781-1796 (December, 1960)

(A. A. Abrikosov, L. P. Gor'kov, JETP, **12**, 1243(1961))



(M. B. Maple, Phys. Lett., **26A**, 513(1968))



Solid State Communications, Vol. 11, pp. 829-834, 1972. Pergamon Press. Printed in Great Britain

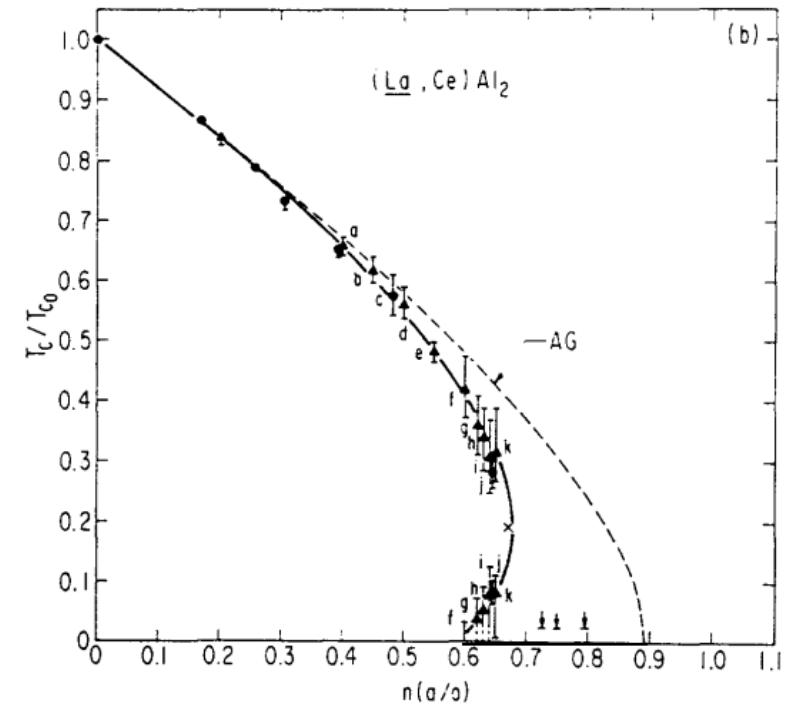
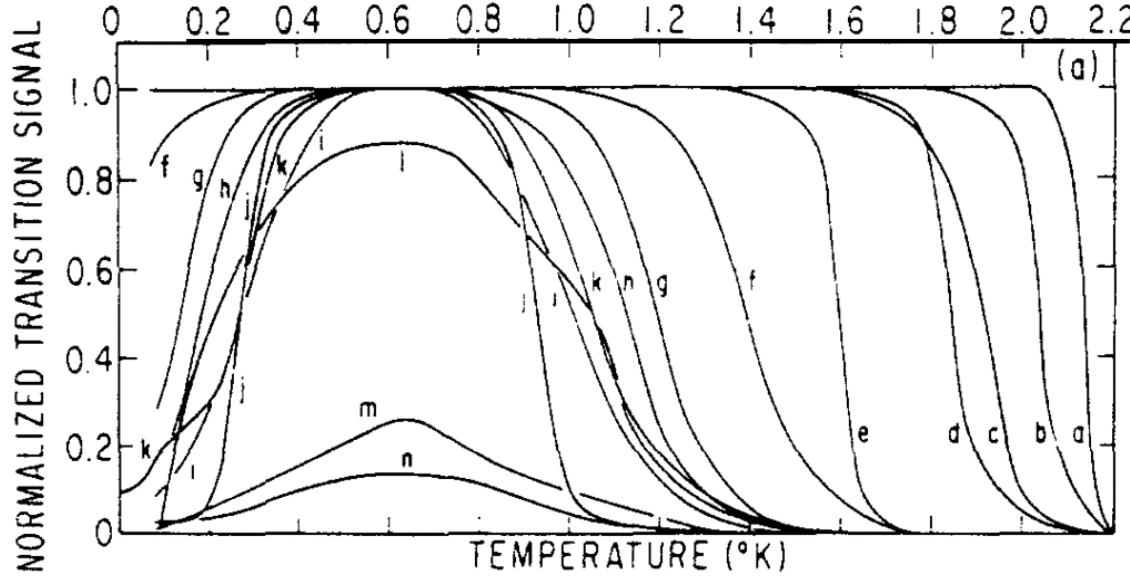
THE RE-ENTRANT SUPERCONDUCTING-NORMAL PHASE BOUNDARY
OF THE KONDO SYSTEM $(La,Ce)Al_2$ *

M.B. Maple, W.A. Fertig, A.C. Mota, L.E. DeLong, D. Wohlleben and R. Fitzgerald

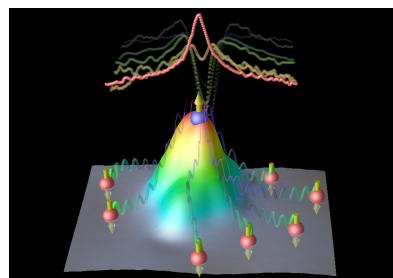
Institute for Pure and Applied Physical Sciences, University of California,
San Diego, La Jolla, California 92037

(Received 5 July 1972 by A.A. Maradudin)

M. B. Maple

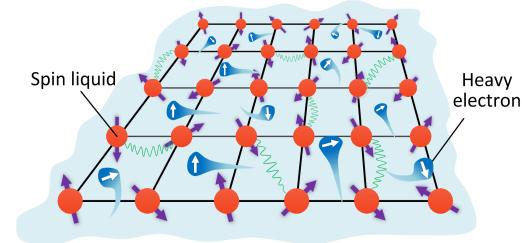
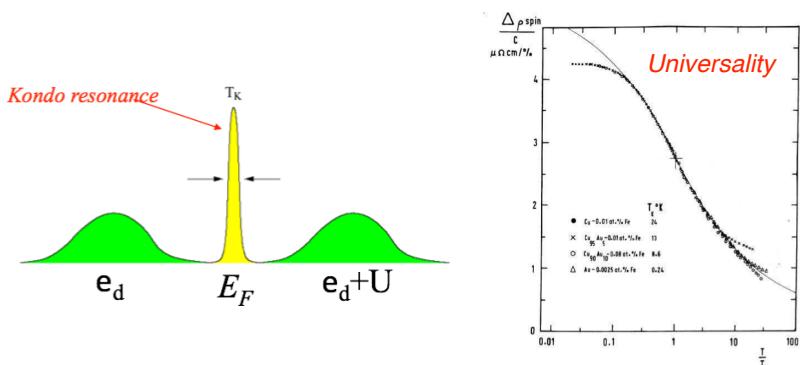


重费米子现象的发现



● Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity **EXP→THEO**
- ✓ 1934-1964 Kondo effect **EXP→THEO**
- ✓ Many approaches: EOM, NRG, QMC ...



● Historic development of heavy fermions

- ✓ CeAl_3 @ 1975 @ $\gamma = 1.62 \text{ J/mol K}^2$

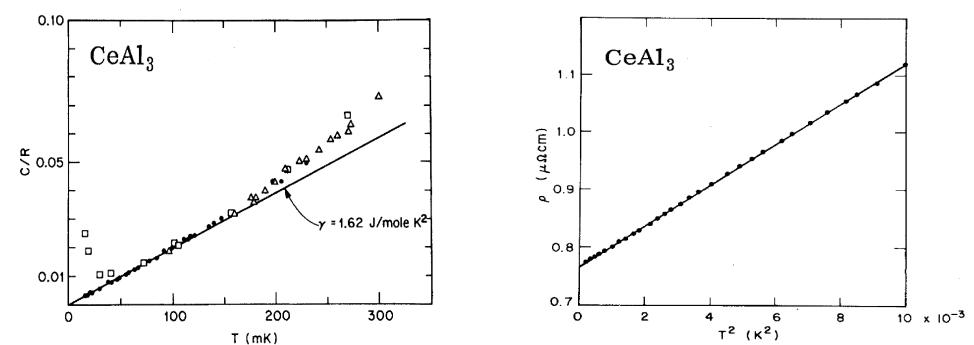
VOLUME 35, NUMBER 26 PHYSICAL REVIEW LETTERS 29 DECEMBER 1975

4f-Virtual-Bound-State Formation in CeAl_3 at Low Temperatures

K. Andres and J. E. Graebner
Bell Laboratories, Murray Hill, New Jersey 07974

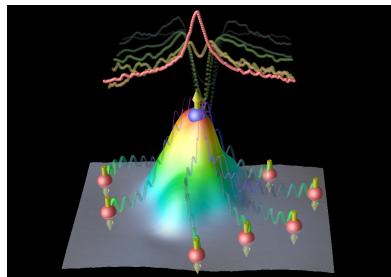
and

H. R. Ott
*Laboratorium für Festkörperphysik, Eidgenössische Technische Hochschule,
 Hönggerberg, Zürich, Switzerland*
(Received 25 August 1975)



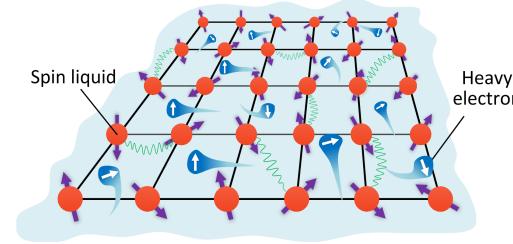
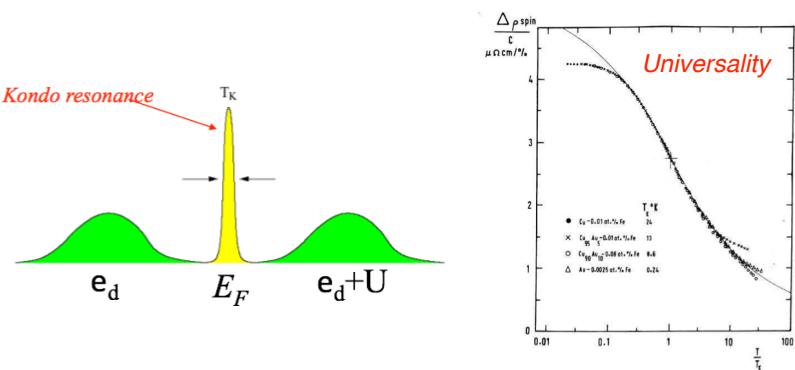
$$\gamma_{\text{Cu}} ; 0.7 \text{ mJ mol}^{-1} \text{ K}^{-2}$$

重费米子现象的发现



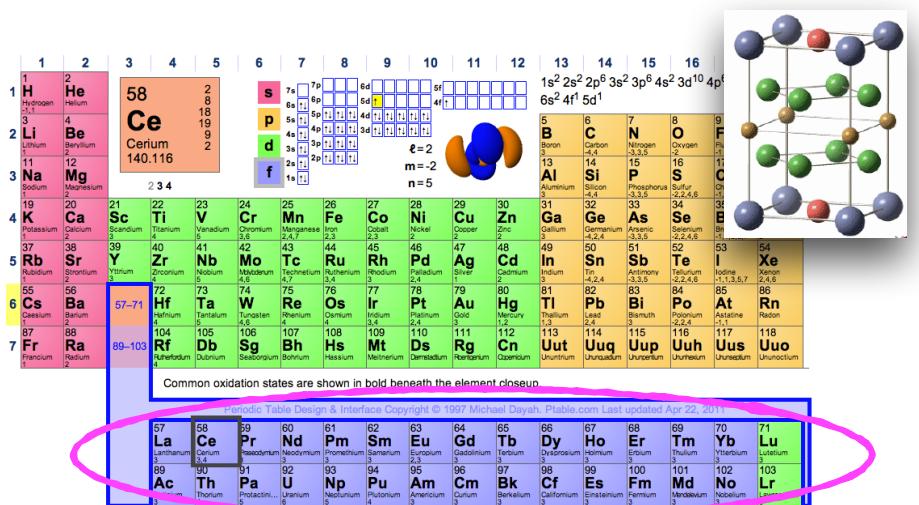
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- ✓ Liquification & Resistance measurement
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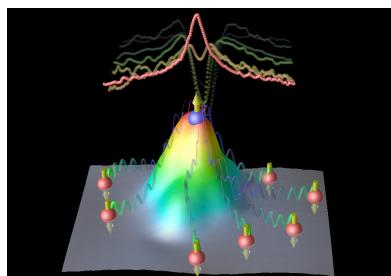
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- ✓ Ce, Yb, U, ..., 4f/5f intermetallics



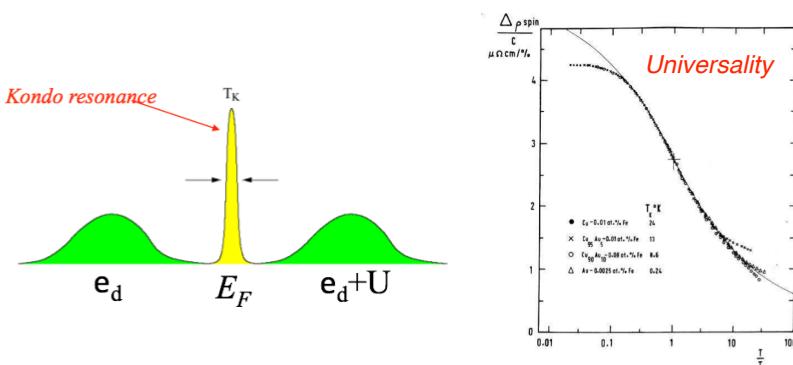
Some d-electron systems: LiV₂O₄, CaCu₃I₄O₁₂ ...

重费米子现象的发现

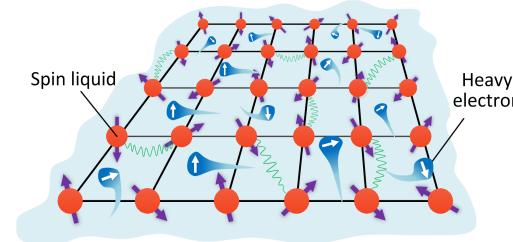


● Historic development of Kondo

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- ✓ 1911-1957 Superconductivity $\text{EXP} \rightarrow \text{THEO}$
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- ✓ Many approaches: EOM, NRG, QMC ...

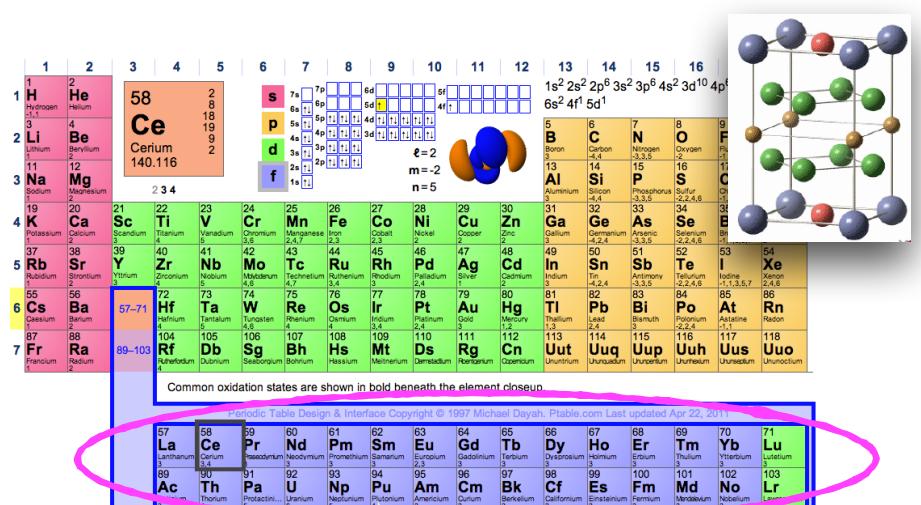


金/铜等

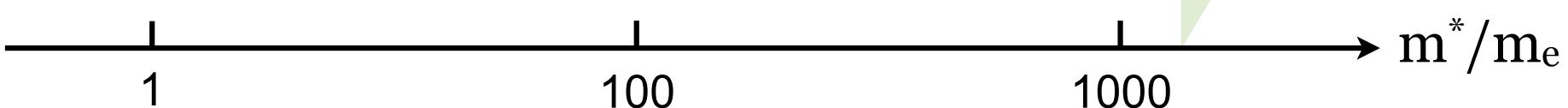


● Historic development of heavy fermions

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- ✓ Ce, Yb, U, ..., 4f/5f intermetallics



CeAl₃ ~ 2000



重费米子超导的发现

PHYSICAL REVIEW B VOLUME 11, NUMBER 1 1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides

E. Bucher,* J. P. Maita, G. W. Hull, R. C. Fulton, and A. S. Cooper
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 14 March 1974)

UBe₁₃

We tried to detect any possible magnetic ordering below 1 K. Instead we found a sharp superconducting transition at 0.97 K, which was reduced by about 0.3 K only in a field of 60 kOe. This suggests that the superconductivity is not an intrinsic property of UBe₁₃, but probably linked with precipitated filaments. Subsequent powdering did not shift nor reduce the superconducting signal, although calibration with a Pb cylinder showed that the signal of UBe₁₃ was only about 50% of the expected full signal. From the fact that none of the other MBe₁₃ phases showed superconductivity down to 0.45 K, one is tempted to conclude that the superconductivity and perhaps also the susceptibility tail at low temperature is due to precipitated U filaments.

Z. Physik B 31, 7–17 (1978)

Transport Properties of LaCu₂Si₂ and CeCu₂Si₂ Between 1.5 K and 300 K***

W. Franz, A. Grickel, F. Steglich, and D. Wohleben
II. Physikalisches Institut der Universität zu Köln, Köln, Fed. Rep. Germany

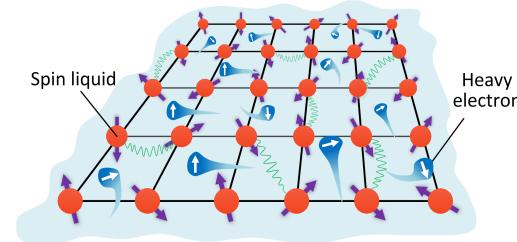
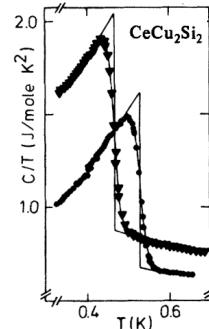
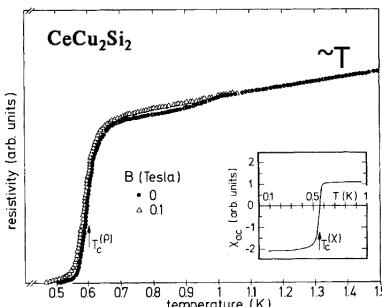
Received May 23, 1978

CeCu₂Si₂

* The resistivity measurement of W. Lieke below 1.5 K showed a continuous drop down to about 0.6 K and then a superconducting transition. The superconductivity could be suppressed at 0.3 K by application of a magnetic field of 3 T. At this field the resistivity was 41 $\mu\Omega$ cm and it did not change upon further increase of the field. We therefore consider 41 $\mu\Omega$ cm as representative of the residual resistivity of this sample. In order to check whether the superconductivity was a bulk property of CeCu₂Si₂ or due to a second phase forming a network through grain boundaries the specific heat was measured down to 0.3 K by C.D. Bredl and the static Meissner effect (on bulk and powdered samples) in a SQUID magnetometer by R.F. Hoyt and A.C. Mota down to 30 mK. Both measurements indicate the absence of bulk superconductivity. According to the static Meissner effect less than 0.1 % of the sample volume is superconducting.



(F. Steglich)



Historic development of heavy fermions

- ✓ CeAl₃ $\gamma=1.62 \text{ J/mol K}^2$ @ 1975
- ✓ CeCu₂Si₂ $T_c=0.5 \text{ K}$ @ 1979

PHYSICAL REVIEW LETTERS

17 DECEMBER 1979

Specific Heat in the Presence of Strong Pauli Paramagnetism: CeCu₂Si₂

F. Steglich
Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

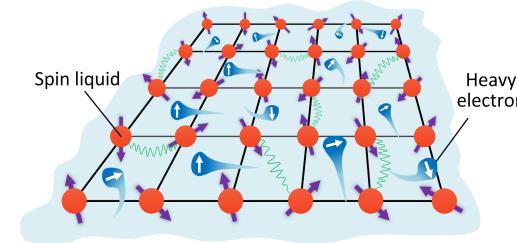
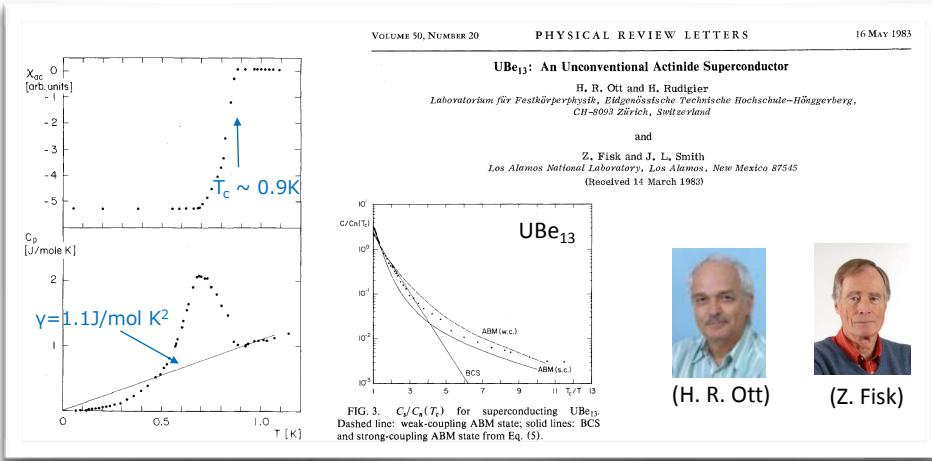
J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz
II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

H. Schäfer
Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany
(Received 10 August 1979; revised manuscript received 7 November 1979)

The size of the specific-heat jump at T_c , in proportion to γT_c , suggests that Cooper-pair states are formed by these heavy fermions. Since the Debye temperature, Θ , is of the order of 200 K,⁵ we find $T_c < T_F < \Theta$ with $T_c/T_F \approx T_F/\Theta \approx 0.05$. This suggests that CeCu₂Si₂ (i) behaves as a “high-temperature superconductor” and (ii) cannot be described by conventional theory of superconductivity which assumes a typical phonon frequency $k_B\Theta/h \ll k_B T_F/h$, the characteristic frequency of the fermions.

重费米子超导的发现



Historic development of heavy fermions

- ✓ CeAl₃ $\gamma=1.62 \text{ J/mol K}^2$ @ 1975
- ✓ CeCu₂Si₂ $T_c=0.5 \text{ K}$ @ 1979

NUMBER 25 PHYSICAL REVIEW LETTERS 17 DECEMBER 1979

Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu₂Si₂

F. Steglich

Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz
II. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

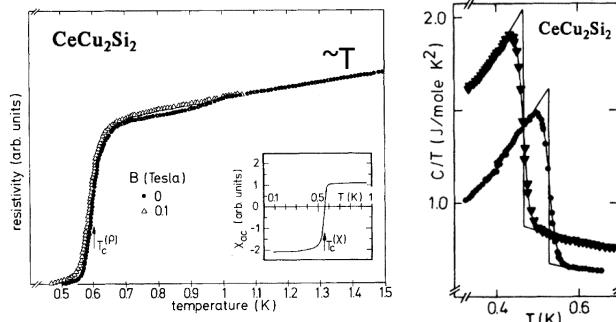
and

H. Schäfer

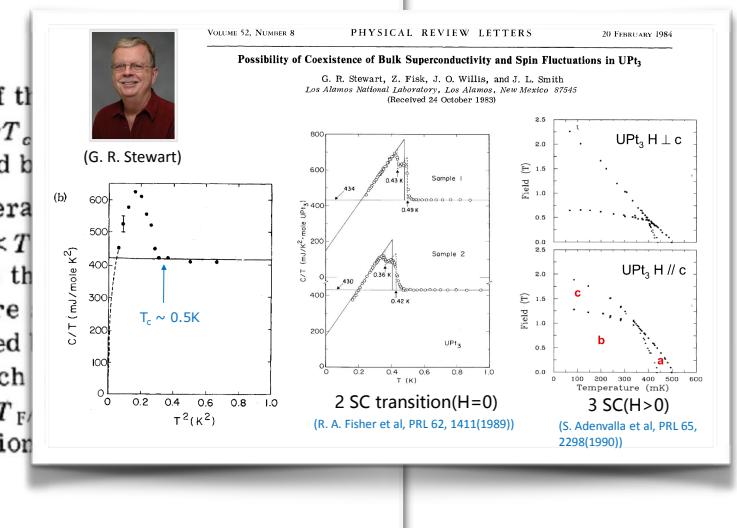
Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany
(Received 10 August 1979; revised manuscript received 7 November 1979)



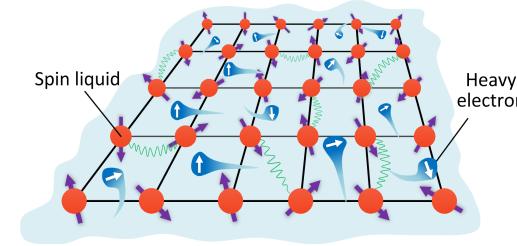
(F. Steglich)



The size of the T_c , in proportion to γT_c , pair states are formed by the Debye temperature of 200 K,⁵ we find $T_c < T_c^*$ ≈ 0.05 . This suggests that as a ‘high-temperature’ (ii) cannot be described by superconductivity which frequency $k_B\Theta/h \ll k_B T_F$ frequency of the fermion



重费米子超导体汇总



● Historic development of heavy fermions

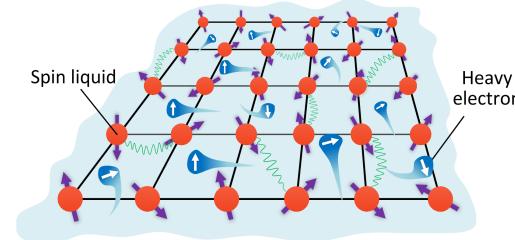
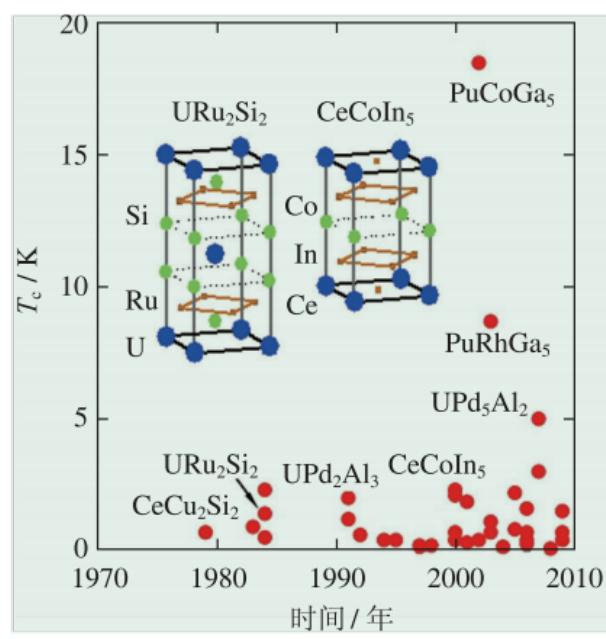
- ✓ $\text{CeAl}_3 \gamma = 1.62 \text{ J/mol K}^2$ @ 1975
- ✓ $\text{CeCu}_2\text{Si}_2 T_c = 0.5 \text{ K}$ @ 1979

Serials	Compounds	T_c (K)	γ	nodes
Ce ₂ X ₂ (AFM SC)	CeCu ₂ Si ₂	0.6-0.7	1000	No
	CeCu ₂ Ge ₂	0.64(10.1GPa)	200	-
	CePd ₂ Si ₂	0.43 (3GPa)	65	-
	CeRh ₂ Si ₂	0.42 (1.06GPa)	23	-
	CeAu ₂ Si ₂	2.5K(22.5GPa)	-	-
	CeNi ₂ Ge ₂	0.3	350	-
Ce _n M _m In _{3n+2m} (AFM SC)	CeIn ₃	0.23 (2.46GPa)	140	Line
	CelrIn ₅	0.4	750	Line
	CeCoIn ₅	2.3	250	Line
	CeRhIn ₅	2.4 (2.3GPa)	430	-
	CePt ₂ In ₇	2.3 (3.1GPa)	340	-
	Ce ₂ RhIn ₈	2.0 (2.3GPa)	400	-
	Ce ₂ PdIn ₈	0.68	550	Line
	Ce ₂ CoIn ₈	0.4	500	-
	Ce ₃ PdIn ₁₁	0.42	290	-
	CePt ₃ Si	0.75	390	Line
Ce-based non-centresymmetric SC	CelrSi ₃	1.65 (2.5GPa)	120	-
	CeRhSi ₃	1.0 (2.6GPa)	120	-
	CeCoGe ₃	0.69 (6.5GPa)	32	-
	CeNiGe ₃	0.43 (6.8GPa)	45	-
Other Ce-based AFM SC	Ce ₂ Ni ₃ Ge ₅	0.26 (4.0GPa)	90	-
	CePd ₅ Al ₂	0.57 (10.8GPa)	56	-

Serials	Compounds	T_c (K)	γ	nodes
Ce ₂ X ₂ (AFM SC)	CeCu ₂ Si ₂	0.6-0.7	1000	No
	CeCu ₂ Ge ₂	0.64(10.1GPa)	200	-
	CePd ₂ Si ₂	0.43 (3GPa)	65	-
	CeRh ₂ Si ₂	0.42 (1.06GPa)	23	-
	CeAu ₂ Si ₂	2.5K(22.5GPa)	-	-
	CeNi ₂ Ge ₂	0.3	350	-
Ce _n M _m In _{3n+2m} (AFM SC)	CeIn ₃	0.23 (2.46GPa)	140	Line
	CelrIn ₅	0.4	750	Line
	CeCoIn ₅	2.3	250	Line
	CeRhIn ₅	2.4 (2.3GPa)	430	-
	CePt ₂ In ₇	2.3 (3.1GPa)	340	-
	Ce ₂ RhIn ₈	2.0 (2.3GPa)	400	-
	Ce ₂ PdIn ₈	0.68	550	Line
	Ce ₂ CoIn ₈	0.4	500	-
	Ce ₃ PdIn ₁₁	0.42	290	-
	CePt ₃ Si	0.75	390	Line
Ce-based non-centresymmetric SC	CelrSi ₃	1.65 (2.5GPa)	120	-
	CeRhSi ₃	1.0 (2.6GPa)	120	-
	CeCoGe ₃	0.69 (6.5GPa)	32	-
	CeNiGe ₃	0.43 (6.8GPa)	45	-
Other Ce-based AFM SC	Ce ₂ Ni ₃ Ge ₅	0.26 (4.0GPa)	90	-
	CePd ₅ Al ₂	0.57 (10.8GPa)	56	-

重费米子超导体汇总

Serials	Compound	T_c / K	
CeM_2X_2 (AFM SC)	$CeCu_2Si_2$	0.5	
	$CeCu_2Ge_2$	0.5	
	$CePd_2Si_2$	1.5	
	$CeRh_2Si_2$	1.5	
	$CeAu_2Si_2$	1.5	
$Ce_nM_mIn_{3n+2m}$ (AFM SC)	$CeIn_3$	0.4	Line
	$CelrIn_5$	0.4	750
	$CeCoIn_5$	2.3	250
	$CeRhIn_5$	2.4 (2.3GPa)	430
	$CePt_2In_7$	2.3 (3.1GPa)	340
	Ce_2RhIn_8	2.0 (2.3GPa)	400
	Ce_2PdIn_8	0.68	550
	Ce_2CoIn_8	0.4	500
	Ce_3PdIn_{11}	0.42	290
Ce-based non-centresymmetric SC	$CePt_3Si$	0.75	390
	$CelrSi_3$	1.65 (2.5GPa)	120
	$CeRhSi_3$	1.0 (2.6GPa)	120
	$CeCoGe_3$	0.69 (6.5GPa)	32
Other Ce-based AFM SC	$CeNiGe_3$	0.43 (6.8GPa)	45
	$Ce_2Ni_3Ge_5$	0.26 (4.0GPa)	90
	$CePd_5Al_2$	0.57 (10.8GPa)	56



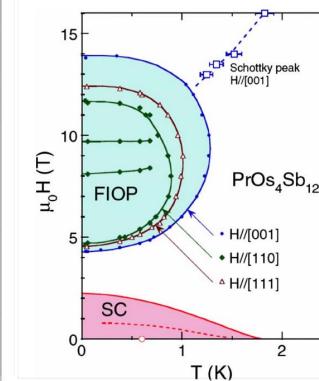
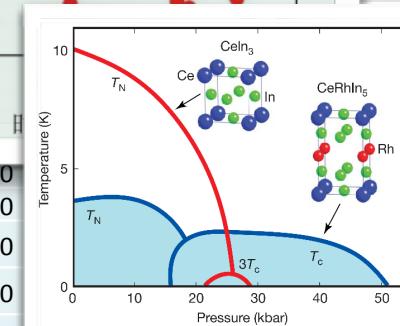
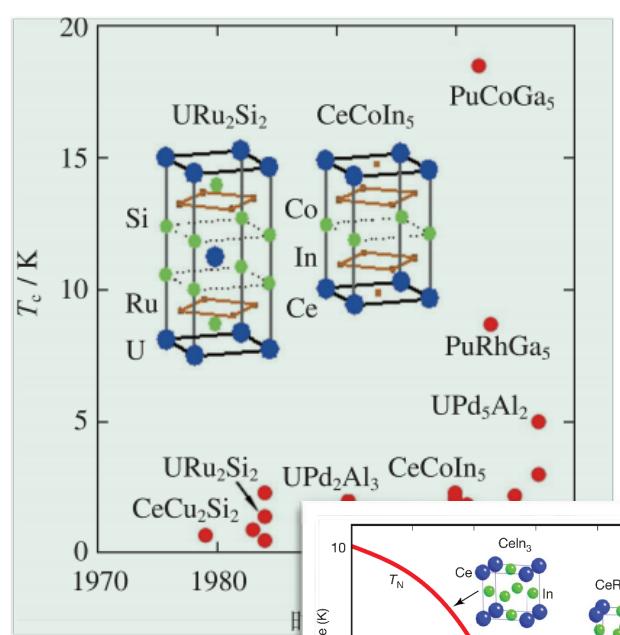
Historic development of heavy fermions

- ✓ $CeAl_3 \gamma=1.62 \text{ J/mol K}^2$ @ 1975
- ✓ $CeCu_2Si_2 T_c=0.5 \text{ K}$ @ 1979

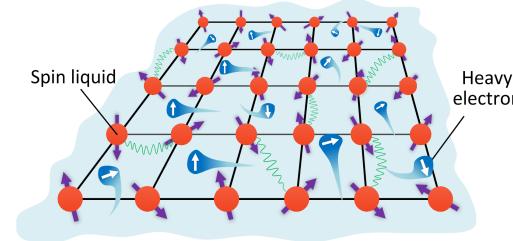
Serials	Compounds	$T_c (K)$	γ	nodes
I AFM SC	UPd_2Al_3	2.0	210	Line
	UNi_2Al_3	1.06	120	-
	UBe_{13}	0.95	1000	No
	U_6Fe	3.8	157	-
	UPt_3	0.53, 0.48	440	Line, Point
	UGe_2	0.8 (~1.2GPa)	34	Line
	$URhGe$	0.3	164	-
	$UCoGe$	0.6	57	Point
	UIr	0.15 (~2.6GPa)	49	-
	U_2PtC_2	1.47	150	-
Hidden order SC	URu_2Si_2	1.5	70	Line
Pr-based SC	$PrOs_4Sb_{12}$	1.85	500	点
	$PrTi_2Al_{20}$	0.2, 1.1(8.7GPa)	100	-
	PrV_2Al_{20}	0.05	90	-
Pu-based SC (Pu-115)	$PuCoGa_5$	18.5	77	Line
	$PuCoIn_5$	2.5	200	Line
	$PuRhGa_5$	8.7	70	Line
	$PuRhIn_5$	1.6	350	Line
Np-based SC	$NpPd_5Al_2$	4.9	200	Point
Yb-based SC	$\beta-YbAlB_4$	0.08	150	-
	$YbRh_2Si_2$	0.002	-	-

重费米子超导与竞争序

Serials	Compound	T_c / K
CeM_2X_2 (AFM SC)	$CeCu_2Si_2$	0.5
	$CeCu_2Ge_2$	0.5
	$CePd_2Si_2$	0.5
	$CeRh_2Si_2$	0.5
	$CeAu_2Si_2$	0.5
$Ce_nM_mIn_{3n+2m}$ (AFM SC)	$CeNi_2Ge_2$	0.5
	$CeIn_3$	0.4
	$CelrIn_5$	0.4
	$CeCoIn_5$	2.3
	$CeRhIn_5$	2.4 (2.3GPa)
	$CePt_2In_7$	2.3 (3.1GPa)
	Ce_2RhIn_8	2.0 (2.3GPa)
	Ce_2PdIn_8	0.68
	Ce_2CoIn_8	0.4
	Ce_3PdIn_{11}	0.42
Ce-based non-centresymmetric SC	$CePt_3Si$	0.75
	$CelrSi_3$	1.65 (2.5GPa)
	$CeRhSi_3$	1.0 (2.6GPa)
	$CeCoGe_3$	0.69 (6.5GPa)
Other Ce-based AFM SC	$CeNiGe_3$	0.43 (6.8GPa)
	$Ce_2Ni_3Ge_5$	0.26 (4.0GPa)
	$CePd_5Al_2$	0.57 (10.8GPa)

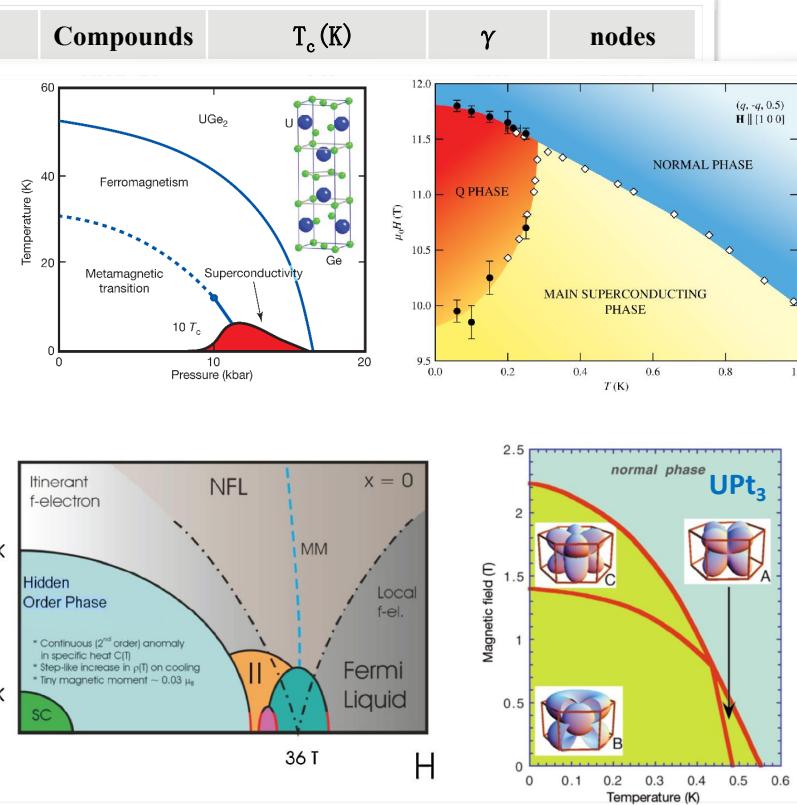


Yb-based SC

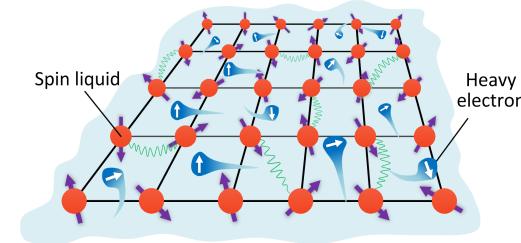
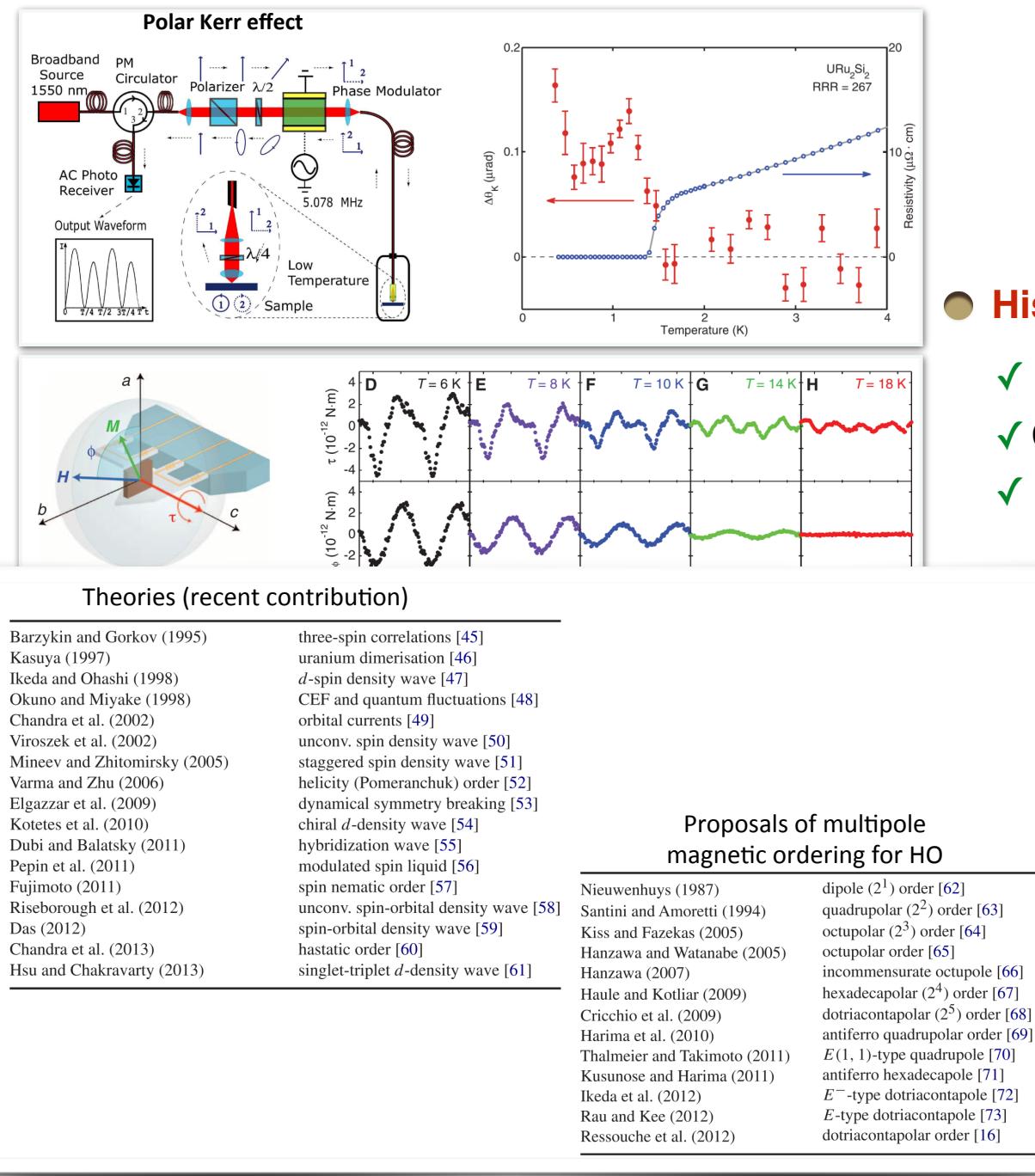


Historic development of heavy fermions

- ✓ $CeAl_3 \gamma = 1.62 \text{ J/mol K}^2$ @ 1975
- ✓ $CeCu_2Si_2 T_c = 0.5 \text{ K}$ @ 1979

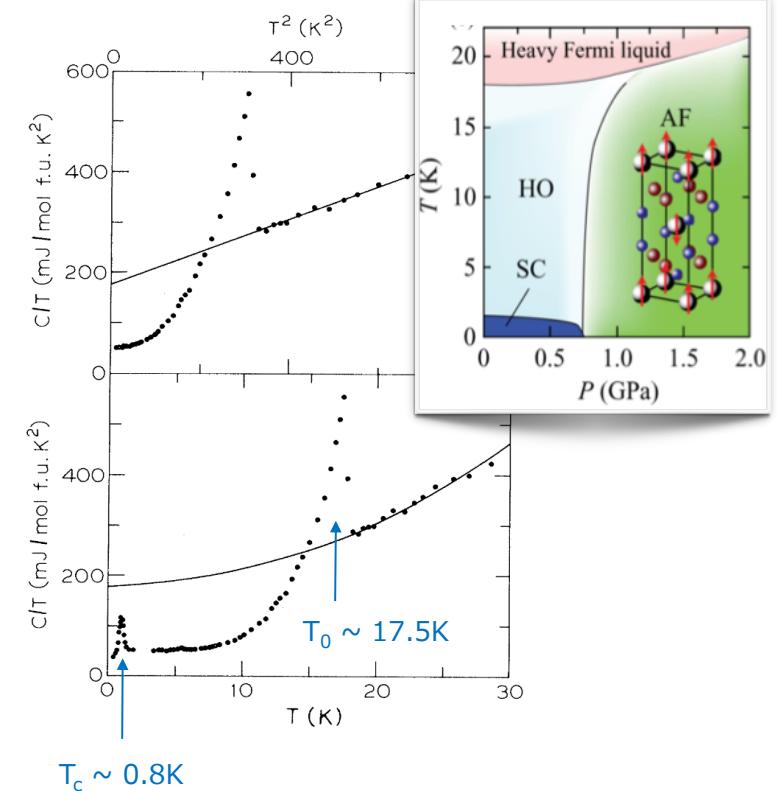


奇异重费米子态：隐藏序

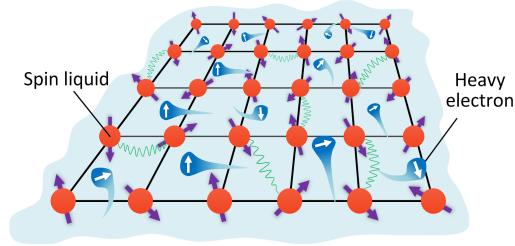


Historic development of heavy fermions

- ✓ CeAl₃ $\gamma=1.62 \text{ J/mol K}^2$ @ 1975
- ✓ CeCu₂Si₂ $T_c=0.5 \text{ K}$ @ 1979
- ✓ URu₂Si₂ Hidden order @ 1985

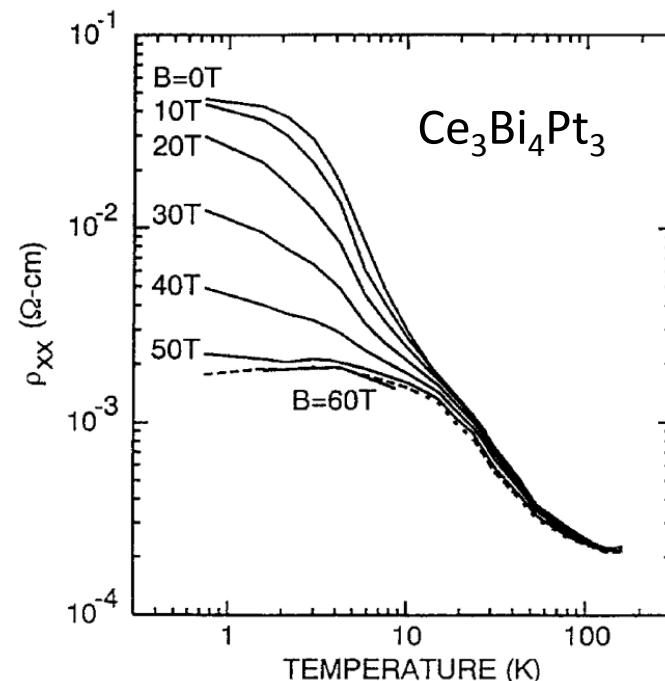


奇异重费米子态：近藤绝缘体

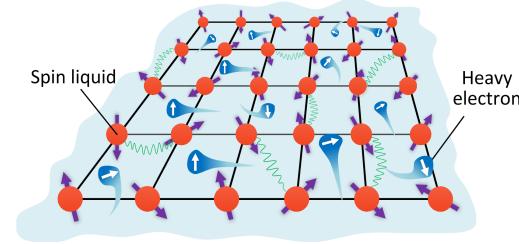


● Historic development of heavy fermions

- ✓ CeAl₃ $\gamma=1.62 \text{ J/mol K}^2$ @ 1975
- ✓ CeCu₂Si₂ $T_c=0.5 \text{ K}$ @ 1979
- ✓ URu₂Si₂ **Hidden order** @ 1985
- ✓ Ce₃Bi₄Pt₃ **Kondo insulator** @ 1992

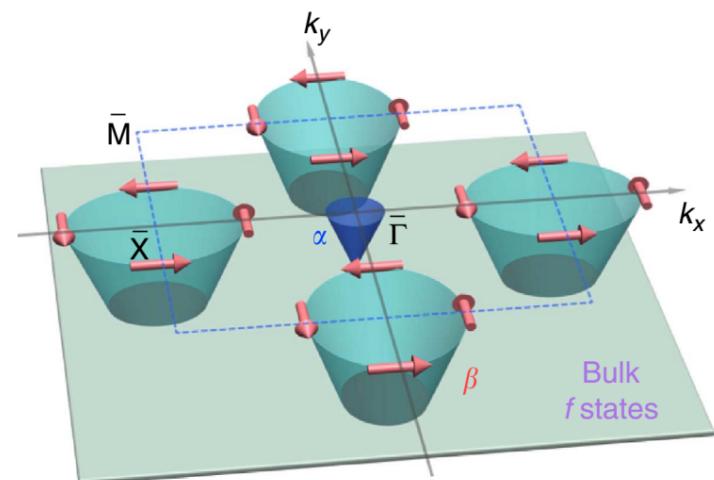


奇异重费米子态：拓扑近藤绝缘体



● Historic development of heavy fermions

- ✓ CeAl₃ $\gamma=1.62 \text{ J/mol K}^2$ @1975
- ✓ CeCu₂Si₂ $T_c=0.5 \text{ K}$ @1979
- ✓ URu₂Si₂ **Hidden order** @1985
- ✓ Ce₃Bi₄Pt₃ **Kondo insulator** @1992
- ✓ SmB₆ **Topological KI** @2010



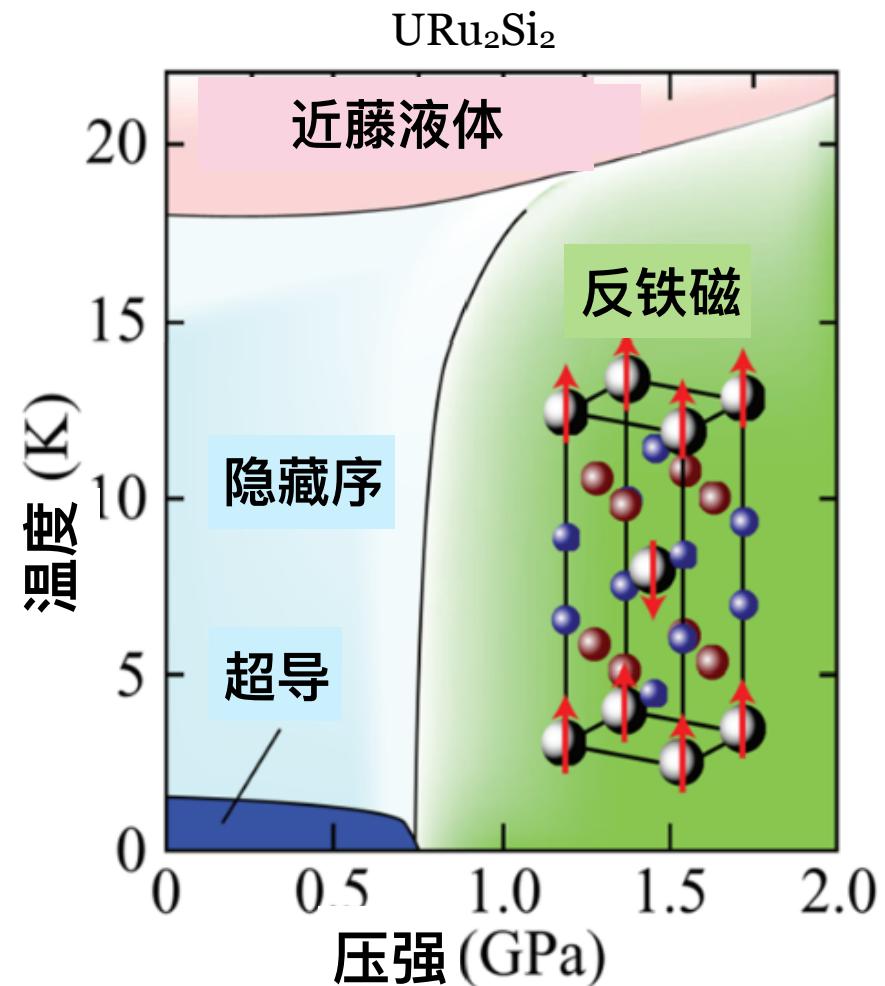
重费米子是强关联电子的model系统

- 能量尺度小，易调控
(磁场、压力、掺杂)

$$E_F \sim 1/m^* \sim 10 \text{ meV}$$

- 竞争尺度多，物理丰富

- 第一个非常规超导体 (CeCu_2Si_2)
- 第一个自旋三重态超导体 (UPt_3)
- 第一个3维拓扑绝缘体 (SmB_6)
- 巨大热电效应 (新型热电器件)
- 非费米液体 (超越传统朗道理论)

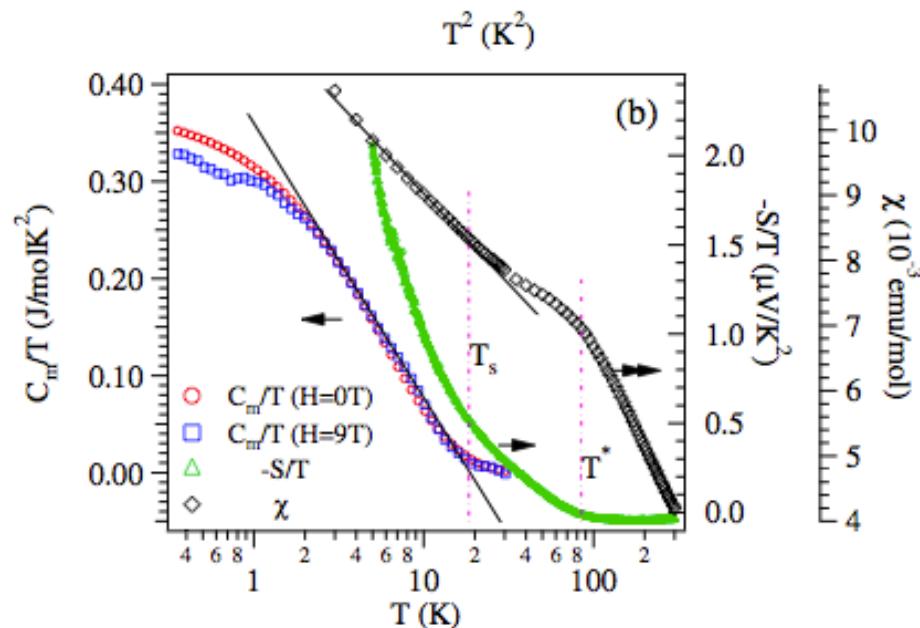
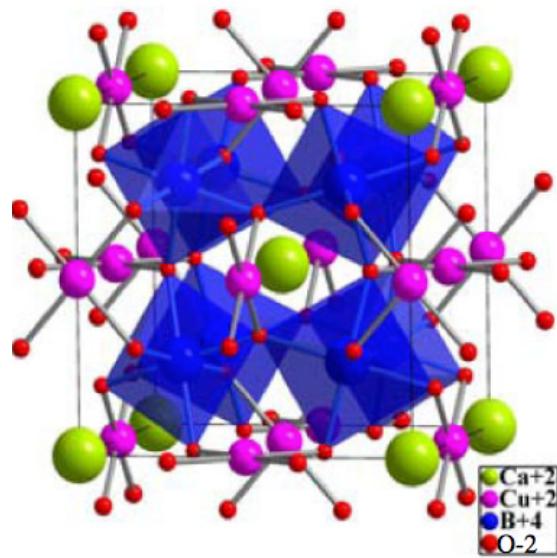


一些典型材料如 CeCoIn_5 系列非常干净，易于研究！
能够帮助我们理解强关联电子体系的本质物理！

Cheng & Yang et al., PRL 111, 176403 (2013)

- 重电子 $m^*/m_e \sim 100$

$\text{CaCu}_3\text{Ir}_4\text{O}_{12}$



极少见的新型d电子重费米子材料，行为完全符合二流体理论的预言

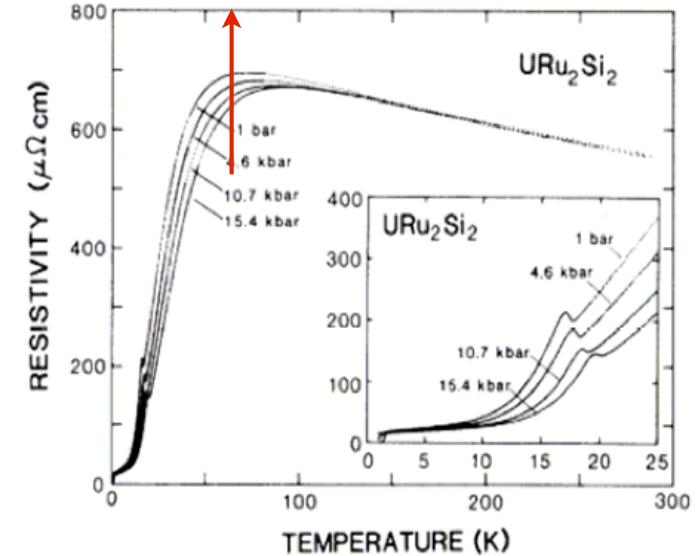
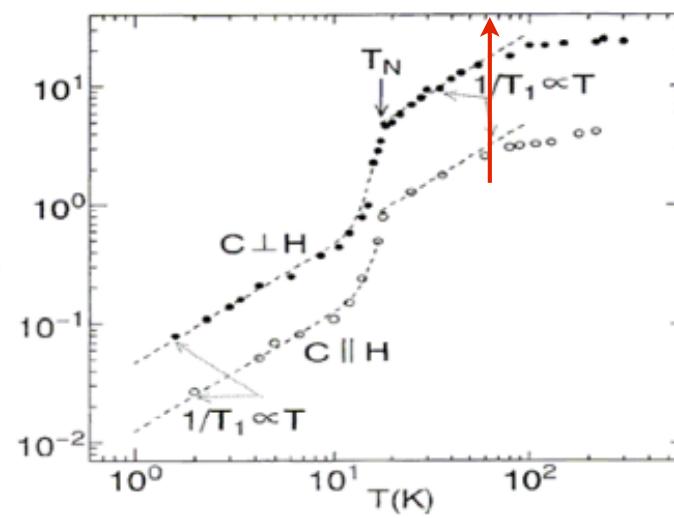
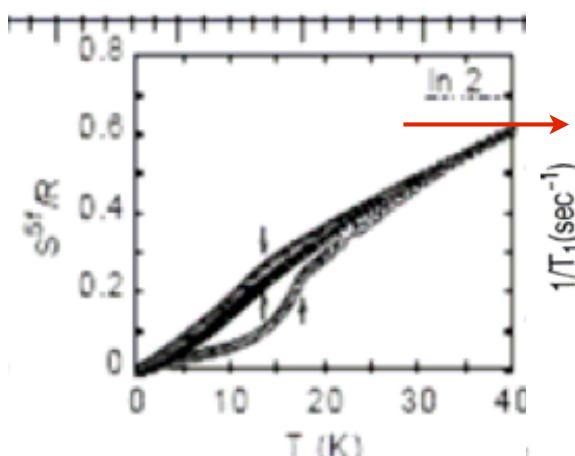
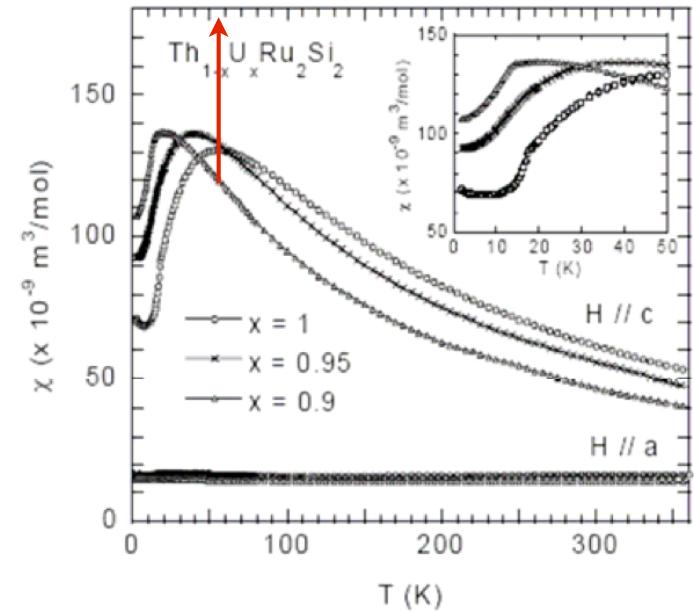
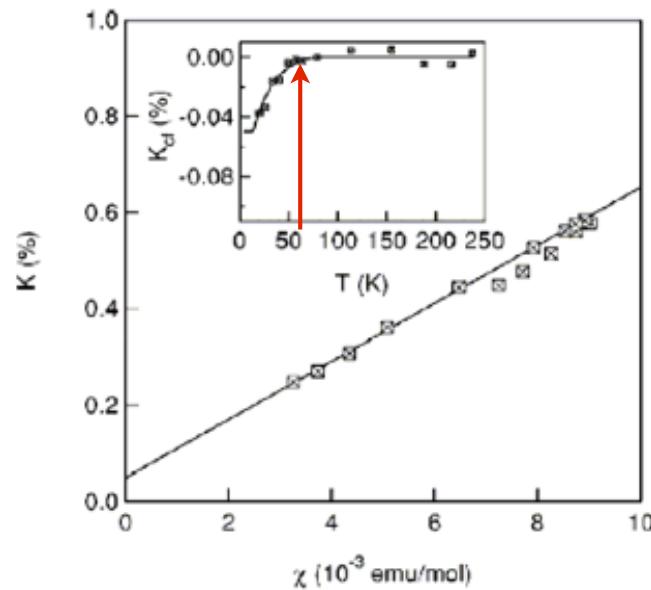
最近，陈仙辉等人发现 AFe_2As_2 等材料也完全符合二流体理论预言，是一种铁基重费米子材料 (PRL 2016)

重费米子金属的一般实验特征：温度演化

URu₂Si₂

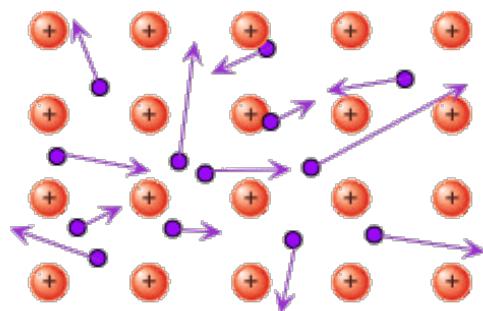
- Resistivity
- Susceptibility
- Knight shift anomaly
- Hall anomaly
- Optical conductivity
- Magnetic entropy
- Point contact spectroscopy
- Neutron/Raman scattering
- NMR spin-lattice relaxation

高温局域f电子 ➡️ 低温巡游重电子

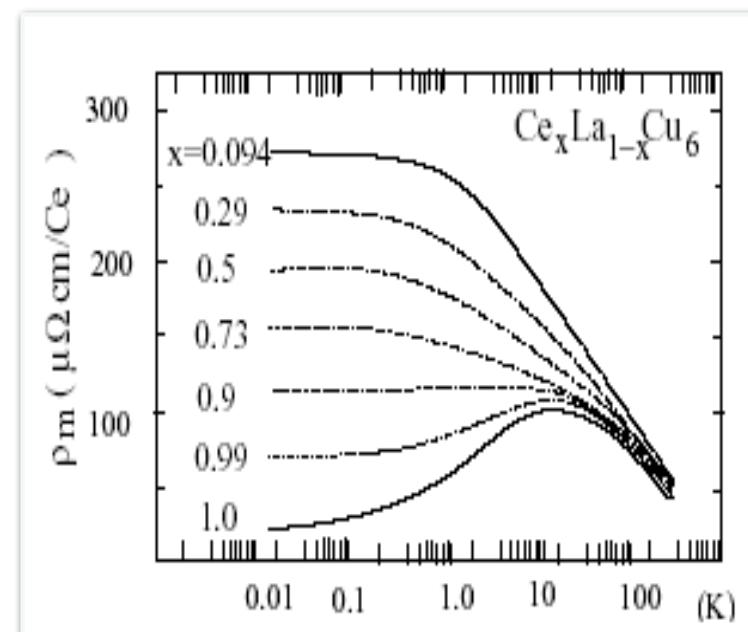
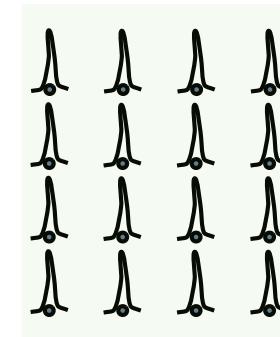


近藤晶格的理论问题

*Incoherent Kondo Scattering
above T^**

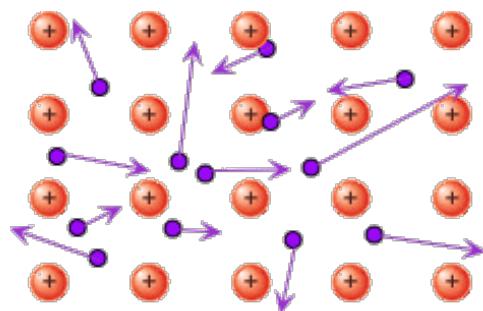


*Coherent behavior
below T^**

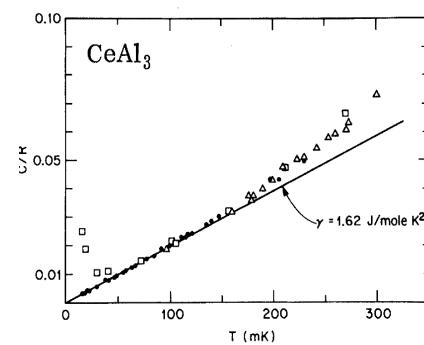
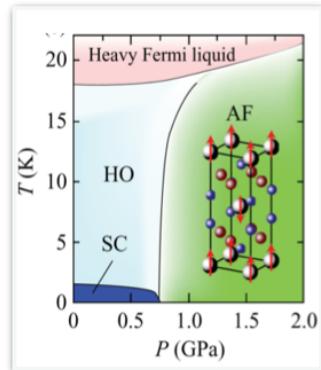
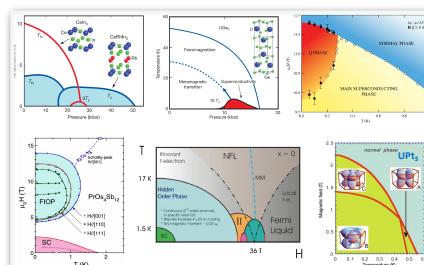
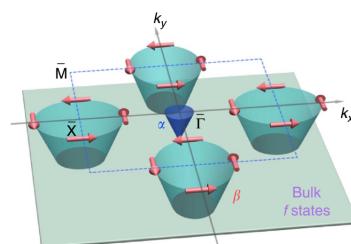
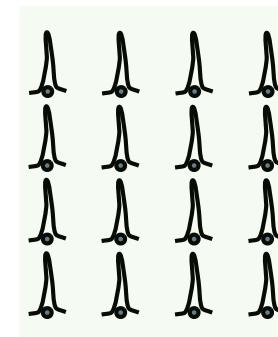


近藤晶格的理论问题

*Incoherent Kondo Scattering
above T^**

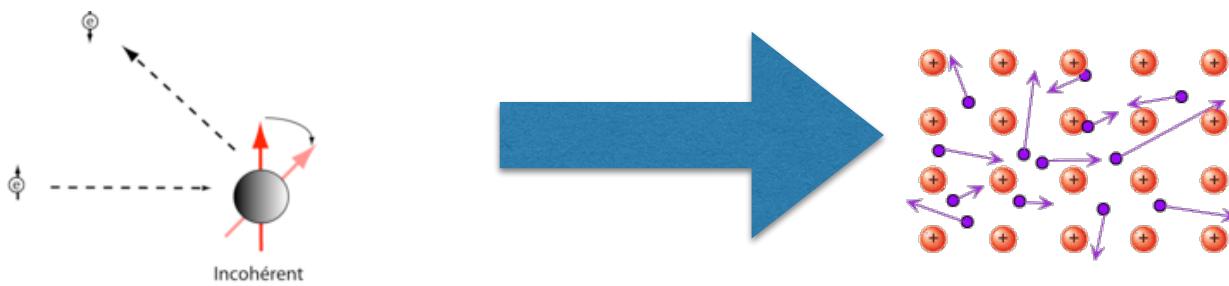


*Coherent behavior
below T^**



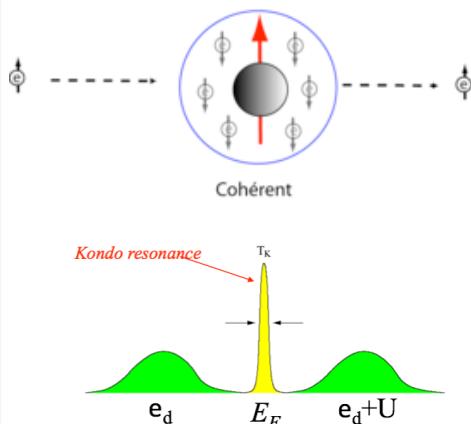
近藤晶格的理论问题

Incoherent Kondo Scattering



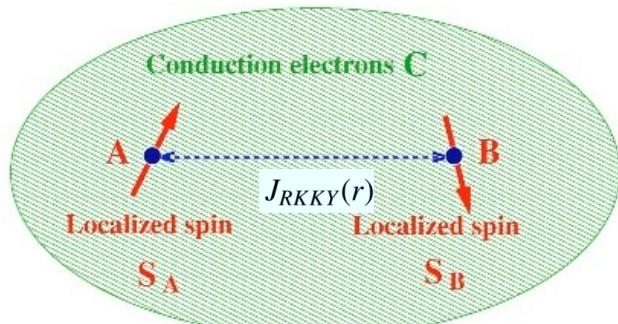
Ce: Ground state doublet

Coherent Kondo Screening



重费米子体系的基本能量标度

RKKY interaction



$$H_{RKKY} = \frac{1}{2} \sum_{\mathbf{x}, \mathbf{x}'} -J_{RKKY}(\mathbf{x} - \mathbf{x}') \vec{\chi}(\mathbf{x} - \mathbf{x}') \vec{S}(\mathbf{x}) \cdot \vec{S}(\mathbf{x}'),$$

$$J_{RKKY}(r) \sim J^2 \rho \frac{\cos 2k_F r}{|r|^3}$$



(S. Doniach)

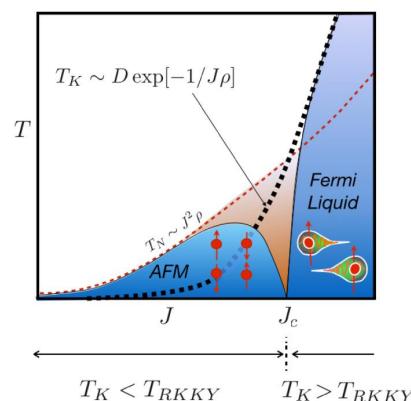
Physica 91B (1977) 231–234 © North-Holland

THE KONDO LATTICE AND WEAK ANTFERROMAGNETISM

S. DONIACH*

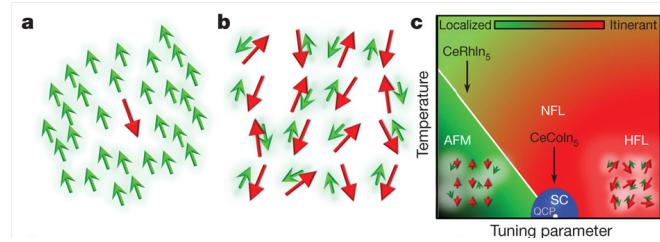
Department of Applied Physics, Stanford University, California 94305, USA

By considering a one-dimensional analog of a system of conduction electrons exchange coupled to a localized spin in each cell of a lattice, it is suggested that a second-order transition from an antiferromagnetic to a Kondo spin-compensated ground state will occur as the exchange coupling constant J is increased to a critical value J_c . For systems in which $J \leq J_c$, a very weak sublattice magnetization may occur as a result of nearly complete spin-compensation.



RKKY

FIGURE 12. Illustrating how the polarization of spin around a magnetic impurity gives rise to Friedel oscillations and induces an RKKY interaction between the spins

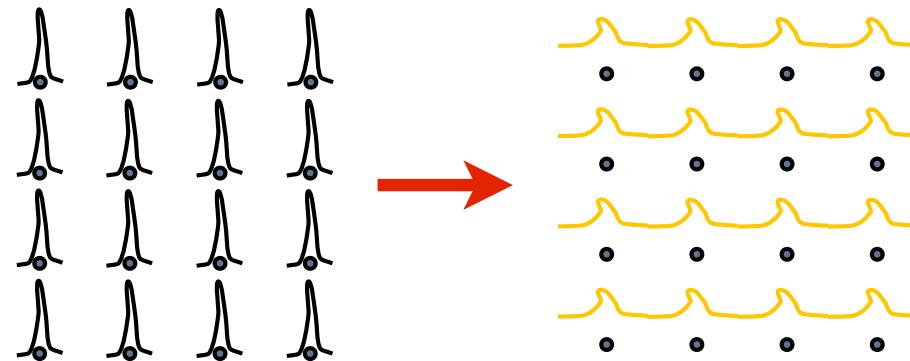


$$H_{KL} = J_H \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_K \sum_i \mathbf{s}_i \cdot \mathbf{s}_i + \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}$$

平均场方法

● Mean-field theory & Hybridization

✓ Lattice coherence @ $T < T^*$



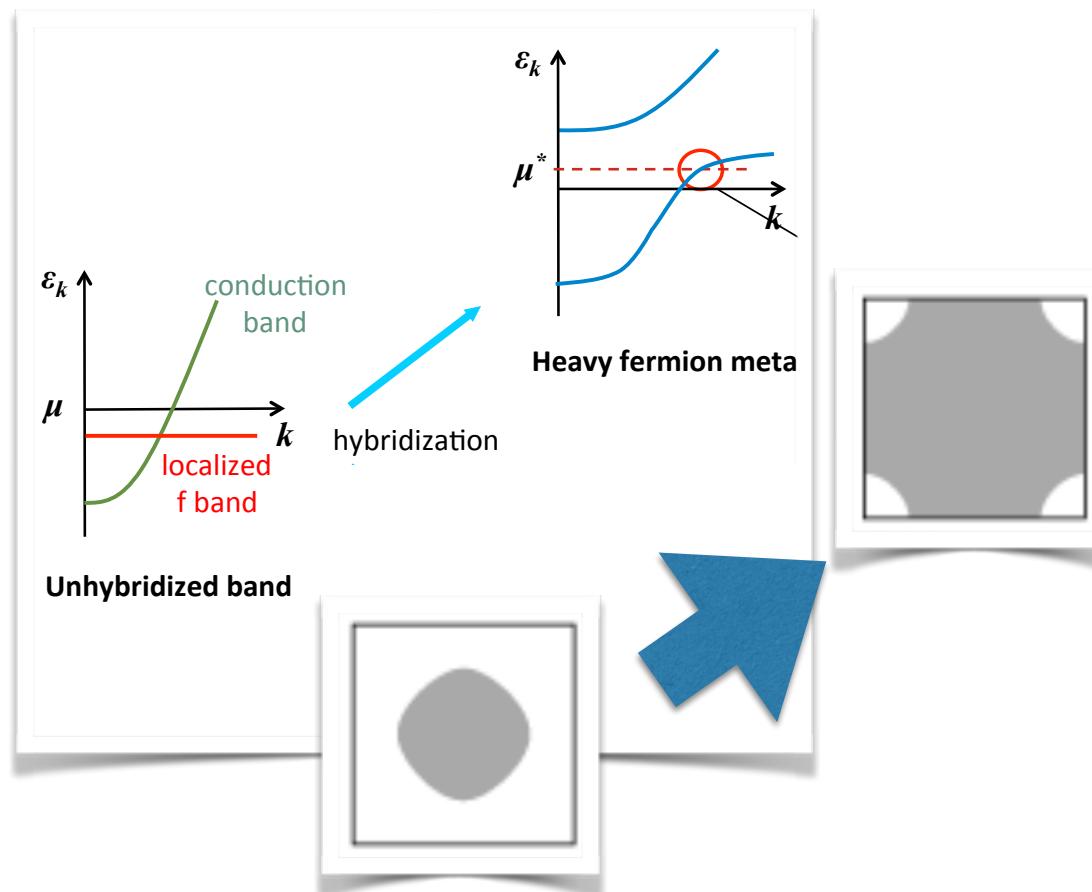
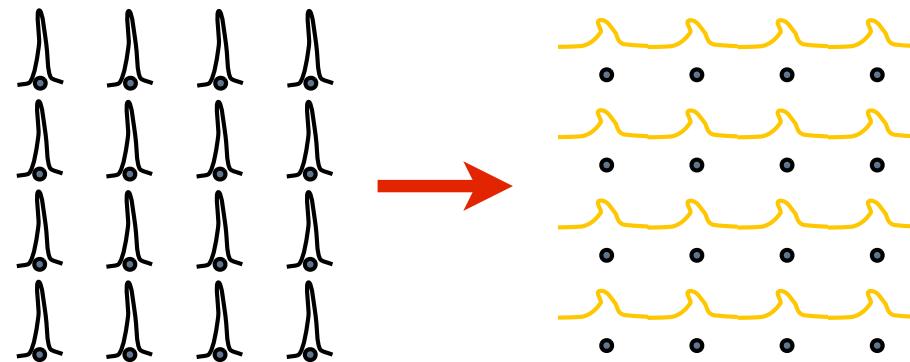
mean-field approach

$$H_{KL} = J_H \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_K \sum_i \mathbf{s}_i \cdot \mathbf{S}_i + \sum_{k\sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}$$

平均场方法

Mean-field theory & Hybridization

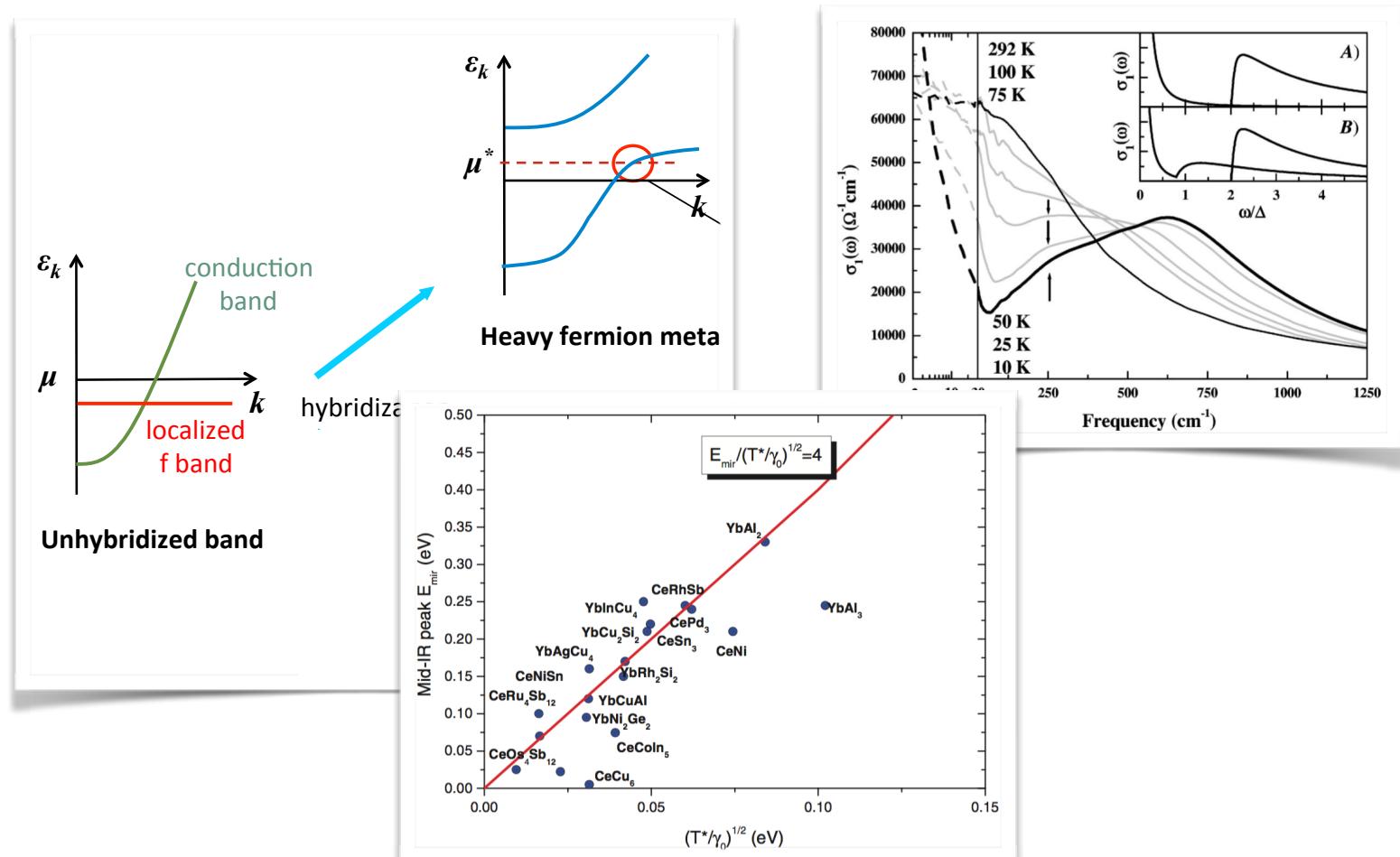
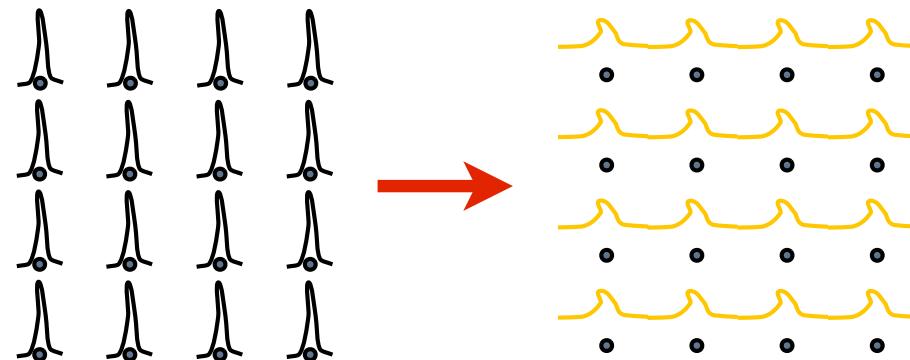
- ✓ Lattice coherence @ $T < T^*$
- ✓ Hybridization & FS change



平均场方法

Mean-field theory & Hybridization

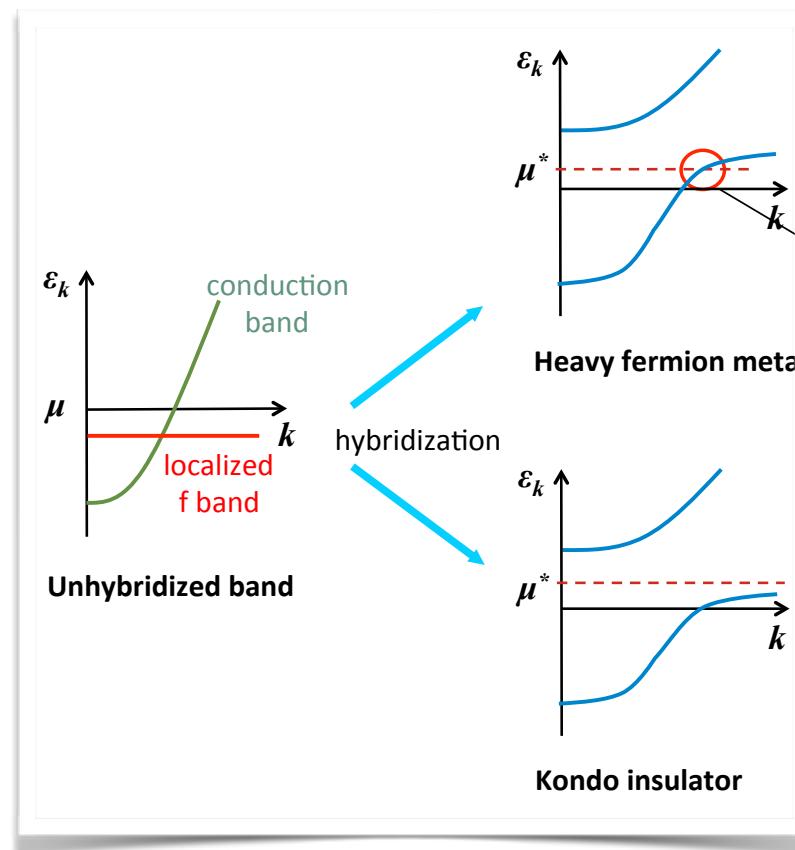
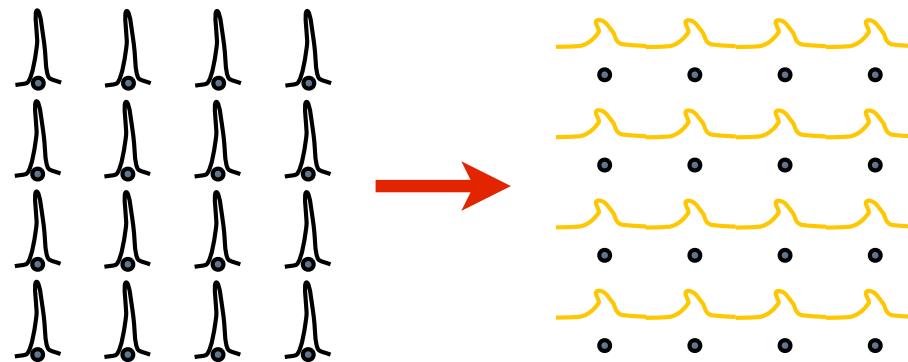
- ✓ Lattice coherence @ $T < T^*$
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap



平均场方法

Mean-field theory & Hybridization

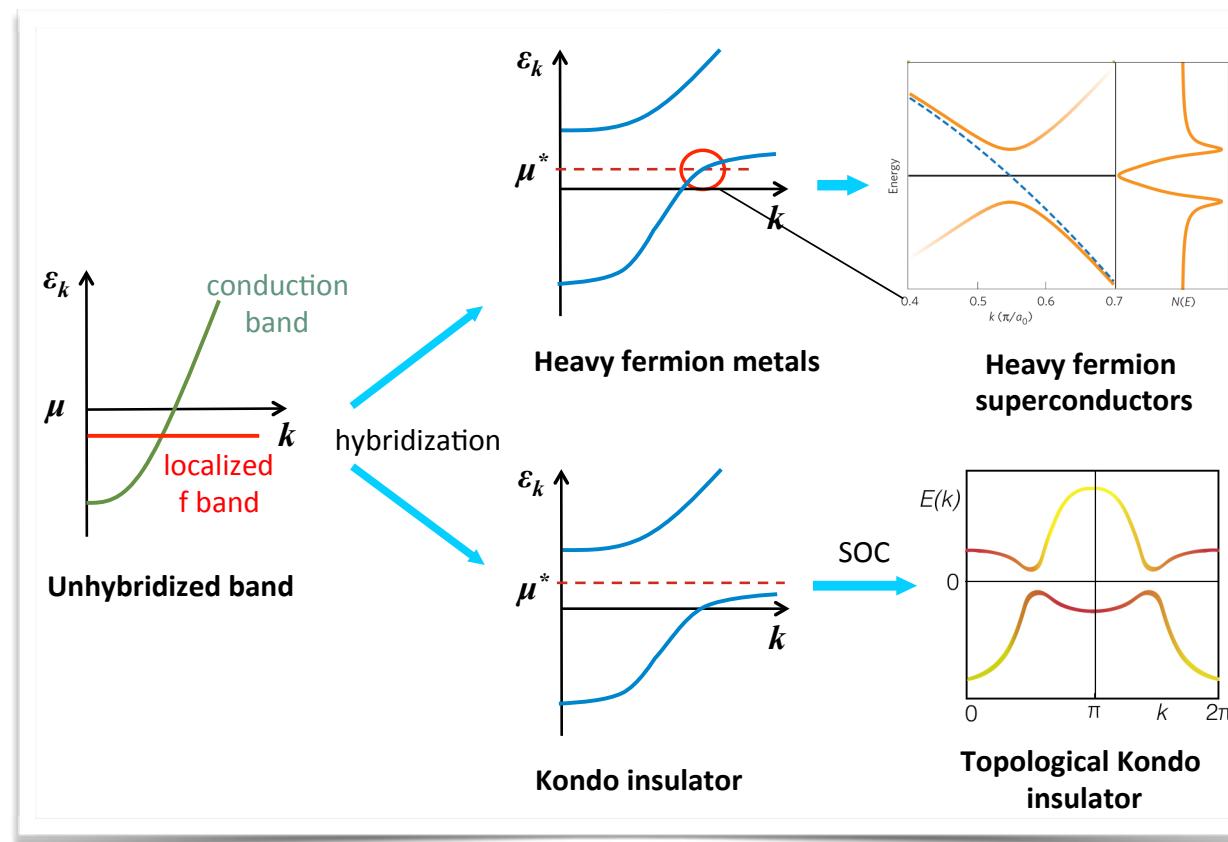
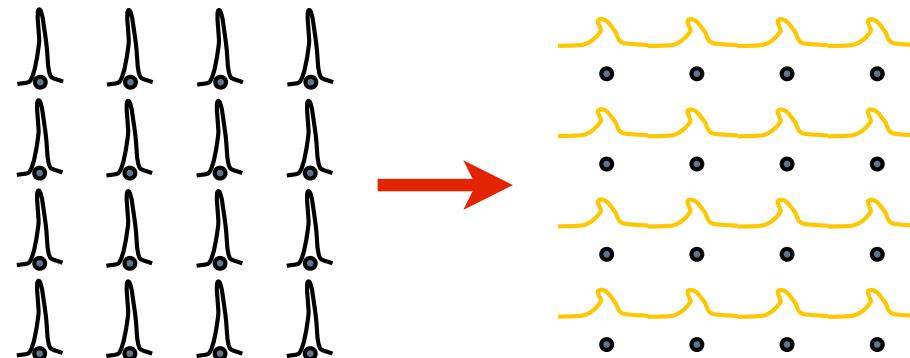
- ✓ Lattice coherence @ $T < T^*$
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap
- ✓ Kondo insulator



平均场方法

Mean-field theory & Hybridization

- ✓ Lattice coherence @ $T < T^*$
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap
- ✓ Ground state: KI & SC & TKI & SL ...



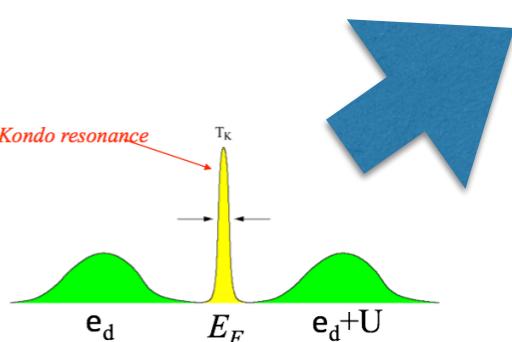
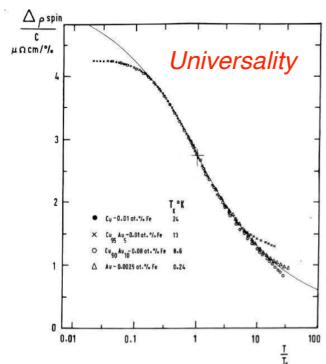
Static Hybridization, only below T^*

动力学平均场方法

● Mean-field theory & Hybridization

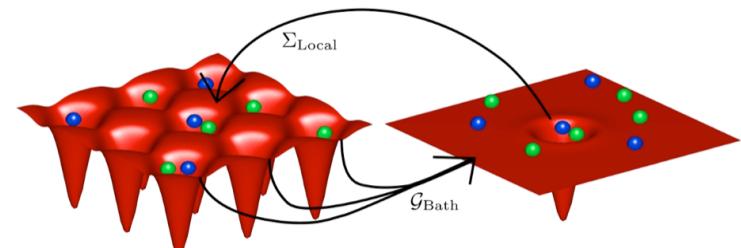
- ✓ Lattice coherence @ $T < T^*$
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap
- ✓ Ground state: KI & SC & TKI & SL ...

No long range magnetic correlations
Hard to describe quantum criticality

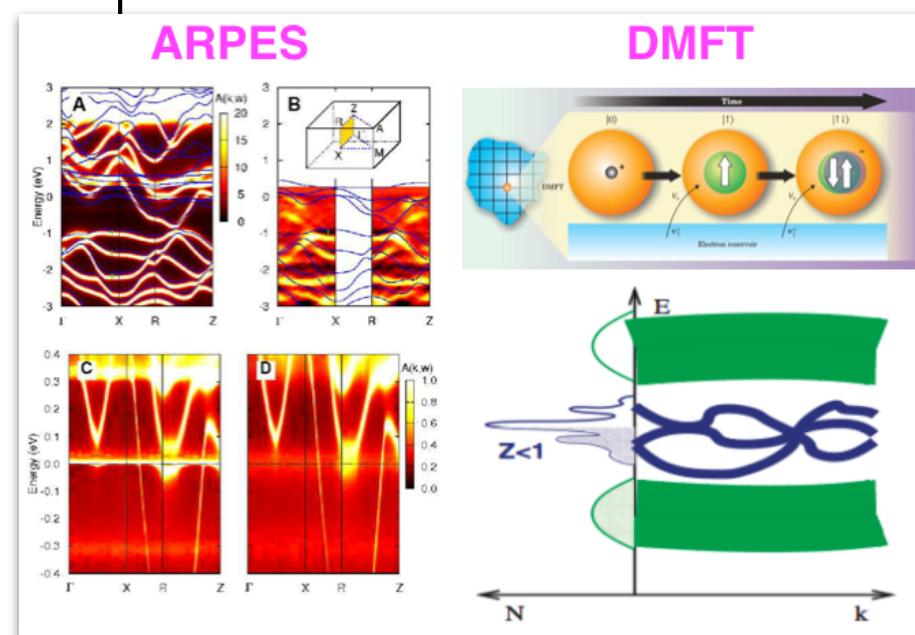


● DMFT & Coherence/Duality

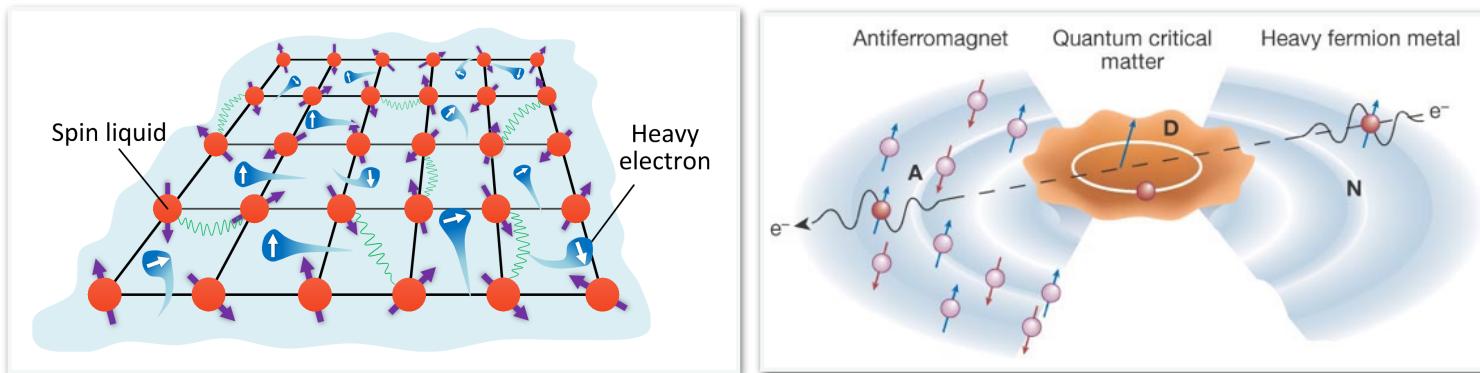
- ✓ Dynamic fluctuations @ $T > T^*$
- ✓ Local/Itinerant crossover @ $T < T^*$
- ✓ Development of coherence @ $T < T^*$
- ✓ Short range correlation (*Extended*)



Dynamical mean-field theory



三、重费米子物理的核心问题

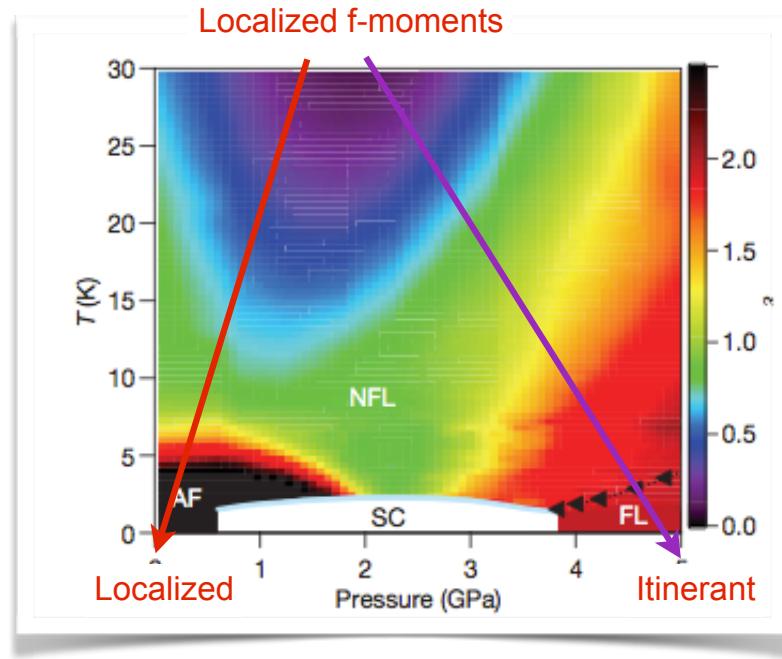


- f电子的局域–巡游转变的问题
- 量子相变与量子临界的问题

参考文献

杨义峰, 重费米子二流体理论, 物理学进展 35, 191 (2015)

f电子的局域–巡游转变



- 局域f电子 @ high T

- ✓ Curie-Weiss susceptibility
- ✓ Incoherence Kondo scattering in resistivity

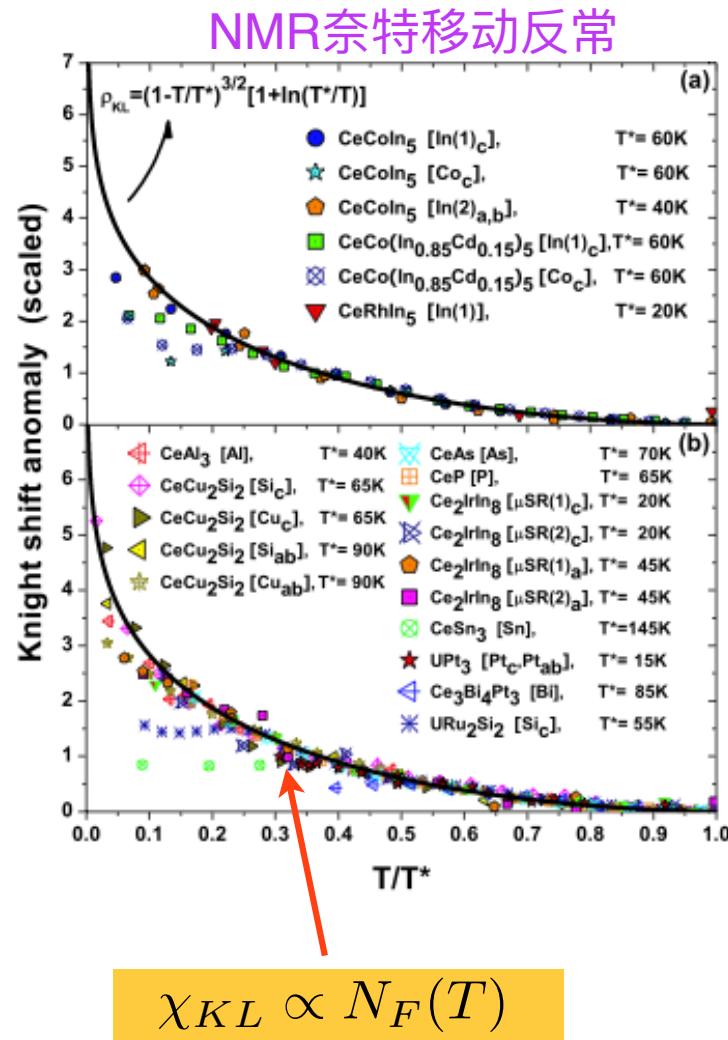
- 巡游f电子 @ low T

- ✓ metallic behavior in resistivity
- ✓ Screened moment in susceptibility

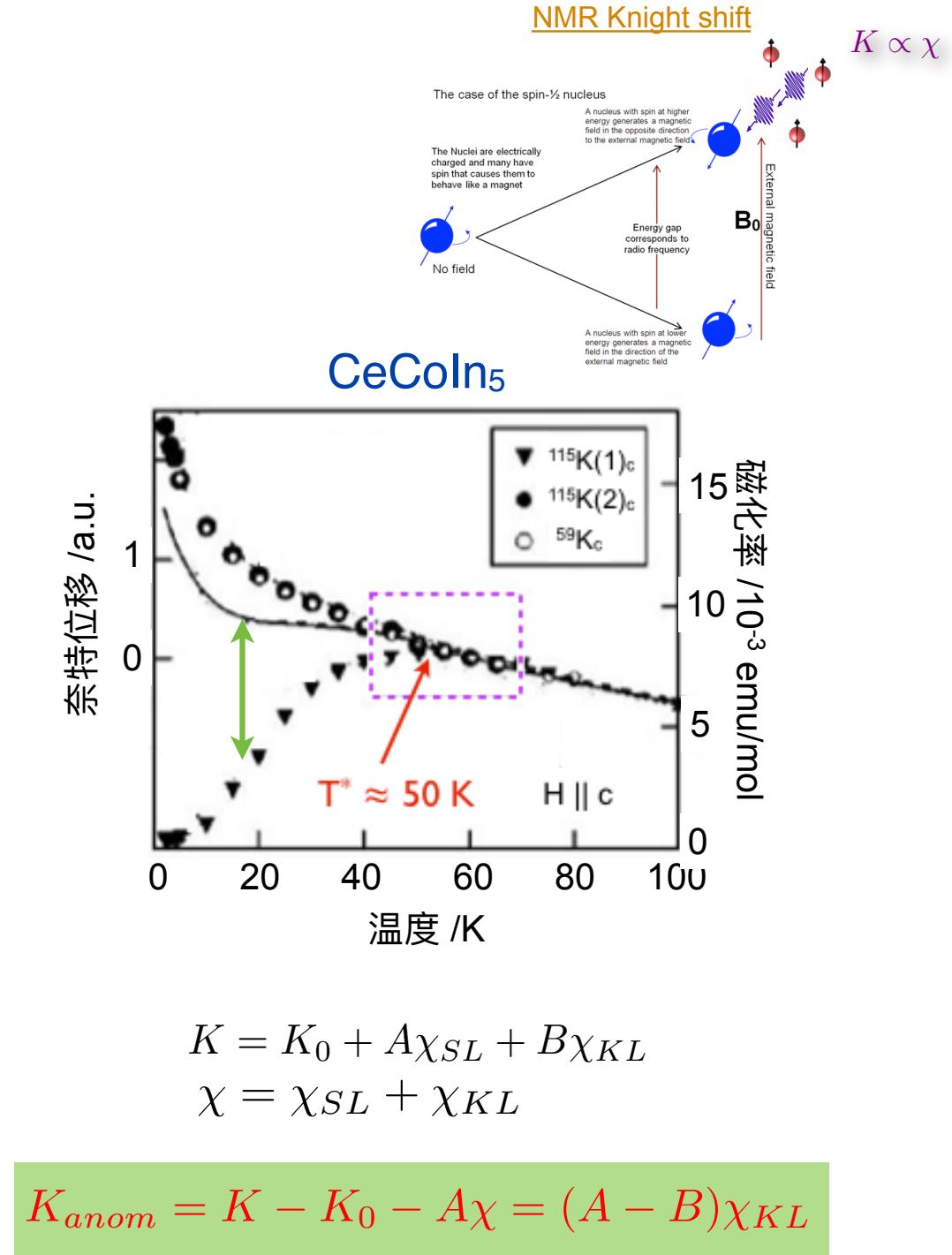
- 如何描述f电子的局域–巡游转变? (二流体理论)

- f电子局域–巡游转变对量子相变有什么影响?

重电子态的普适温度演化超出了传统理论的预期

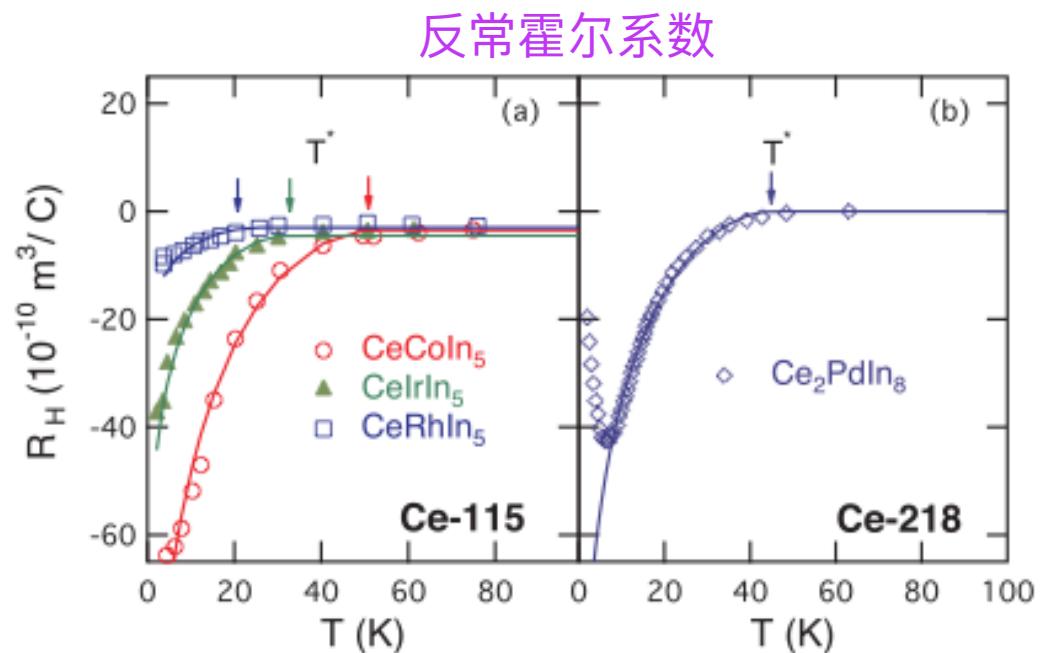
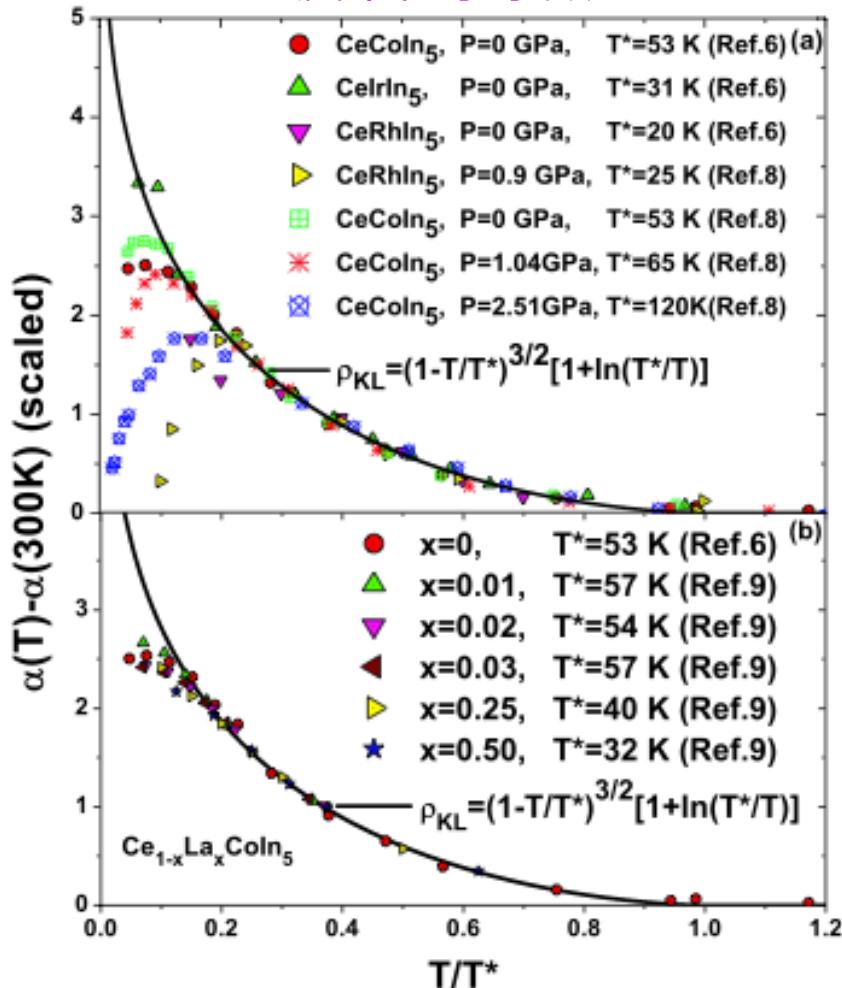


Yang et al., PRL 100, 096404 (2008)
Curro et al, PRB (2004)



霍尔系数中的普适温度演化

反常霍尔系数



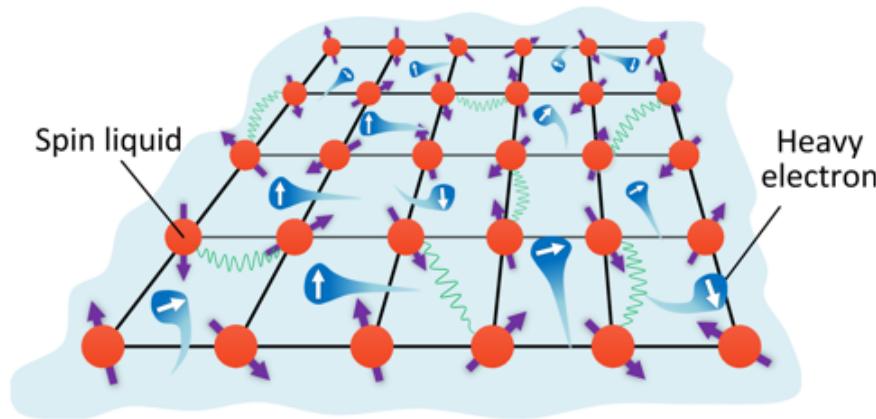
$$R_H \propto \chi_{KL}$$

首次提出重电子的反常霍尔系数公式

Yang, PRB 87, 045102 (2013)

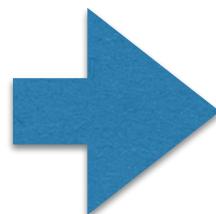
二流体理论

Yang et al., PNAS 109, E3060 (2012)



弱相互作用（高温）时

- 巡游的自由导带电子
- 局域的晶格自旋



二流体理论的核心思想
f 电子呈现局域 – 巡游的量子二重性

强相互作用（低温）时

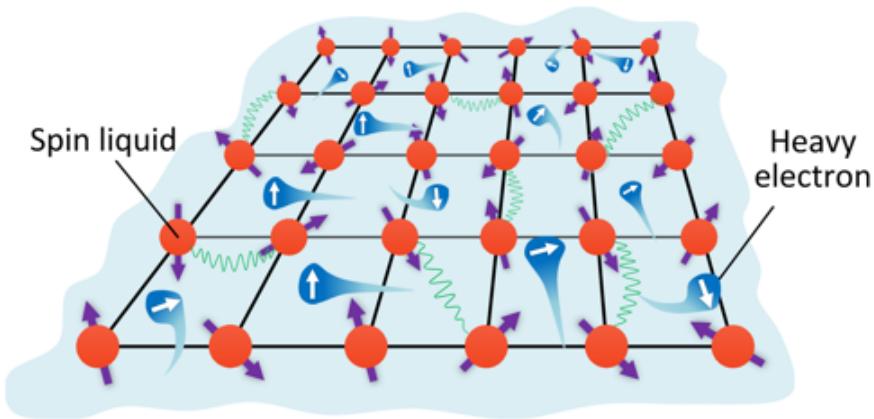
- 重整合后的巡游重电子
- 被部分屏蔽的晶格自旋



体系仍然近似保持为局域和巡游两种流体

二流体理论

Yang et al., PNAS 109, E3060 (2012).



将重费米子问题简化为
局域&巡游 两种共存流体

各种反常实验现象

- 电阻
- 磁性熵
- 磁化率
- 点接触谱(PCS)
- 核磁共振
- 扫描隧道谱(STM)
- 霍尔系数
- 中子散射
- 光电导
- 拉曼散射

简化描述两种流体各自
行为及其相互转化

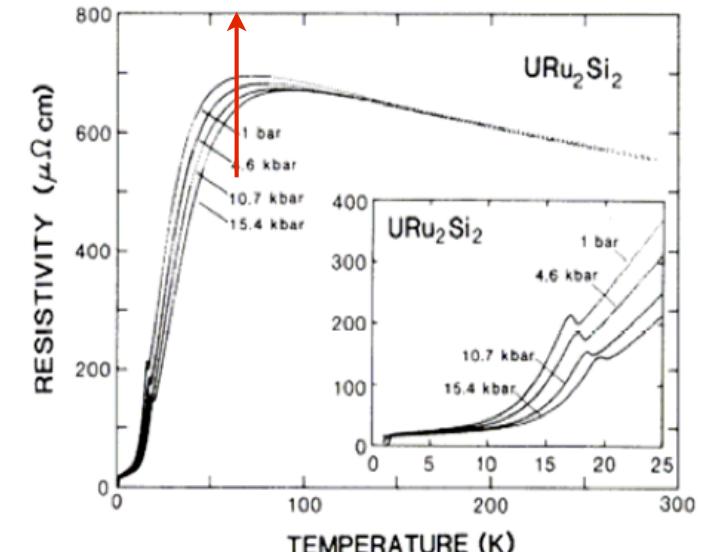
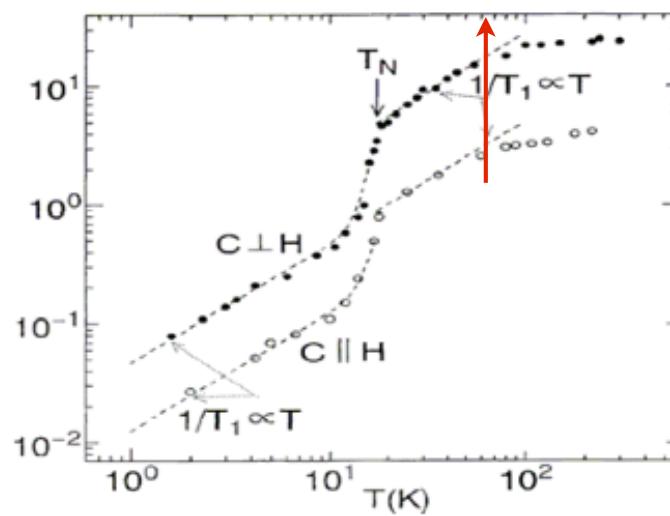
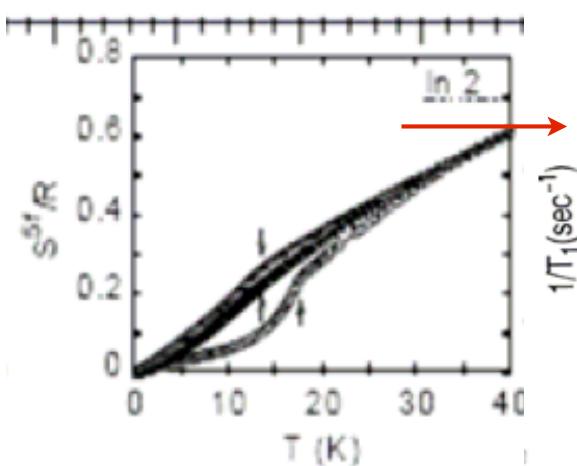
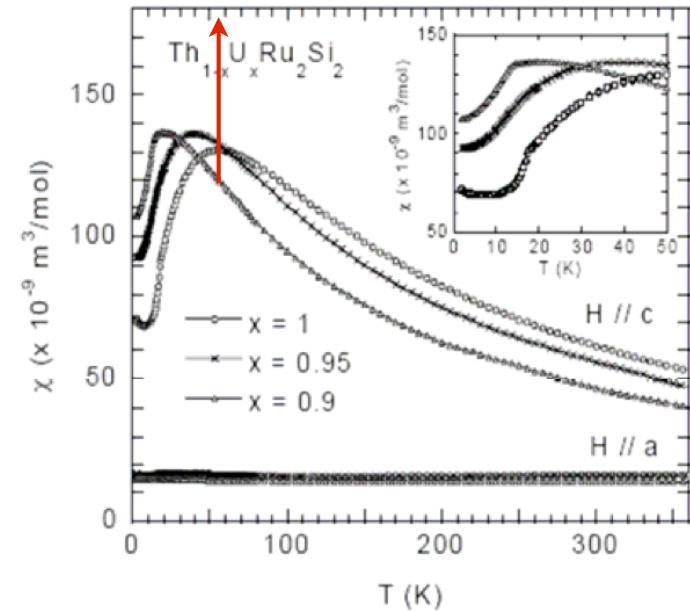
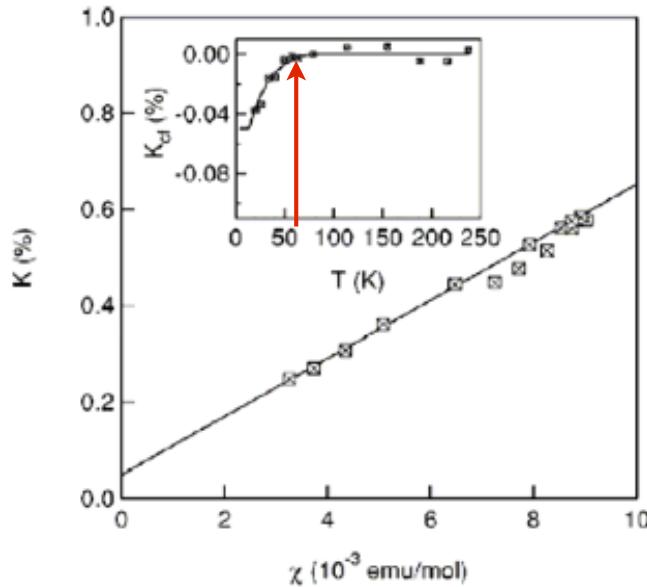
解释复杂多变的实验现象

二流体理论给出了各种反常现象的统一解释

Localized-itinerant crossover at T^*

URu₂Si₂

- Resistivity
- Susceptibility
- Knight shift anomaly
- Hall anomaly
- Optical conductivity
- Magnetic entropy
- Point contact spectroscopy
- Neutron/Raman scattering
- NMR spin-lattice relaxation



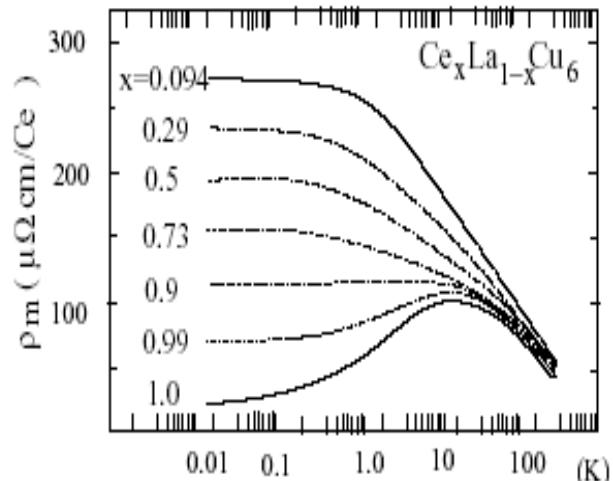


FIGURE 15. Development of coherence in heavy fermion systems. Resistance in $Ce_{1-x}La_xCu_6$ after Onuki and Komatsubara[35]

Table 1 | Experimental T^* , T_K and γ values for a variety of Kondo lattice compounds

Compound	T^* (K)	T_K (K)	γ ($mJ\ mol^{-1}\ K^2$)	$J\rho$	J (meV)	c	Reference
CeRhIn ₅	20 ± 5	0.15	5.7	0.10	40	0.45	6, 8, H.-O.L.*
CeCu ₆	35 ± 5	3.5	8	0.15	43	0.49	9, 10
CeCu ₂ Si ₂	75 ± 20	10	4	0.15	90	0.47	6, 11, 12
CePb ₃	20 ± 5	3	13	0.15	28	0.41	13, 14
CeCoIn ₅	50 ± 10	6.6	7.6	0.16	49	0.55	4, 6, 7
CePd ₂ Si ₂	40 ± 10	9	7.8	0.17	51	0.41	15, 16
CePd ₂ Al ₃	35 ± 10	10	9.7	0.18	43	0.40	17, 18, 19
CeRu ₂ Si ₂	60 ± 10	20	6.68	0.19	66	0.42	20, 21
U ₂ Zn ₁₇	20 ± 5	2.7	12.3	0.15	29	0.41	22, 23
URu ₂ Si ₂	55 ± 5	12	6.5	0.17	62	0.45	6, 24, 25
UBe ₁₃	55 ± 5	20	8	0.19	57	0.43	26, 27
UPd ₂ Al ₃	60 ± 10	25	9.7	0.21	51	0.48	19, 28
YbRh ₂ Si ₂	70 ± 20	20	7.8	0.19	58	0.53	Z.F.†
YbNi ₂ B ₂ C	50 ± 5	20	11	0.21	44	0.47	29

掺杂系统:

$$T_K \sim \rho^{-1} e^{-1/J_K \rho}$$

非磁性系统:

$$\gamma \sim \pi^2 \rho / 3$$

重费米子系统:

$$J_{RKKY} \sim c J_K^2 \rho$$

重电子态的物理起源

统一温标

Supplementary Table I: Estimates of T^* from different methods for a variety of heavy electron compounds. The unit of γ is mJ/mol-(La, Y, Lu, etc) K⁻¹ and that of T_K and all T^* 's is Kelvin. References for all the data sources are given in the text.

Compounds	Optical	Entropy	Resistivity	Susceptibility	Knight shift	Relaxation	Hall	Others	T^*	T_K	γ
CeRhIn ₅		15-20	50	20	10-20	20	20	20	20±5	0.15	5.7
CePb ₃		>10	25	15					20±5	3	13
CeCu ₆	40	30	15	35		40	40	30	35±5	3.5	8
CePd ₂ Si ₂		≥30		40					40±10	9	7.8
CePd ₂ Al ₃		>12	40						35±10	10	9.7
CeCoIn ₅	50-75	50	50	50	50	65	53	60	50±10	6.6	7.6
CeRu ₂ Si ₂		>30		50	60	70		70	60±10	20	6.68
CeCu ₂ Si ₂		>20	<100	75	75				75±20	10	4
U ₂ Zn ₁₇	>6	>15	17-18	30					20±5	2.7	12.3
UBe ₁₃	45-85	50	2.5	50	60				55±5	20	8
URu ₂ Si ₂	40-90	50	70	55	55	60	55		55±5	12	6.5
UPd ₂ Al ₃	50	>11	90	50		60			60±10	25	9.7
YbNi ₂ B ₂ C		50	45	50					50±5	20	11
YbRh ₂ Si ₂	80	>40	100	70			90		70±20	20	7.8
CeAl ₂		17	20						20±5	>0.36	5.46-9.55
CePtSi _{0.9} Ge _{0.1}		≥12		20	15				20±10		
CePtSi		≥15	30	20	20				25±5		
CeAl ₃	10	>10	35-40	40	40	40	40		40±5	>0.2	3.8-4.95
CeIrIn ₅	>30	>15	<50	50			30		40±10		
Ce ₆₅ Al ₁₀ Cu ₂₀ Co ₅		>30	40	70					50±10		3.44
CeP		>20	80	70	65		80		70±10	≤1.7	0.8
CeAs	≤80		60	80	70		80		70±10		1.0
Ce ₃ Bi ₄ Pt ₃				80	85	100		100	90±10		10
CePd ₃	≤150		130	130					130±20		0.28-3.48
CeSn ₃	150			140	145				145±5		11.66
UPt ₃	20	20		20	15	15	25	20	20±5		
YbCuAl		>20	70	40	30			40	35±5		
YbAl ₃	80-160	≥110		120				>50	120±10		3.8

RKKY (inter-site exchange coupling) origin of T^*

- T^* is given by the RKKY exchange interaction
- The RKKY interaction governs the emergence of heavy electrons

$$T^* = cJ^2\rho$$

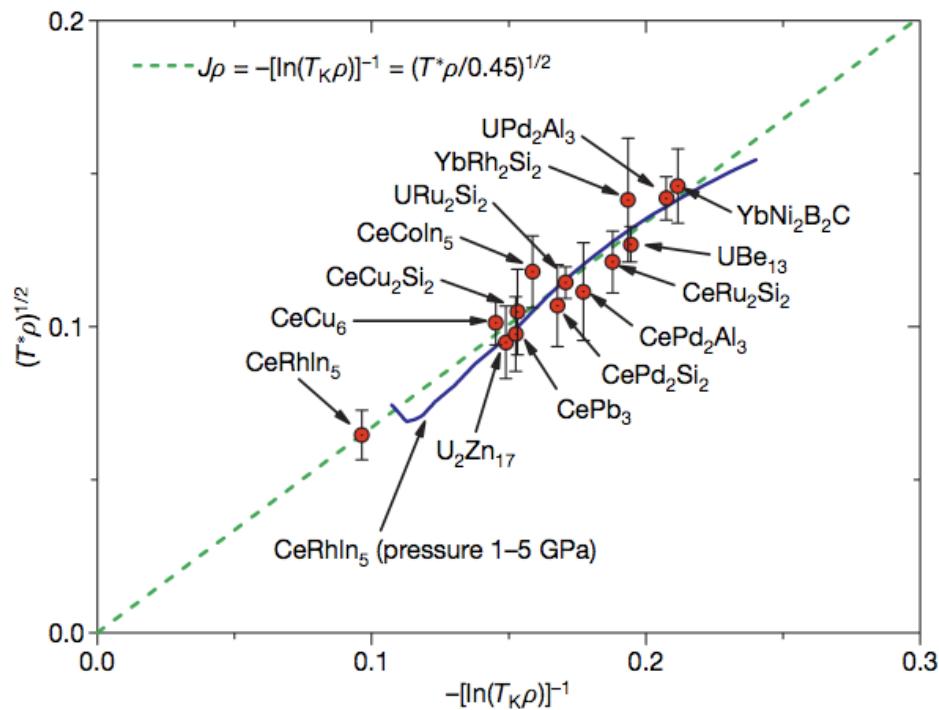


Figure 1 | Confirmation of T^* given by the intersite RKKY interaction for a variety of Kondo lattice materials. The solid line shows T^* (resistivity peak) of CeRhIn₅ under pressure from 1 GPa (lower left) to 5 GPa (upper right).

In mean-field theory or DMFT, the onset of coherence is given by the Kondo scale!

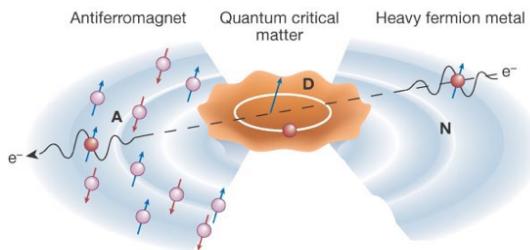
Hence Kondo lattice physics is different from the single-ion Kondo physics!

A new scaling different from Kondo scaling !!!

Yang et al, Nature 454, 611 (2008).

This demonstrates that the heavy electron emergence is collective and originates in the coupling between neighboring f-electrons.

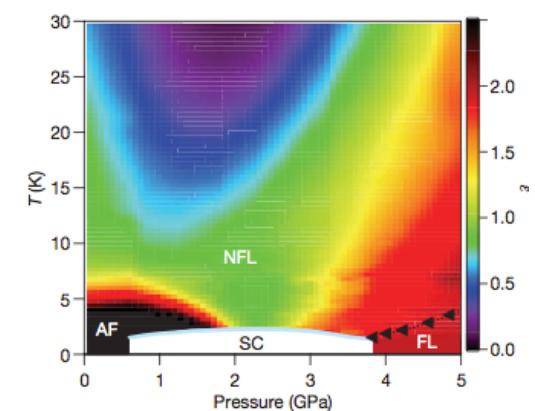
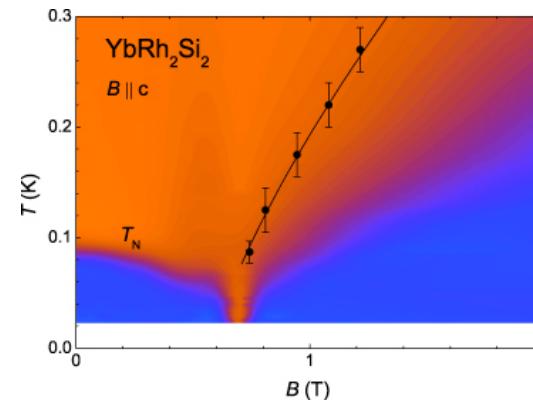
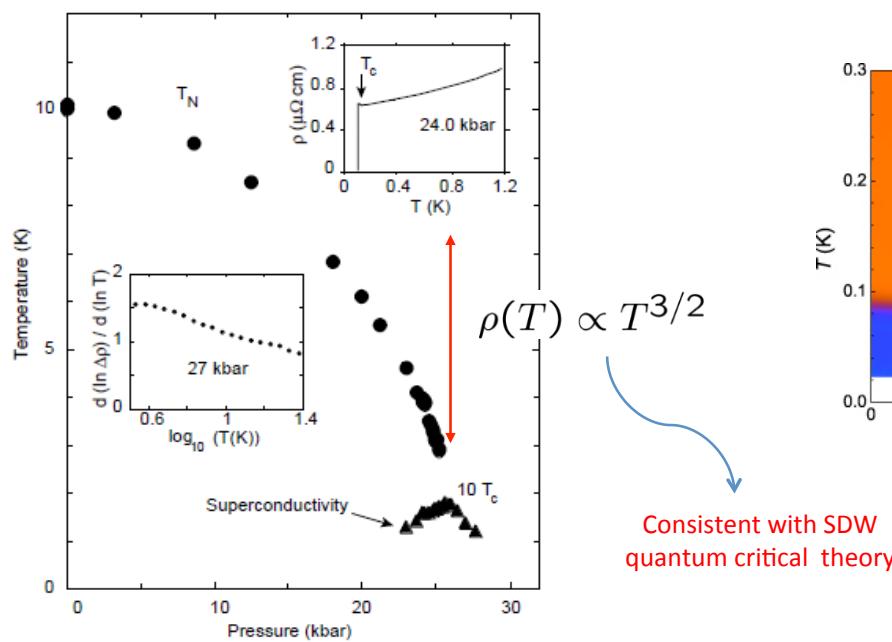
量子相变与量子临界的问题



● Quantum criticality @ low T

- ✓ RKKY & Magnetism
 - ✓ SDW scenario (Hertz/Millis/Moriya)

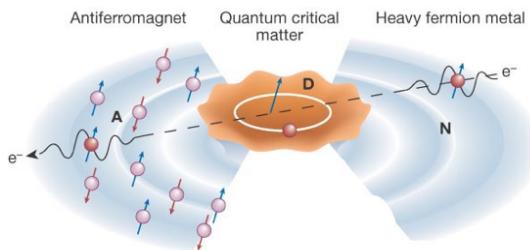
- Power-law scaling in resistivity, specific heat, susceptibility
 - Peak at finite wave vector in dynamic susceptibility
 - **No ω/T scaling ...**



The SDW predictions are violated

Long-wavelength Spin density wave (SDW) fluctuations

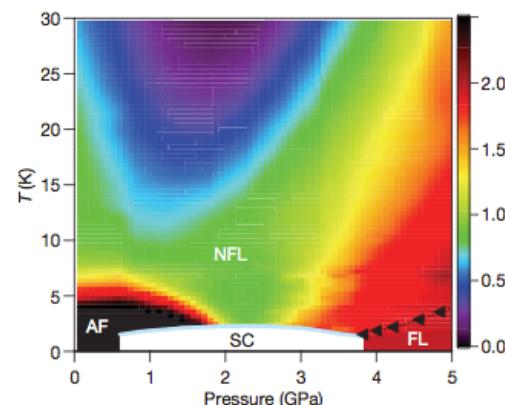
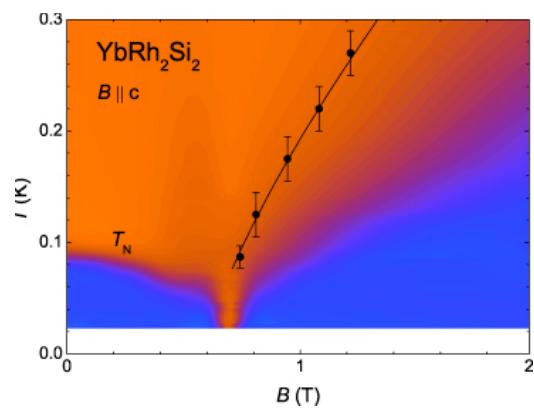
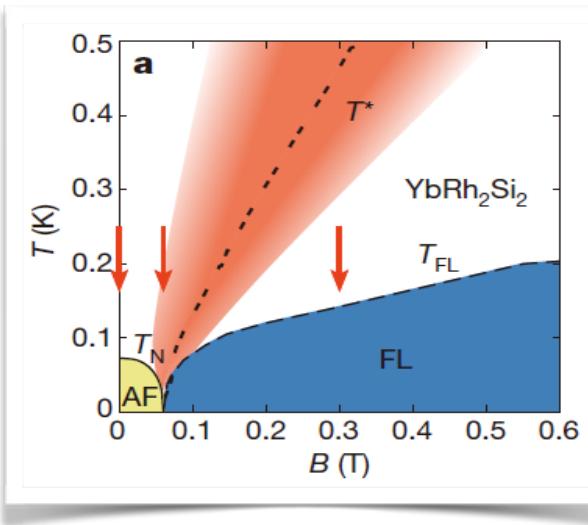
量子相变与量子临界的问题



Quantum criticality @ low T

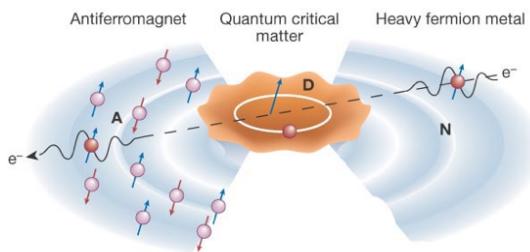
- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001

AFM quantum phase transition
accompanied with
suppression of Kondo screening
(Fermi surface change)



The SDW predictions are violated

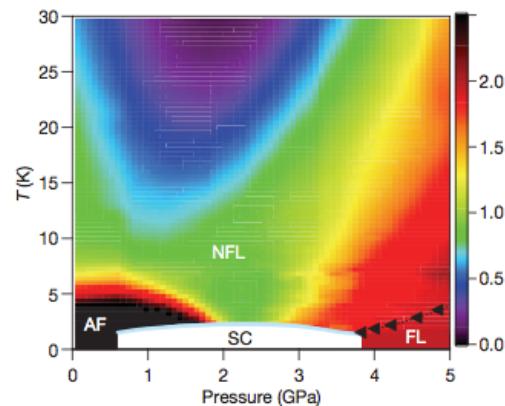
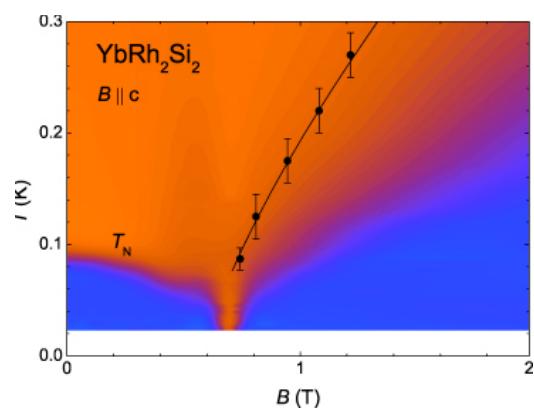
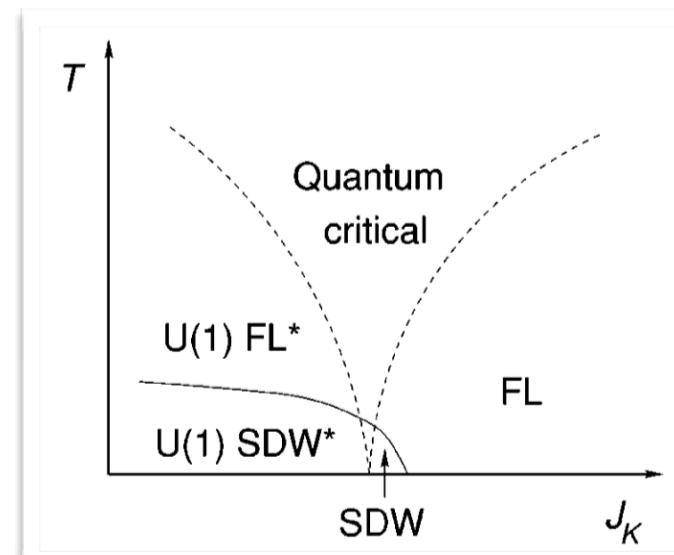
量子相变与量子临界的问题



Fractionalized excitations near QCP
spinon/spinon SDW ...

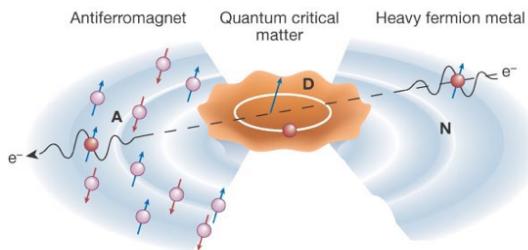
Quantum criticality @ low T

- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001
- ✓ Fractionalization @2004



The SDW predictions are violated

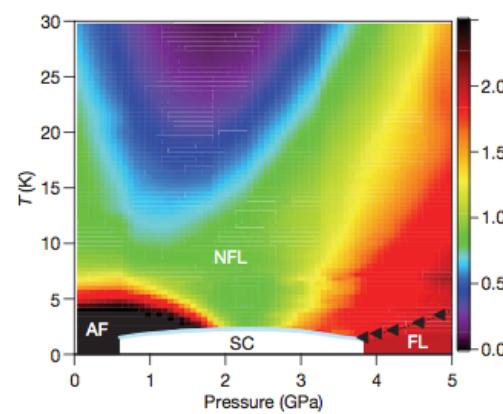
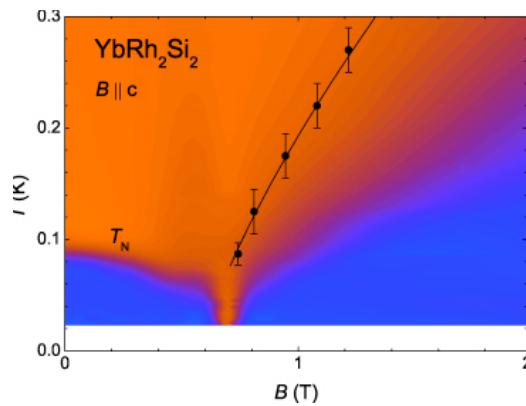
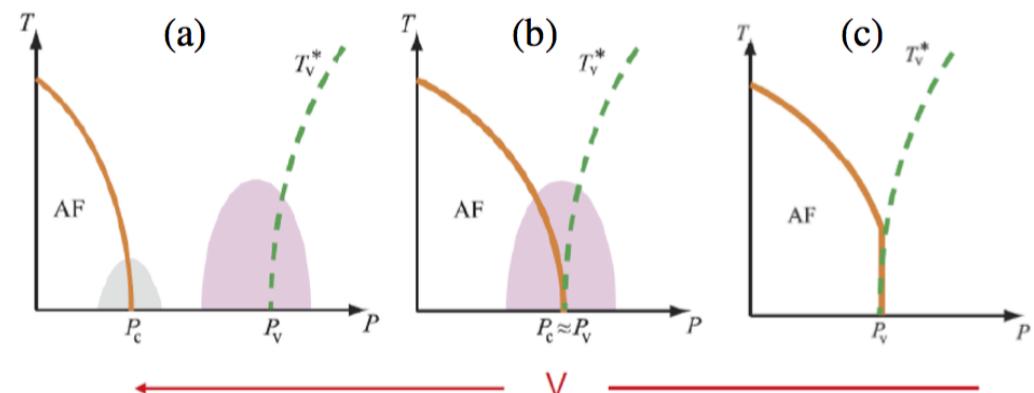
量子相变与量子临界的问题



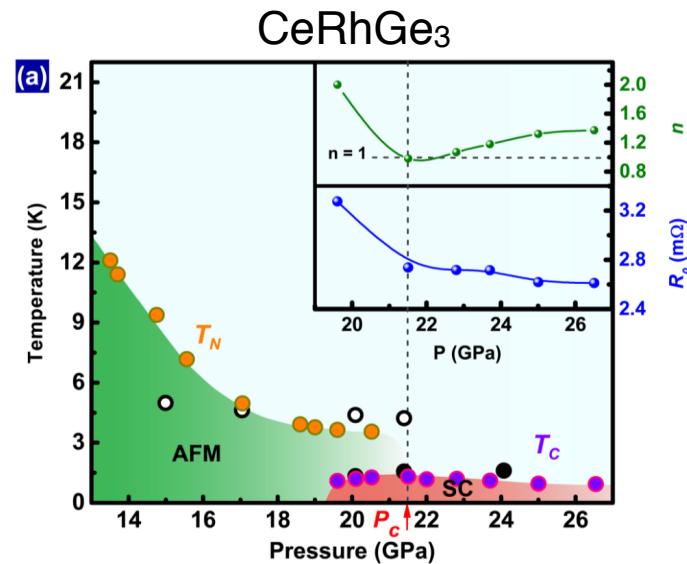
Quantum criticality @ low T

- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001
- ✓ Fractionalization @2004
- ✓ Critical valence fluctuations @2010

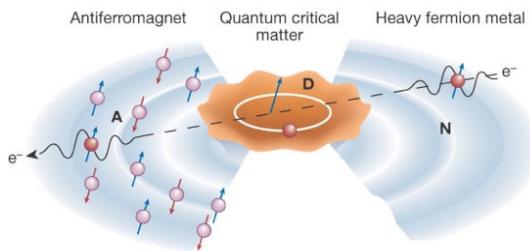
Valence transition & abrupt FS change
with hybridization on both sides



The SDW predictions are violated



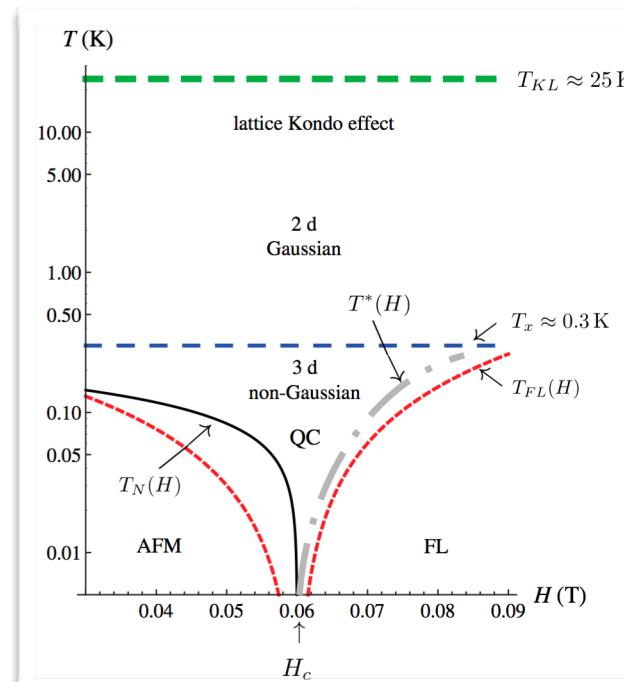
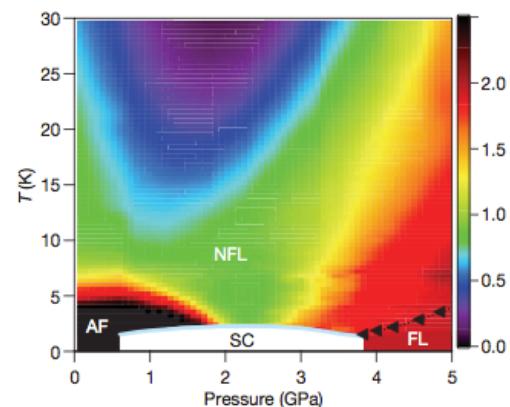
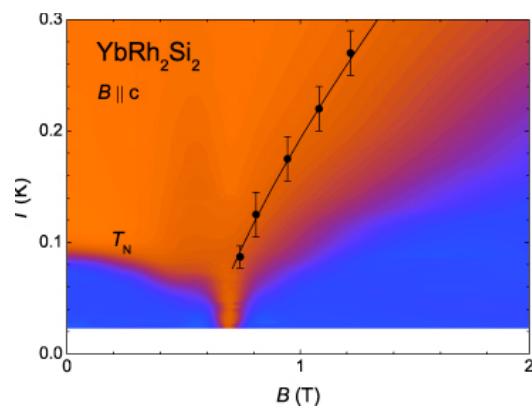
量子相变与量子临界的问题



Heavy quasiparticles still exist
but become critical: $N(E) \sim |E|^\alpha$

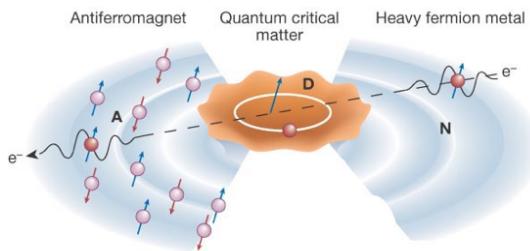
Quantum criticality @ low T

- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001
- ✓ Fractionalization @2004
- ✓ Critical valence fluctuations @2010
- ✓ Critical quasiparticle @2011



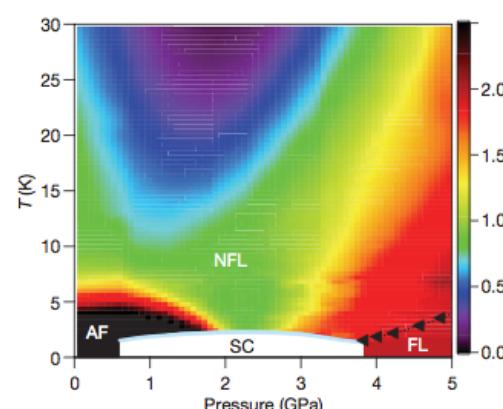
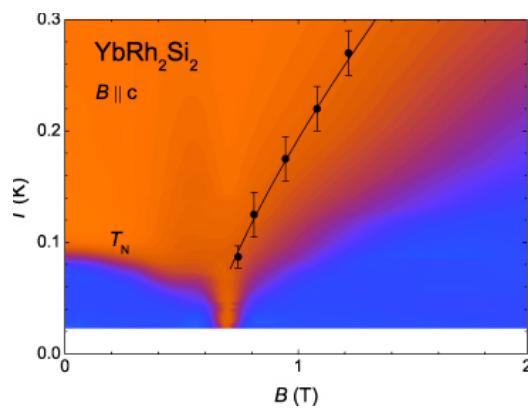
The SDW predictions are violated

量子相变与量子临界的问题



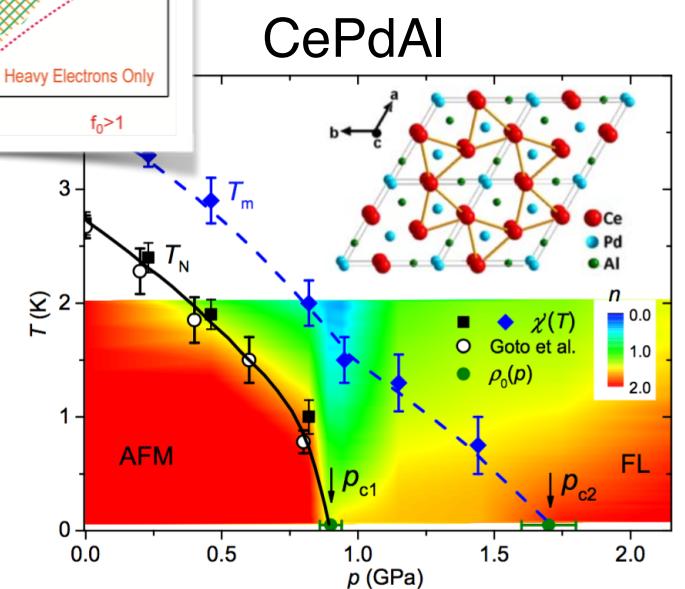
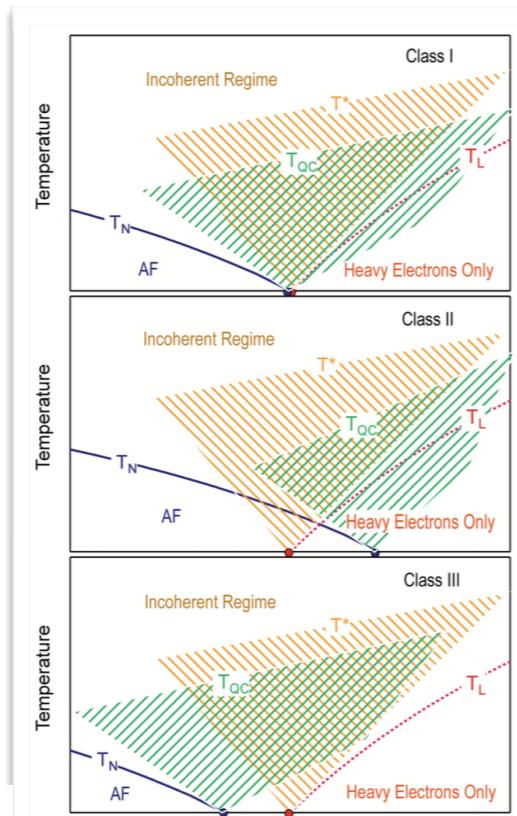
Quantum criticality @ low T

- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001
- ✓ Fractionalization @2004
- ✓ Critical valence fluctuations @2010
- ✓ Critical quasiparticle @2011
- ✓ Magnetic & hybridization fluc. @2017

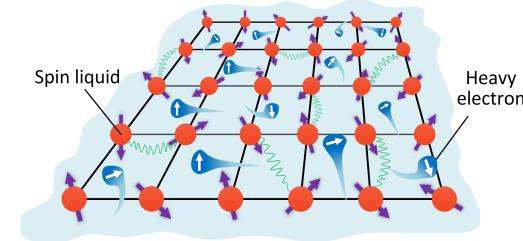
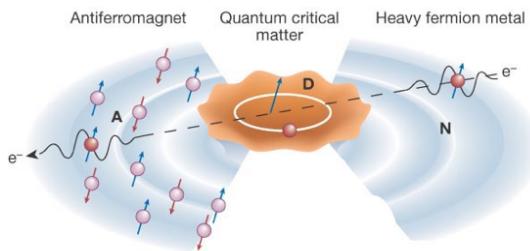


The SDW predictions are violated

Interplay of two types of quantum critical fluctuations

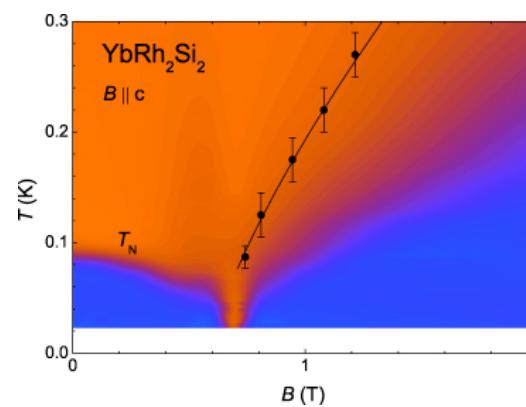


f电子的局域-巡游转变



Quantum criticality @ low T

- ✓ RKKY & Magnetism
- ✓ SDW scenario (Hertz/Millis/Moriya)
- ✓ Kondo breakdown @2001
- ✓ Fractionalization @2004
- ✓ Critical valence fluctuations @2010
- ✓ Critical quasiparticle @2011
- ✓ Magnetic & hybridization fluc. @2017



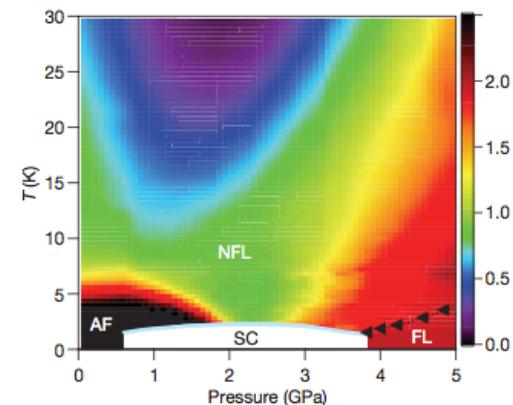
Non-Fermi liquid @ high T

- ✓ Onset of coherence (What is T^* ?)
- ✓ Local/Itinerant crossover (Two fluids?)
- ✓ New scaling (Different from Kondo?)
- ✓ Two types of quantum fluctuations?



New Quantum Critical Matter

SC, Hidden order, TI ...

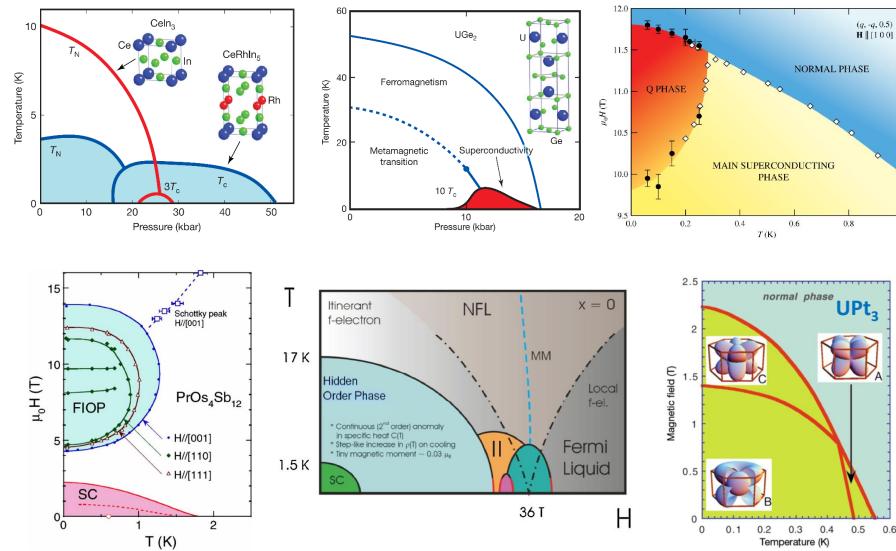


第二部分

重费米子超导的发展与机理研究

- 重费米子超导与竞争序
- 重费米子超导机理与前沿问题

一、重费米子超导与竞争序



参考文献

李宇, 杨义峰, 重费米子超导与竞争序, 物理学报 64, 217401 (2015)

1979, CeCu₂Si₂(1st HF SC)

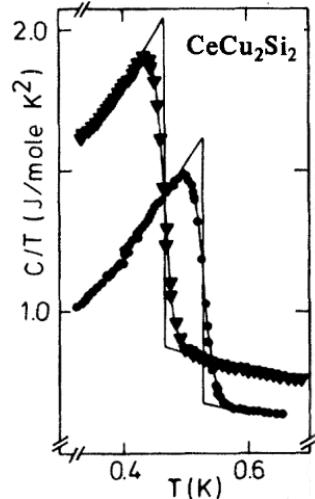
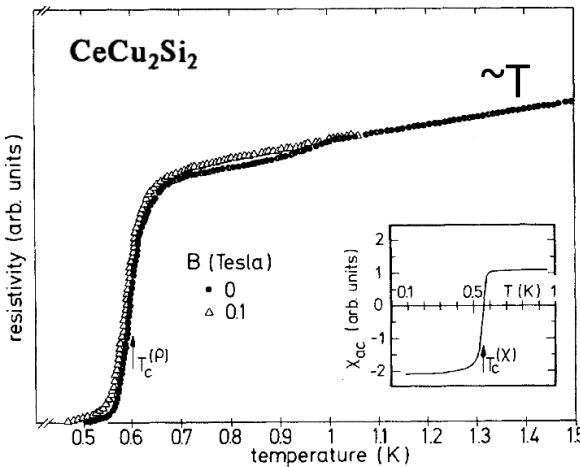
VOLUME 43, NUMBER 25

PHYSICAL REVIEW LETTERS

17 DECEMBER 1979



(F. Steglich)



Superconductivity in the Presence of Strong Pauli Paramagnetism: CeCu₂Si₂

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and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz

I. Physikalisches Institut, Universität zu Köln, D-5000 Köln 41, West Germany

and

H. Schäfer

Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

(Received 10 August 1979; revised manuscript received 7 November 1979)

The size of the specific-heat jump at T_c , in proportion to γT_c , suggests that Cooper-pair states are formed by these heavy fermions. Since the Debye temperature, Θ , is of the order of 200 K,⁵ we find $T_c < T_F < \Theta$ with $T_c/T_F \approx T_F/\Theta \approx 0.05$. This suggests that CeCu₂Si (i) behaves as a ‘high-temperature superconductor’ and (ii) cannot be described by conventional theory of superconductivity which assumes a typical phonon frequency $k_B\Theta/h \ll k_B T_F/h$, the characteristic frequency of the fermions.

Passed the HF SC by! (1975, UBe₁₃) (1978, CeCu₂Si₂)

PHYSICAL REVIEW B VOLUME 11, NUMBER 1 1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides

E. Bucher,* J. P. Maita, G. W. Hull, R. C. Fulton, and A. S. Cooper

Bell Laboratories, Murray Hill, New Jersey 07974

(Received 14 March 1974)

UBe₁₃

We tried to detect any possible magnetic ordering below 1 °K. Instead we found a sharp superconducting transition at 0.97 °K, which was reduced by about 0.3 °K only in a field of 60 kOe. This suggests that the superconductivity is not an intrinsic property of UBe₁₃, but probably linked with precipitated filaments. Subsequent powdering did not shift nor reduce the superconducting signal, although calibration with a Pb cylinder showed that the signal of UBe₁₃ was only about 50% of the expected full signal. From the fact that none of the other MBe₁₃ phases showed superconductivity down to 0.45 °K, one is tempted to conclude that the superconductivity and perhaps also the susceptibility tail at low temperature is due to precipitated U filaments.

Z. Physik B 31, 7–17 (1978)

Zeitschrift
für Physik B
© by Springer-Verlag 1978

Transport Properties of LaCu₂Si₂ and CeCu₂Si₂ Between 1.5 K and 300 K***

W. Franz, A. Grießel, F. Steglich, and D. Wohlleben

II. Physikalisches Institut der Universität zu Köln, Köln, Fed. Rep. Germany

Received May 23, 1978

CeCu₂Si₂

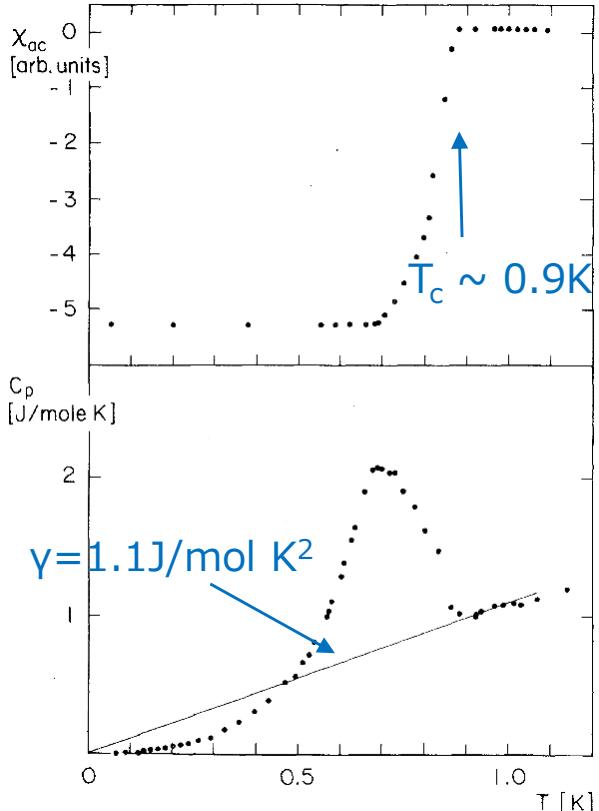
* The resistivity measurement of W. Lieke below 1.5 K showed a continuous drop down to about 0.6 K and then a superconducting transition. The superconductivity could be suppressed at 0.3 K by application of a magnetic field of 3 T. At this field the resistivity was 41 μΩ cm and it did not change upon further increase of the field. We therefore consider 41 μΩ cm as representative of the residual resistivity of this sample. In order to check whether the superconductivity was a bulk property of CeCu₂Si₂ or due to a second phase forming a network through grain boundaries the specific heat was measured down to 0.3 K by C.D. Bredl and the static Meissner effect (on bulk and powdered samples) in a SQUID magnetometer by R.F. Hoyt and A.C. Mota down to 30 mK. Both measurements indicate the absence of bulk superconductivity. According to the static Meissner effect less than 0.1 % of the sample volume is superconducting.

1983, UBe₁₃

VOLUME 50, NUMBER 20

PHYSICAL REVIEW LETTERS

16 MAY 1983



UBe₁₃: An Unconventional Actinide Superconductor

H. R. Ott and H. Rudigier

*Laboratorium für Festkörperphysik, Eidgenössische Technische Hochschule-Hönggerberg,
CH-8093 Zürich, Switzerland*

and

Z. Fisk and J. L. Smith

Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 14 March 1983)

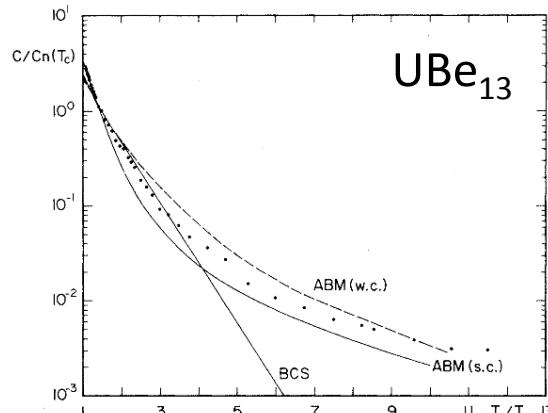


FIG. 3. $C_s/C_n(T_c)$ for superconducting UBe₁₃. Dashed line: weak-coupling ABM state; solid lines: BCS and strong-coupling ABM state from Eq. (5).



(H. R. Ott)



(Z. Fisk)

(H. R. Ott, et al, PRL 52, 1915(1984))

- $C_V \propto T^3$ below T_c , and analogy to He³, UBe₁₃ is the 1st p-wave SC. (H. R. Ott, et al, PRL(1984))
 (But it's incorrect due to inherent symmetry of crystal lattice and strong spin-orbit coupling) (Thompson, PPT, 2007)

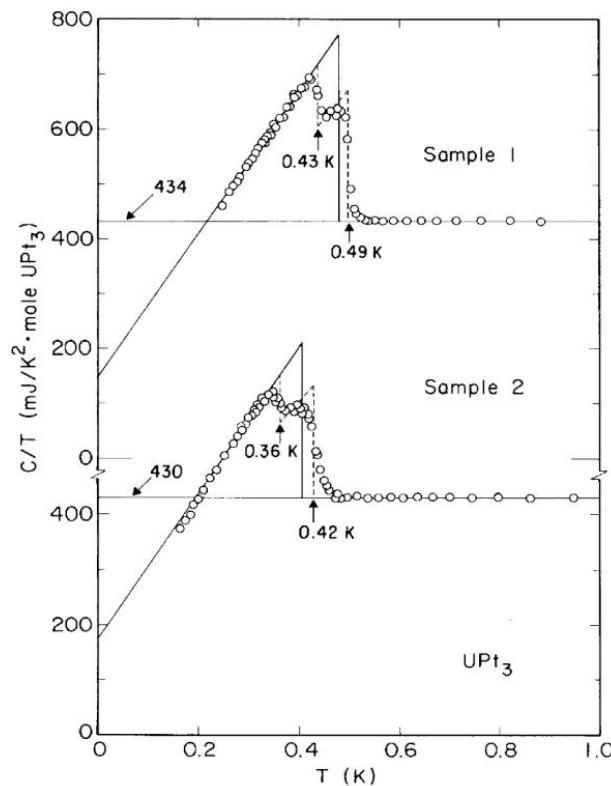
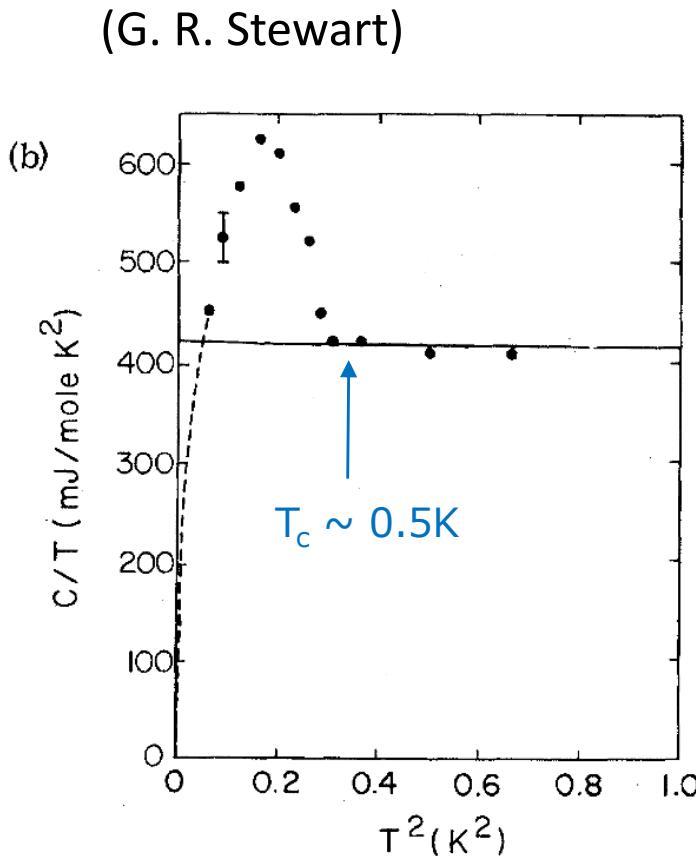


Possibility of Coexistence of Bulk Superconductivity and Spin Fluctuations in UPt₃

G. R. Stewart, Z. Fisk, J. O. Willis, and J. L. Smith

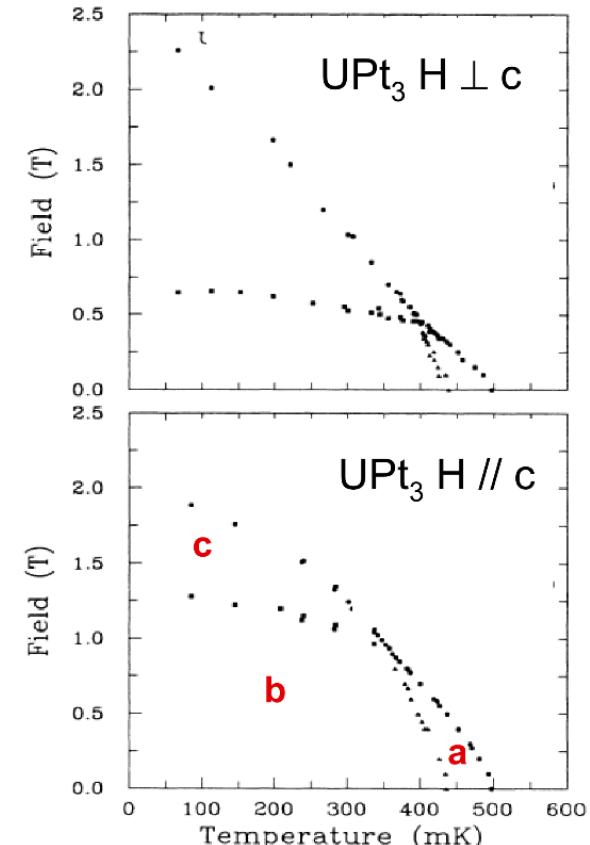
Los Alamos National Laboratory, Los Alamos, New Mexico 87545

(Received 24 October 1983)



2 SC transition($H=0$)

(R. A. Fisher et al, PRL 62, 1411(1989))



3 SC($H>0$)

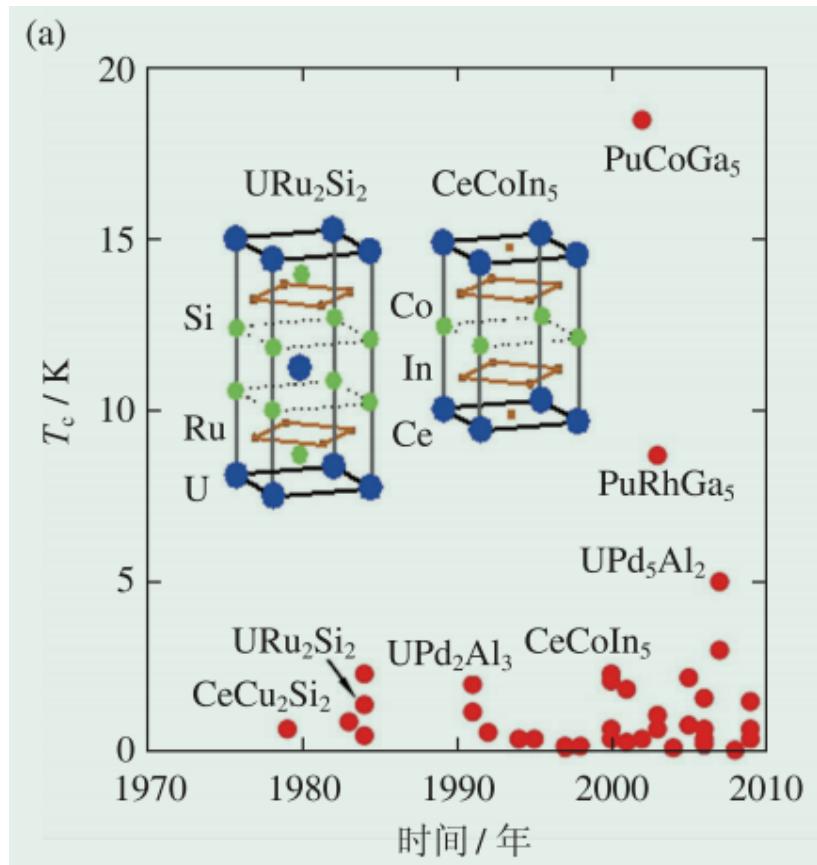
(S. Adenwalla et al, PRL 65, 2298(1990))

Serials	Compounds	T _c (K)	γ	nodes
CeM ₂ X ₂ (AFM SC)	CeCu ₂ Si ₂	0.6-0.7	1000	No
	CeCu ₂ Ge ₂	0.64(10.1GPa)	200	-
	CePd ₂ Si ₂	0.43 (3GPa)	65	-
	CeRh ₂ Si ₂	0.42 (1.06GPa)	23	-
	CeAu ₂ Si ₂	2.5K(22.5GPa)	-	-
	CeNi ₂ Ge ₂	0.3	350	-
Ce _n M _m In _{3n+2m} (AFM SC)	CeIn ₃	0.23 (2.46GPa)	140	Line
	CeIrIn ₅	0.4	750	Line
	CeCoIn ₅	2.3	250	Line
	CeRhIn ₅	2.4 (2.3GPa)	430	-
	CePt ₂ In ₇	2.3 (3.1GPa)	340	-
	Ce ₂ RhIn ₈	2.0 (2.3GPa)	400	-
	Ce ₂ PdIn ₈	0.68	550	Line
	Ce ₂ CoIn ₈	0.4	500	-
	Ce ₃ PdIn ₁₁	0.42	290	-
Ce-based non-centresymmetric SC	CePt ₃ Si	0.75	390	Line
	CeIrSi ₃	1.65 (2.5GPa)	120	-
	CeRhSi ₃	1.0 (2.6GPa)	120	-
	CeCoGe ₃	0.69 (6.5GPa)	32	-
Other Ce-based AFM SC	CeNiGe ₃	0.43 (6.8GPa)	45	-
	Ce ₂ Ni ₃ Ge ₅	0.26 (4.0GPa)	90	-
	CePd ₅ Al ₂	0.57 (10.8GPa)	56	-

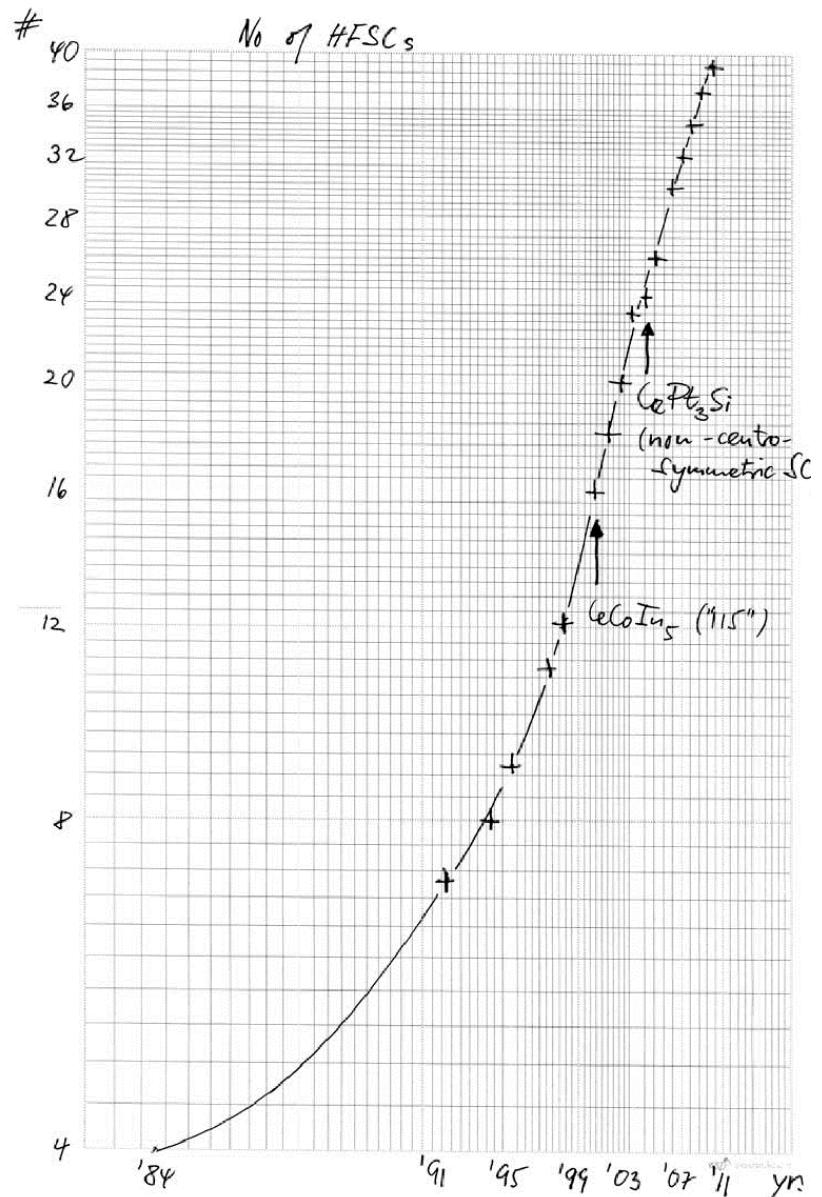
Serials	Compounds	T _c (K)	γ	nodes
U-based AFM SC	UPd ₂ Al ₃	2.0	210	Line
	UNi ₂ Al ₃	1.06	120	-
	UBe ₁₃	0.95	1000	No
	U ₆ Fe	3.8	157	-
	UPt ₃	0.53, 0.48	440	Line, Point
U-based FM SC	UGe ₂	0.8 (~1.2GPa)	34	Line
	URhGe	0.3	164	-
	UCoGe	0.6	57	Point
	UIr	0.15 (~2.6GPa)	49	-
	U ₂ PtC ₂	1.47	150	-
Hidden order SC	URu ₂ Si ₂	1.5	70	Line
Pr-based SC	PrOs ₄ Sb ₁₂	1.85	500	点
	PrTi ₂ Al ₂₀	0.2, 1.1(8.7GPa)	100	-
	PrV ₂ Al ₂₀	0.05	90	-
	PuCoGa ₅	18.5	77	Line
Pu-based SC (Pu-115)	PuCoIn ₅	2.5	200	Line
	PuRhGa ₅	8.7	70	Line
	PuRhIn ₅	1.6	350	Line
	NpPd ₅ Al ₂	4.9	200	Point
Yb-based SC	β -YbAlB ₄	0.08	150	-
	YbRh ₂ Si ₂	0.002	-	-

(γ —
mJ·mol⁻¹·K⁻²)

Discovery of HF Superconductors

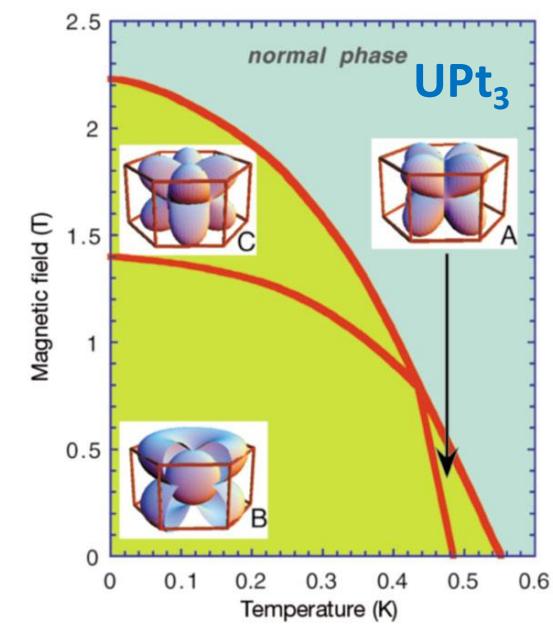
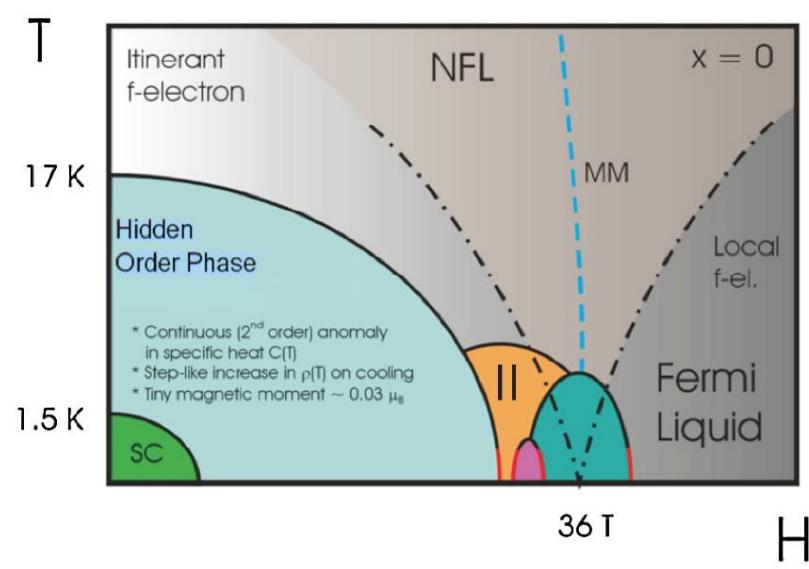
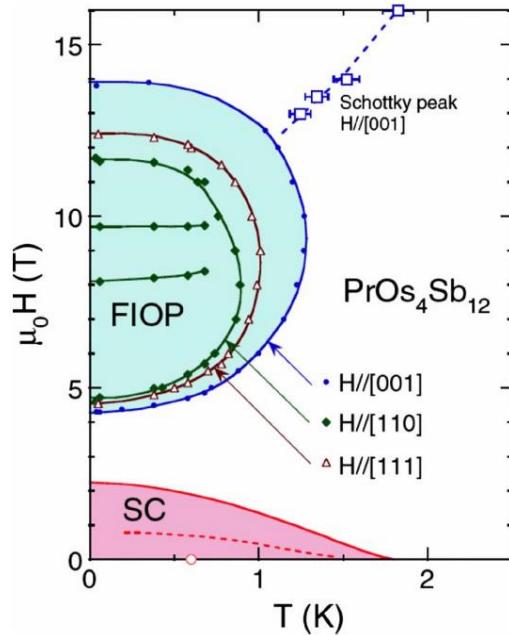
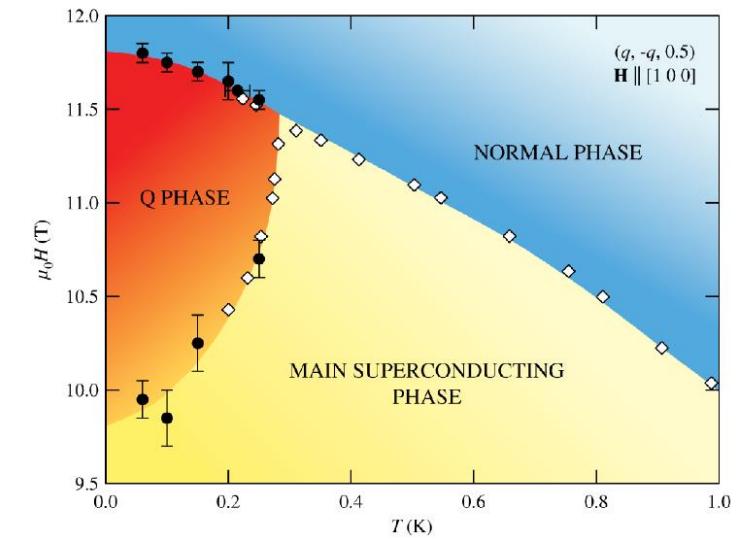
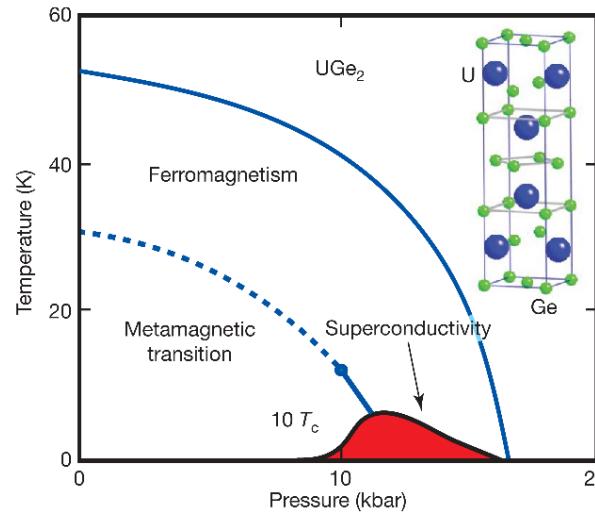
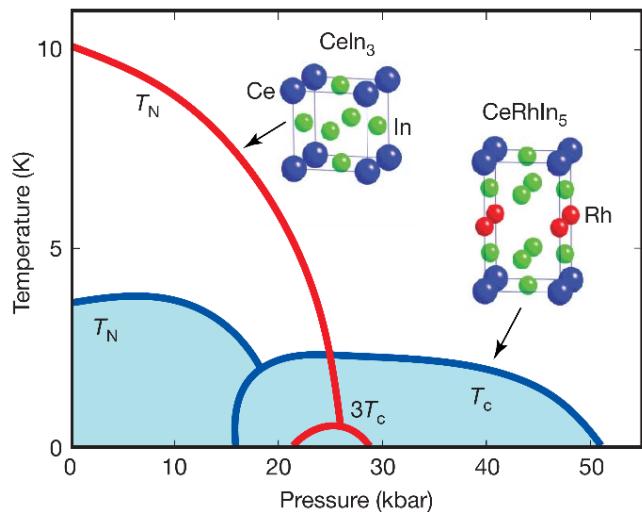


Yang, 物理 43, 80 (2014)



(F. Steglich, PPT)

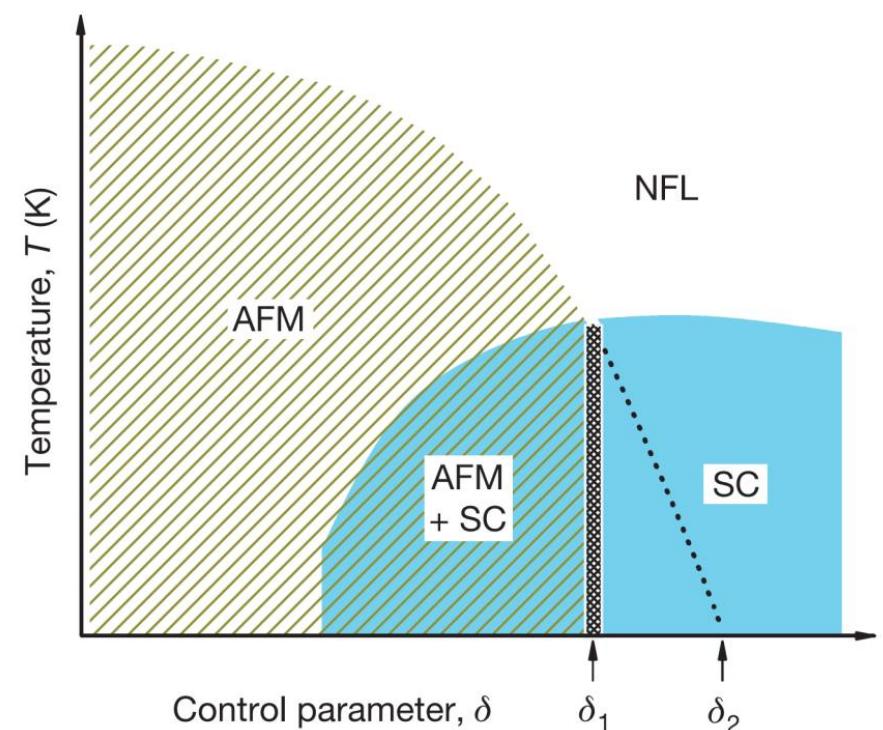
Varieties of phase diagram in HF SC



$Ce_nM_mX_{3n+2m}$: basic properties

Compound	Ground State	T_N, T_c (K)
CeCoIn ₅	SC	2.3
CeRhIn ₅	AFM/SC (P)	3.8/2.4 (2.3 GPa)
CeIrIn ₅	SC	0.4
CePt ₂ In ₇	AFM/SC (P)	5.5/2.3 (3.1 GPa)
Ce ₂ CoIn ₈	SC	0.4
Ce ₂ RhIn ₈	AFM/SC (P)	2.8/2.0 (2.3 GPa)
Ce ₂ PdIn ₈	SC	0.7
Ce ₃ PdIn ₁₁	AFM + SC	1.6 + 1.5 + 0.4 (SC)
PuCoIn ₅	SC	2.5
PuRhIn ₅	SC	1.6
PuCoGa ₅	SC	18.5
PuRhGa ₅	SC	8.7

(White, Thompson, Maple, Physica C(2015))

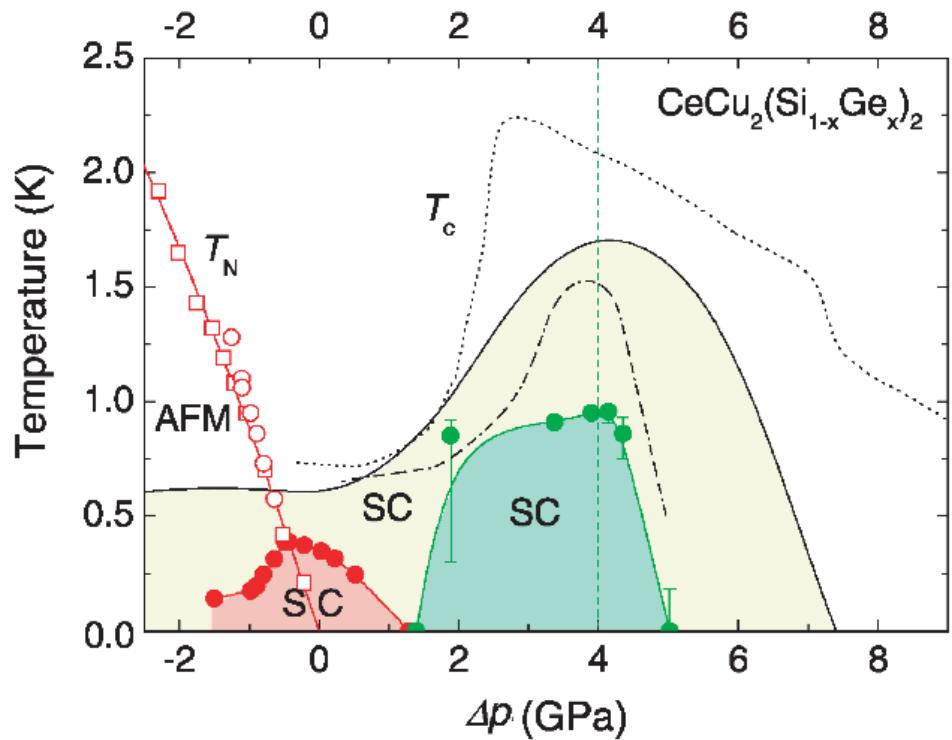
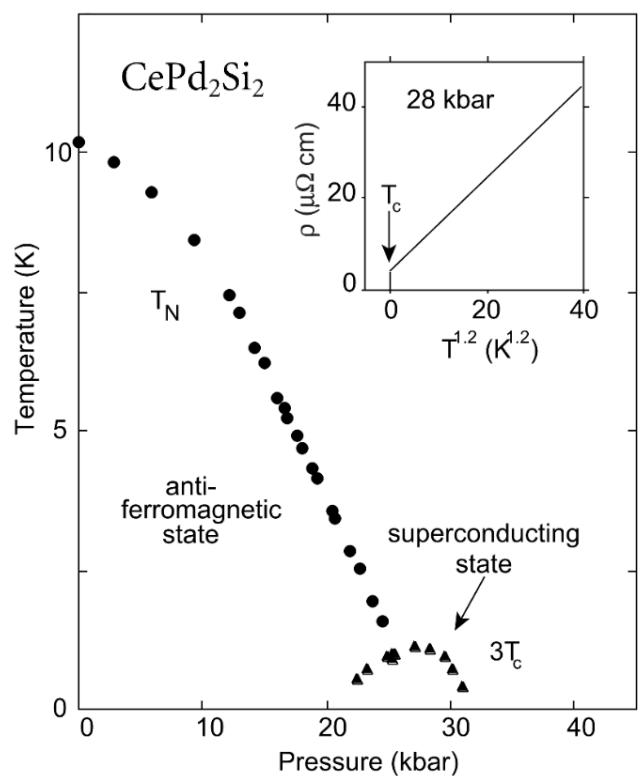


(T. Park et al, nature 440, 65(2006))

CeM₂X₂: at the border of Antiferromagnetism

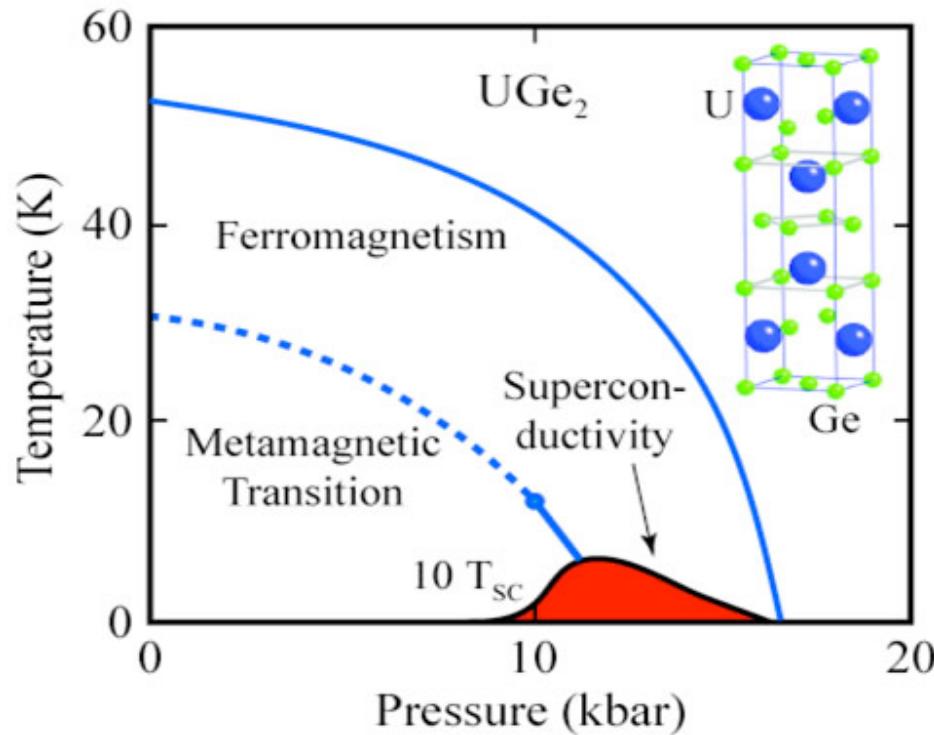
	CeCu ₂ Si ₂	CeCu ₂ Ge ₂	CePd ₂ Si ₂	CeRh ₂ Si ₂	CeNi ₂ Ge ₂	CeNiGe ₃	Ce ₂ Ni ₃ Ge ₅	CePd ₅ Al ₂
State	AF, SC	AF, SC	AF, SC	AF, SC	ISC	AF, SC	AF, SC	AF, SC
T _N (K)	0.8	4.15	10	36, 25		5.5	5.1, 4.5	3.9, 2.9

• Valence transition



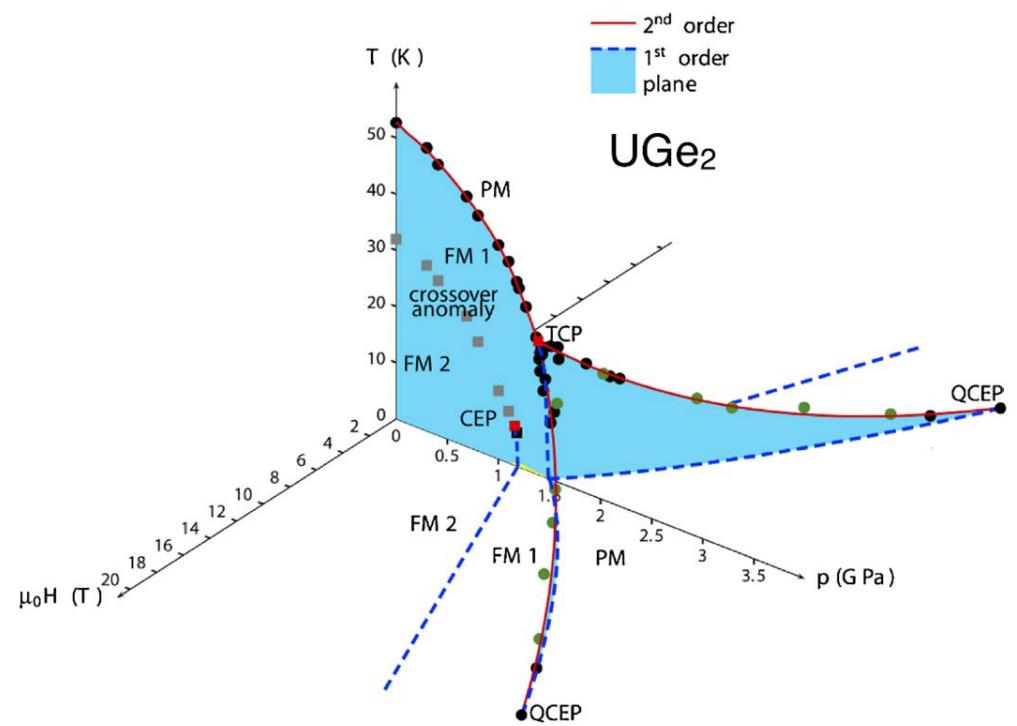
(H. Yuan et al, Science, 302, 2104(2003))

UGe₂:



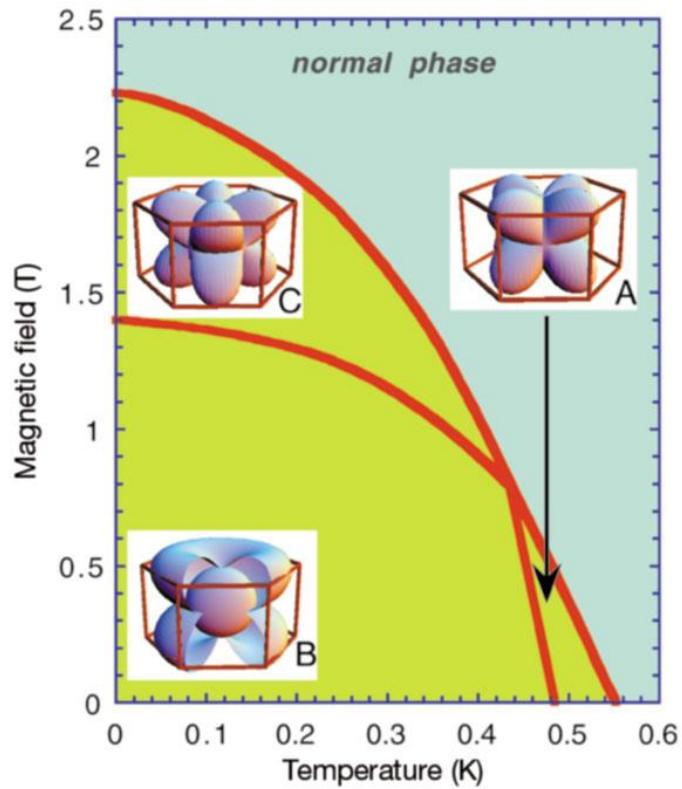
(A. Huxley et al, JPCM 15, S1945(2003))

- 2 distinct ferromagnetic phases.
- SC well below the Curie temperature.
- SC and FM do not merely coexist, but also implying the emergence of SC depend on the presence of FM.

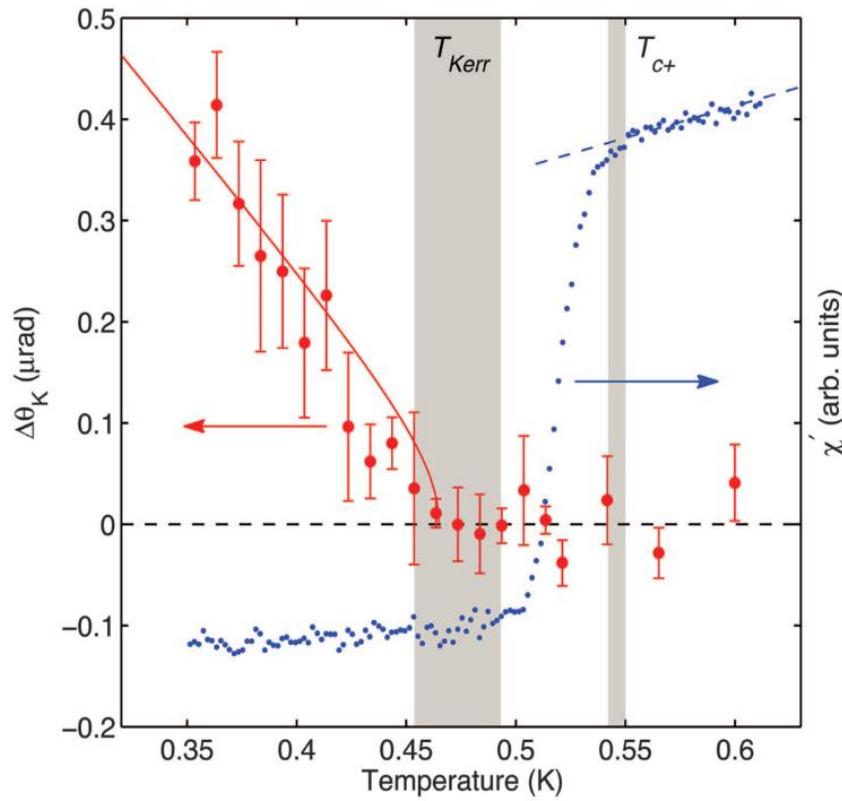


(D. Aoki et al, JPSJ 83, 061011(2014))

UPt₃: time-reversal symmetry breaking(Kerr effect)

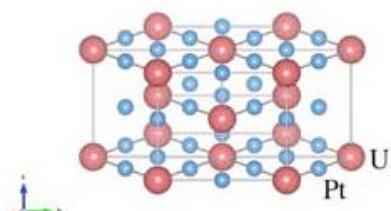


(A. Huxley et al, Nature 406, 160(2000))



(E. R. Schemm et al, Science 345, 190(2014))

B phase: complex-two component order parameter and time-reversal symmetry breaking (nonzero polar Kerr effect as $T < 0.45\text{K}$ (E. R. Schemm et al, Science 345, 190(2014)))



利用极化光探测时间反演对称破缺

PrT₂X₂₀ compounds: Quadrupolar

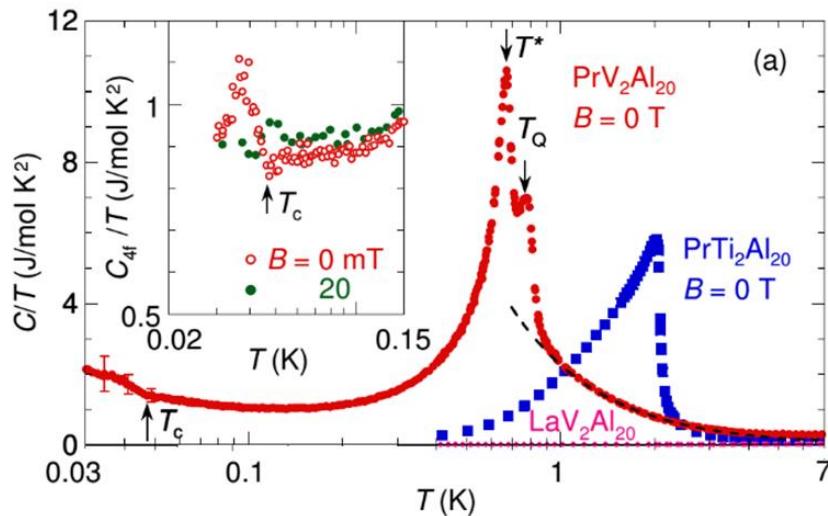
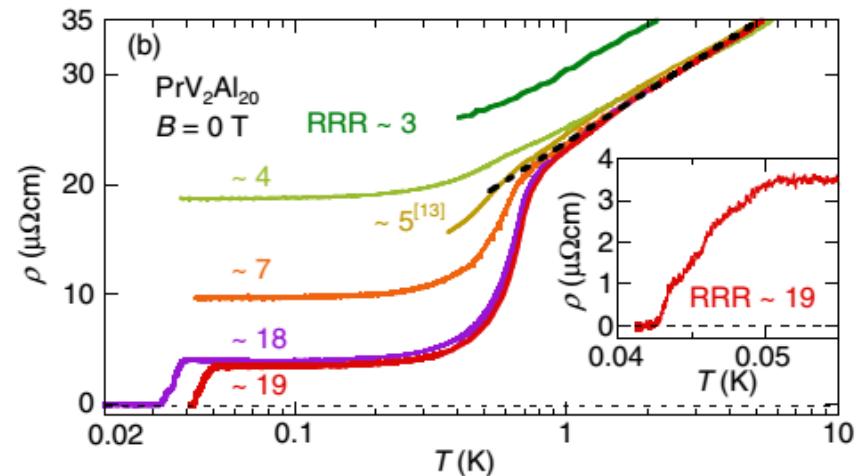
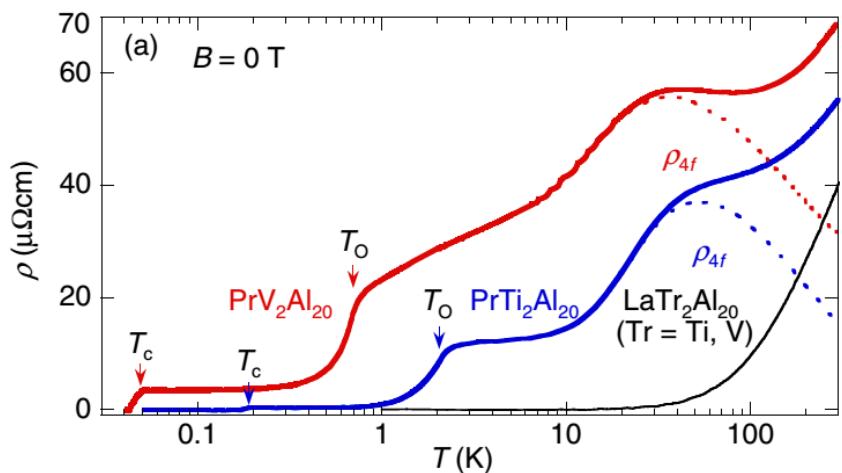
- PrT₂X₂₀: 1st discovered in 2010
- Pr: non-Kramer ground state, nonmagnetic, but quadrupolar.

PrT₂X₂₀ (X=Al and Zn) family is appropriate for revealing characteristics of the quadrupoles such as quadrupole order, multi-channel Kondo effect, exotic superconductivity.

PrT₂X₂₀ (T: Transition metal, X: Al, Zn)

	Lattice parameter (Å)	Structural transition	CEF ground state	Quadrupole order	SC transition
PrRu ₂ Zn ₂₀	14.3467(4)	T _S =138 K	Singlet (T < T _S)	—	— (>0.04 K)
PrRh ₂ Zn ₂₀	14.2702(3)	T _S =170 ~470 K	Γ ₂₃ doublet (T) (T < T _S)	AFQ T _Q =0.06 K	T _c =0.06 K
PrOs ₂ Zn ₂₀	14.365(5)	T _S =87 K	?	— (>0.4 K)	— (>0.4 K)
PrIr ₂ Zn ₂₀	14.2729(2)	—	Γ ₃ doublet	AFQ T _Q =0.11 K	T _c =0.05 K
PrTi ₂ Al ₂₀	14.723(7)	—	Γ ₃ doublet	FQ T _Q =2 K	T _c =0.2 K (a. p.) T _c =1 K (~8 GPa)
PrV ₂ Al ₂₀	14.591(2)	—	Γ ₃ doublet	AFQ T _Q =0.6 K	T _c =0.05 K
PrNb ₂ Al ₂₀	14.7730(3)	—	Γ ₃ doublet	—	—

$\text{PrT}_2\text{X}_{20}$: Quadrupole quantum criticality

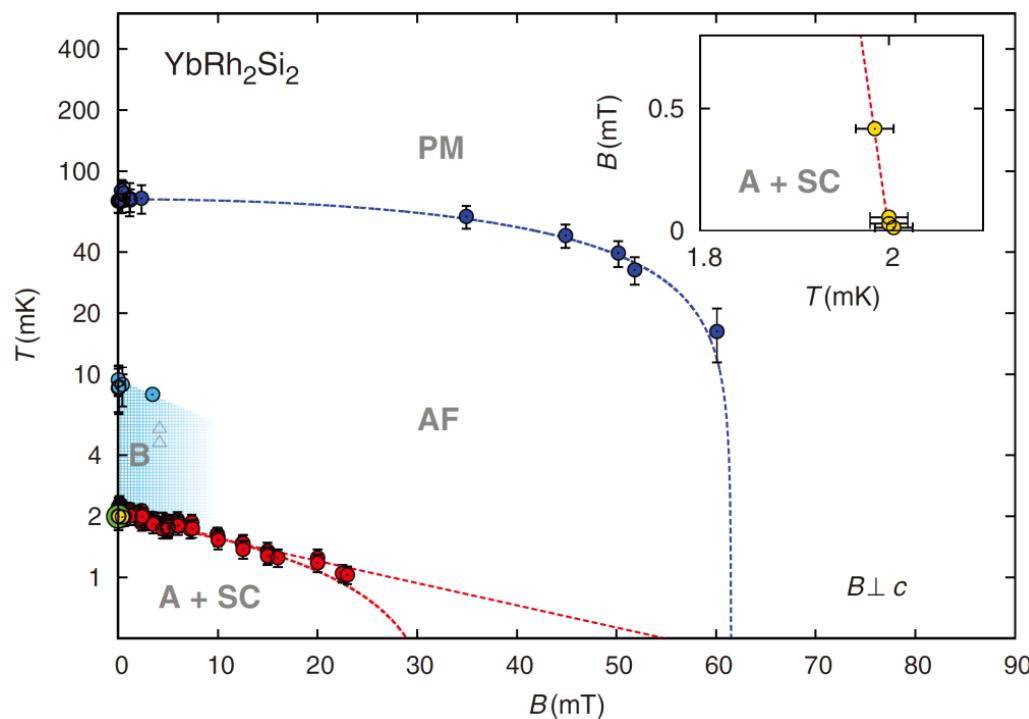


T_Q — quadrupole ordering;
 T^* — octapole ordering

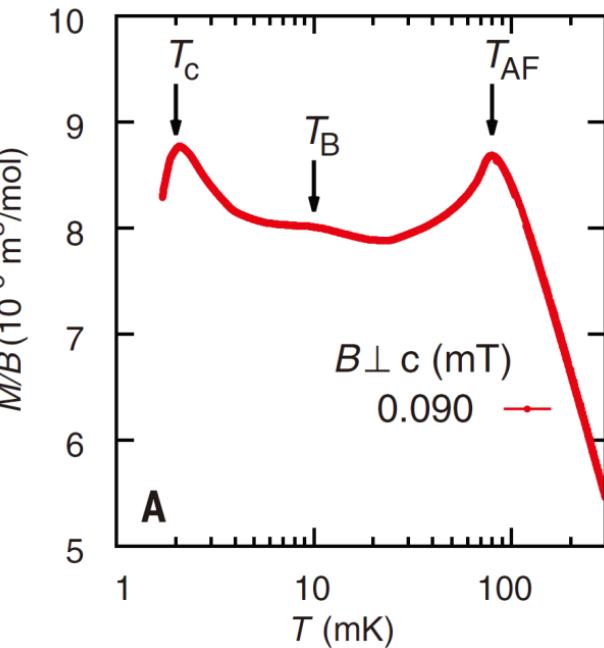
Quadrupole fluctuation mediated superconductivity? Quadrupole Kondo effect? Orbital quantum criticality?

Emergence of superconductivity in the canonical heavy-electron metal YbRh_2Si_2

Erwin Schuberth,^{1,2,*} Marc Tippmann,^{1,3} Lucia Steinke,^{1,2} Stefan Lausberg,² Alexander Steppke,² Manuel Brando,² Cornelius Krellner,^{2,4} Christoph Geibel,² Rong Yu,^{5,6} Qimiao Si,^{7,*} Frank Steglich^{2,8,9,*}



E. Schuberth *et al.*, Science **351**, 485 (2016)



- $T_c = 2 \text{ mK}$, $T_{AF} = 70 \text{ mK}$.
 - **A-phase: nuclear antiferromagnetic order**
 - **“Nuclear Kondo effect”** —— the formation of a singlet state between the nuclear and conduction electron spins.
- responsible for Cooper pairing ?

Spin-fluctuation theory

Ferromagnetic spin fluctuation theory on ^3He (1970s)



VOLUME 54, NUMBER 12

PHYSICAL REVIEW LETTERS

25 MARCH 1985

1985

Attractive Interaction and Pairing in Fermion Systems with Strong On-Site Repulsion

J. E. Hirsch

Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 19 November 1984)

Antiferromagnetic interaction could provide the driving force for anisotropic singlet pairing.

RAPID COMMUNICATIONS

PHYSICAL REVIEW B

1986

VOLUME 34, NUMBER 9

1 NOVEMBER 1986

Spin-fluctuation-mediated even-parity pairing in heavy-fermion superconductors

K. Miyake,* S. Schmitt-Rink, and C. M. Varma

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 27 June 1986; revised manuscript received 11 August 1986)

PHYSICAL REVIEW B

VOLUME 34, NUMBER 11

1 DECEMBER 1986

Possible superconductivity in nearly antiferromagnetic itinerant fermion systems

M. T. Béal-Monod and C. Bourbonnais

Laboratoire de Physique des Solides, Bâtiment 510, Université Paris-Sud, 91405 Orsay, France

V. J. Emery

*Physics Department, Brookhaven National Laboratory, Upton, New York 11973
and Laboratoire de Physique des Solides, Université Paris-Sud, 91405 Orsay, France*

(Received 12 May 1986)

RAPID COMMUNICATIONS

PHYSICAL REVIEW B

VOLUME 34, NUMBER 11

1 DECEMBER 1986

d-wave pairing near a spin-density-wave instability

D. J. Scalapino, E. Loh, Jr.,* and J. E. Hirsch†

Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

(Received 23 June 1986)

MSV

t-J model with AFM interaction

BBE

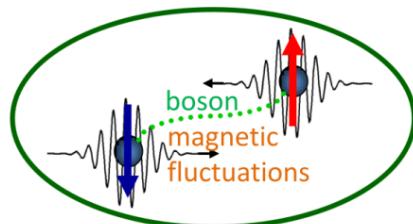
Hubbard model with a contact interaction(close to AFM QCP)

SLH

Spin-fluctuation scenario



(D. Pines)

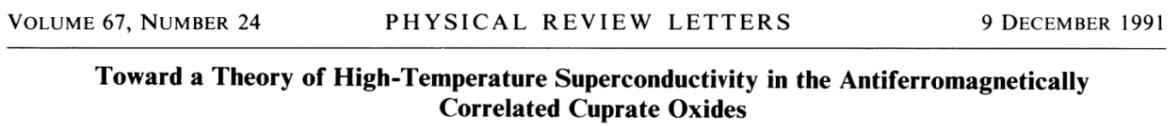
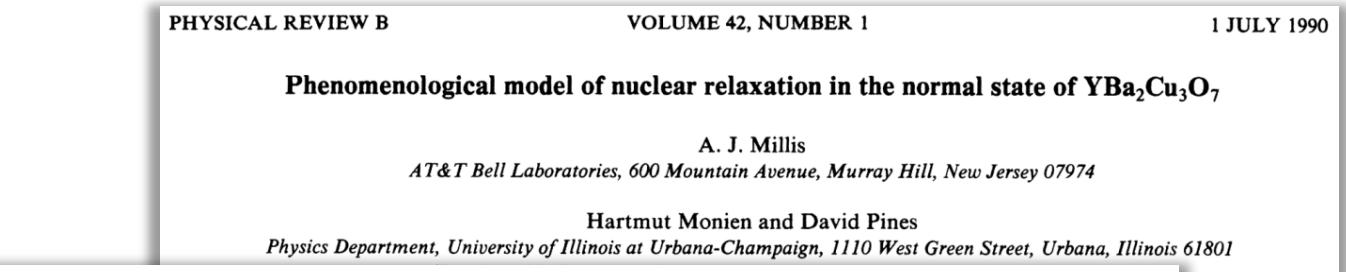


$$\chi_{\text{AF}}(\mathbf{q}, \omega) = \frac{\chi_Q}{1 + \xi^2(\mathbf{Q} - \mathbf{q})^2 - i(\omega/\omega_{\text{SF}})}$$

$$T_c = \alpha \hbar \omega_{\text{SF}}(T_c) \frac{\xi^2(T_c)}{a^2} \exp \left(-\frac{1}{\lambda(T_c)} \right)$$

$$\equiv \alpha \frac{\Gamma(T_c)}{\pi^2} \exp \left(-\frac{1}{\lambda(T_c)} \right),$$

$$\lambda(T_c) = \eta g_{\text{eff}}^2(T_c) \chi_0(T_c) N(0)$$



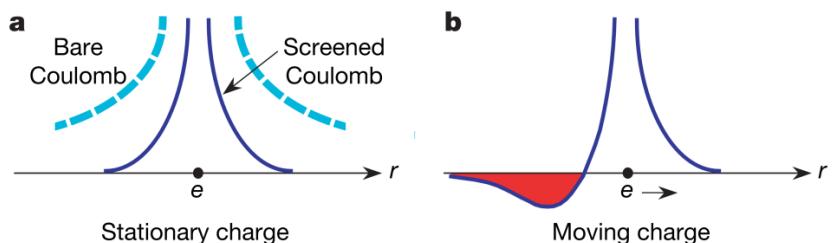
Vol 450 | 20/27 December 2007 | doi:10.1038/nature06480

nature

REVIEWS

Superconductivity without phonons

P. Monthoux^{1,2}, D. Pines^{3,4} & G. G. Lonzarich⁵



Spin-fluctuation scenario

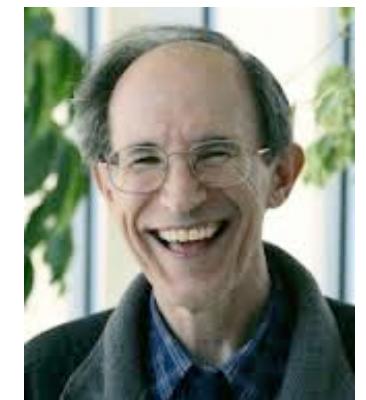
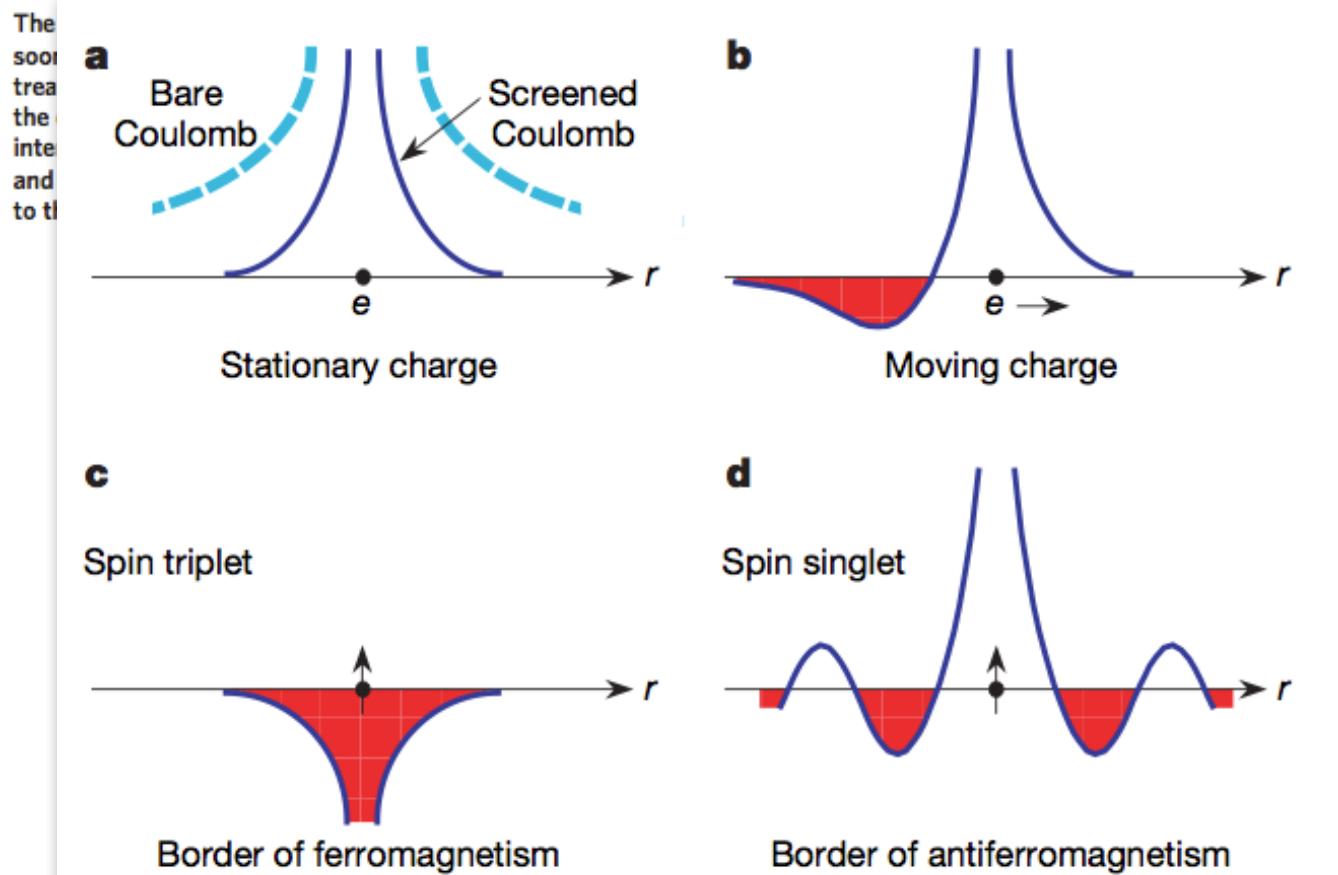
Vol 450 | 20/27 December 2007 | doi:10.1038/nature06480

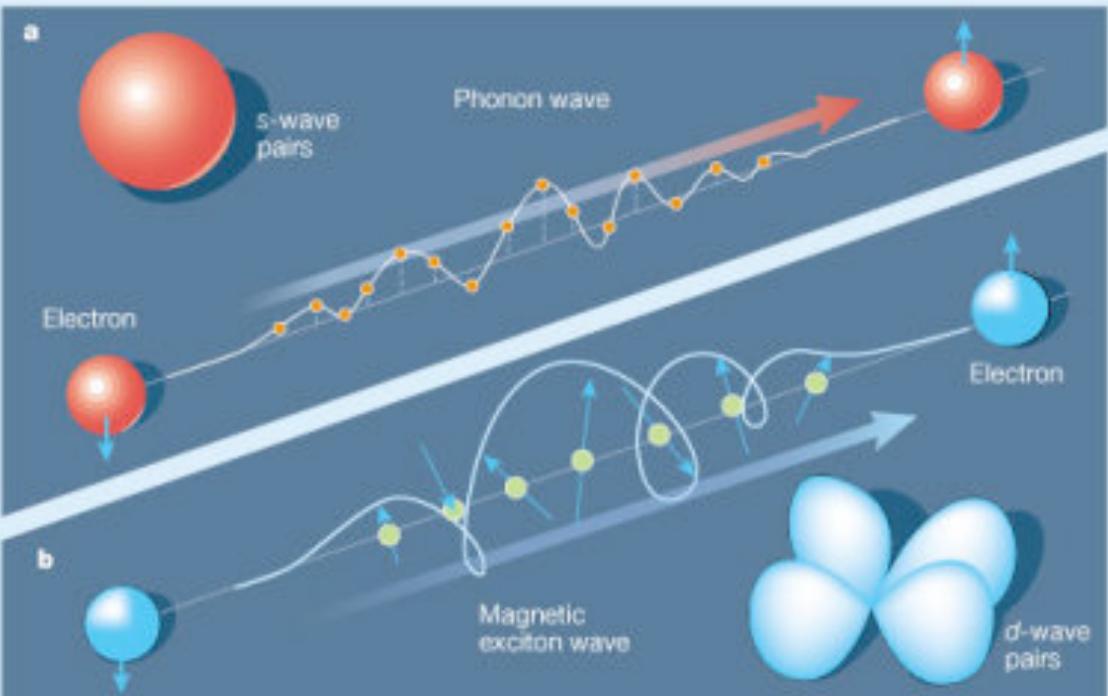
nature

REVIEWS

Superconductivity without phonons

P. Monthoux^{1,2}, D. Pines^{3,4} & G. G. Lonzarich⁵





Pairing mechanism

- **The majority of HF compounds**: heavy quasiparticle pairing due to the exchange of **spin-fluctuation**(‘paramagnon’)
- **Ge-doped CeCu₂Si₂(2nd SC domain)**: **valence fluctuation** of Ce(density fluctuation) induced SC pairing
- **U-based HF compounds**: pairing due to the exchange of weakly damped propagating **magnetic excitons**
- **Pr-skutterudite** : **quadrupolar** degrees of freedom is crucial.
- **Yb-doped CeCoIn₅**: may be captured by the **Composite pairing** mechanism

二、重费米子超导机理与前沿问题

A phenomenological model for HF SC

Given the complexity of the phase diagram and pairing mechanism, one may ask if there exists **a general framework** for heavy fermion superconductivity.

- 📌 **Pairing Electrons**

The f-electrons may be itinerant, or both itinerant and localized

- 📌 **Pairing Force**

Quantum critical fluctuations: spin, valence, orbital, ...

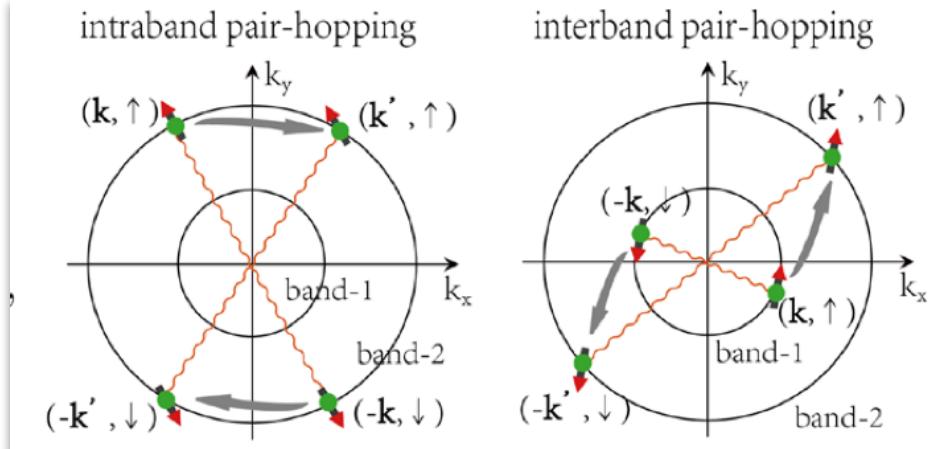
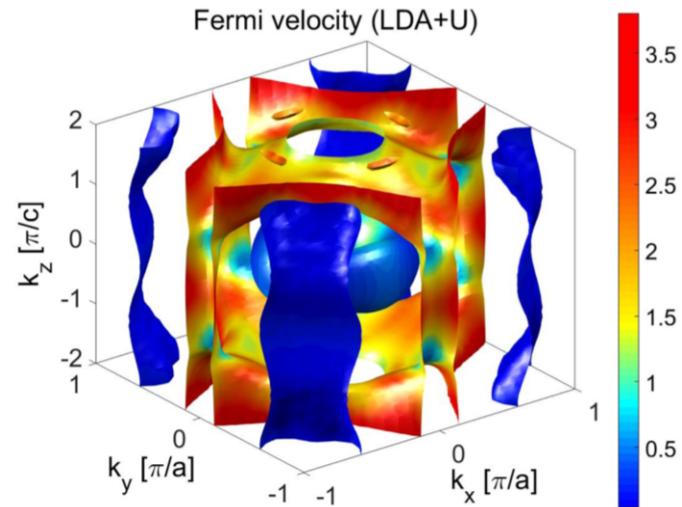
- 📌 **Realistic Fermi Surfaces**

Multiple Fermi surfaces: inter- and intra-band scatterings

Our Proposal

Realistic band structures + quantum critical pairing interactions

Multiband Eliashberg theory

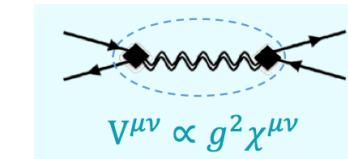


Multiband Linearized Eliashberg equations

$$Z_\mu(\mathbf{k}, i\omega_n) = 1 + \frac{\pi T}{\omega_n} \sum_{\nu, m} \oint_{\text{FS}_\nu} \frac{d\mathbf{k}'_\parallel}{(2\pi)^3 v_{\nu, \mathbf{k}'_\parallel}} \text{sgn}(\omega_m) V^{\mu\nu}(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_m), \quad \rightarrow \text{Renormalization } (Z \gg 1)$$

$$\lambda \phi_\mu(\mathbf{k}, i\omega_n) = -C\pi T \sum_{\nu, m} \oint_{\text{FS}_\nu} \frac{d\mathbf{k}'_\parallel}{(2\pi)^3 v_{\nu, \mathbf{k}'_\parallel}} \frac{V^{\mu\nu}(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_m)}{|\omega_m Z_\nu(\mathbf{k}', i\omega_m)|} \phi_\nu(\mathbf{k}', i\omega_m) \quad \rightarrow \text{Gap equations at } T_c$$

($C=1$ for singlet-pairing, while $C=-1/3$ for triplet-pairing.)

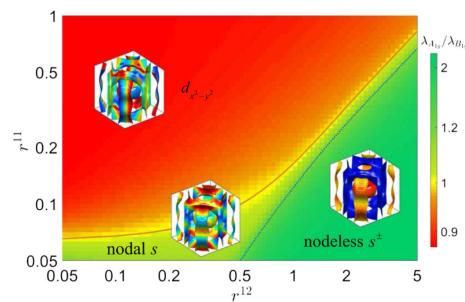


Fermi surface topology

QC pairing interactions

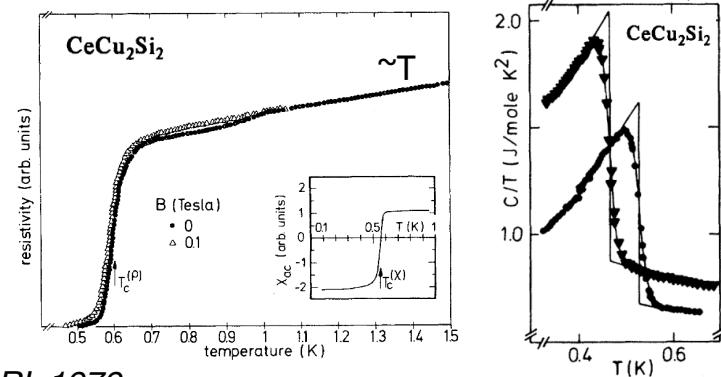
$$V^{\mu\nu}(\mathbf{q}, \nu_n) = \frac{V_0^{\mu\nu}}{1 + \xi^2 (\mathbf{q} - \mathbf{Q})^2 + |\nu_n / \omega_{sf}|^\alpha}$$

Possible nodeless s^\pm -wave superconductivity in CeCu_2Si_2



Y. Li & YY et al., PRL 120, 217001 (2018)

Experiments



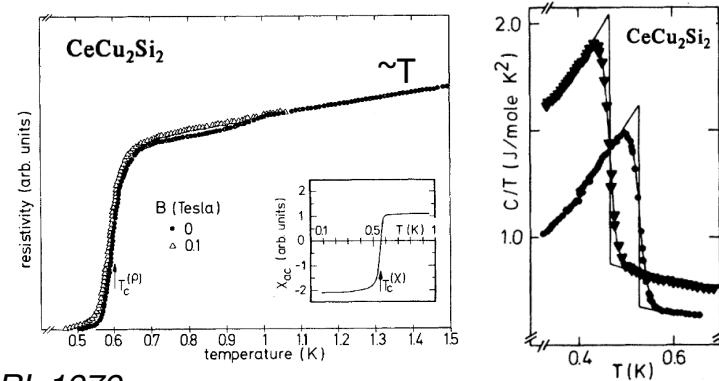
Steglich et al., PRL 1979

● First unconventional superconductor

- ✓ $T_c/T_F \sim T_F/\Theta \sim 0.05$
- ✓ High temperature superconductivity
- ✓ Beyond conventional BCS theory

*For a long time, it was thought to have a d-wave nodal gap,
and many experiments seem also to support it!*

Experiments

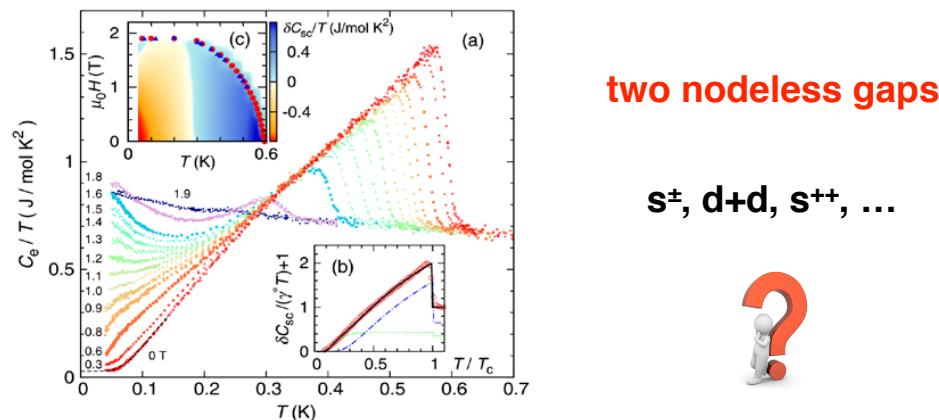


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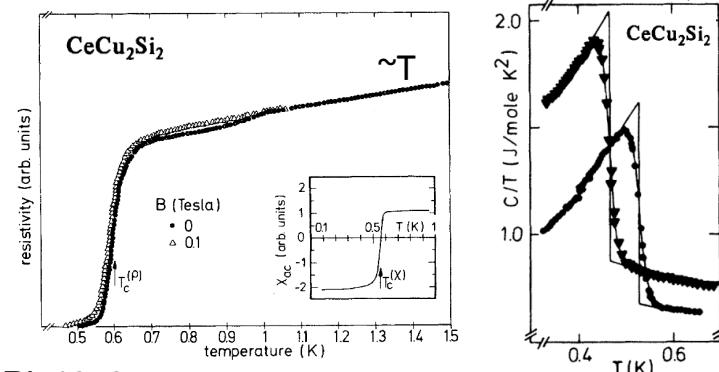
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Kittaka et al., PRL 2014 & many others (penetration depth, STM, ...)

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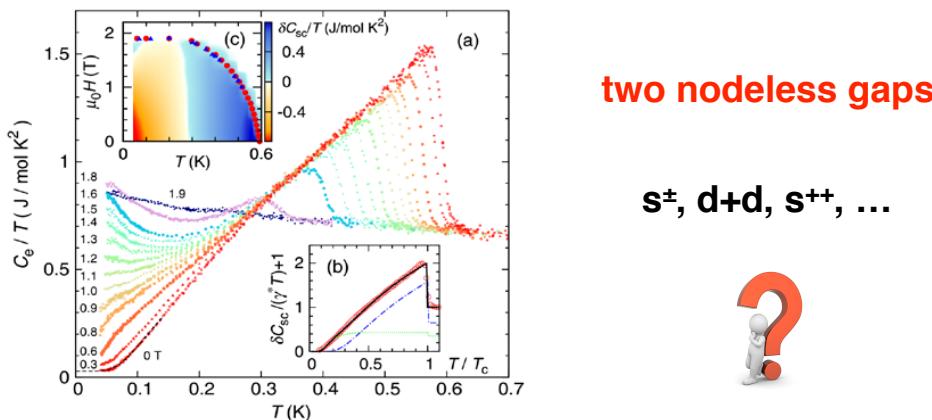


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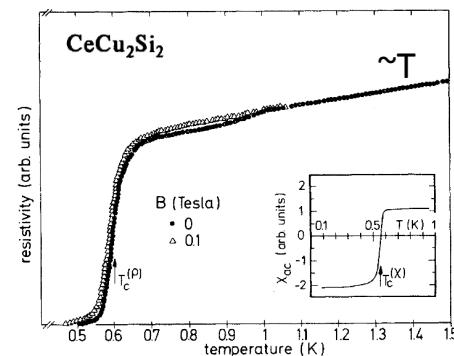
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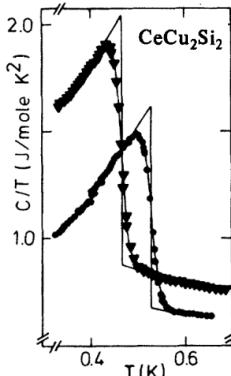
What is missing in theory?

Kittaka et al., PRL 2014 & many others (penetration depth, STM, ...)

Experiments



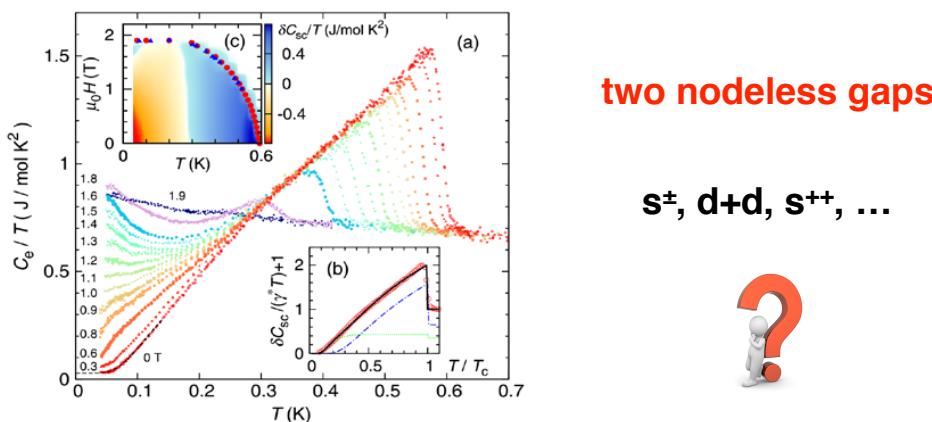
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two nodeless gaps

$s^\pm, d+d, s^{++}, \dots$



Kittaka et al., PRL 2014 & many others (penetration depth, STM, ...)

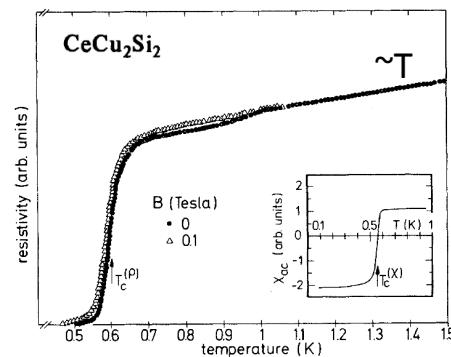
What is missing in theory?

- ✓ Band structures
- ✓ Pairing glues



Gap equation

Experiments

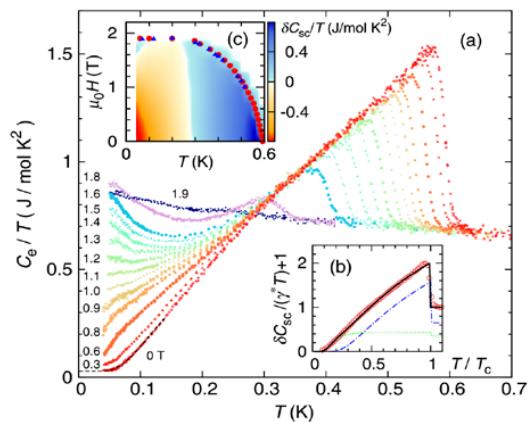


Steglich et al., PRL 1979

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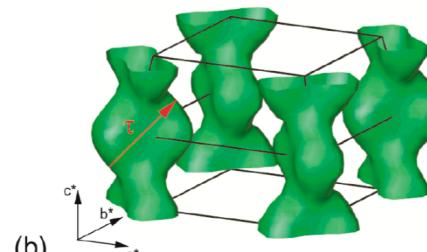
Kittaka et al., PRL 2014 & many others (penetration depth, STM, ...)

What is missing in theory?

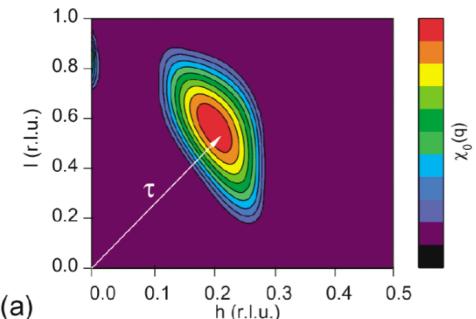
- ✓ Band structures
- ✓ Pairing glues



Gap equation



(b)



(a)

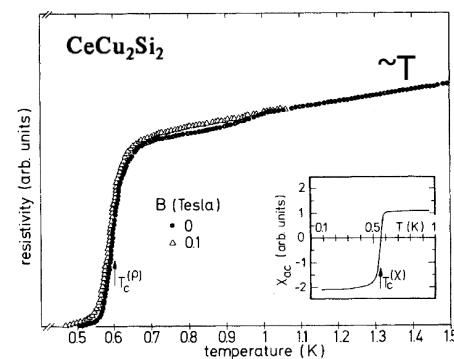
Zwicknagl & Stockert et al., PRL 2004; 2008 ...

Nested heavy band + QC magnetic fluctuations



d-wave gap symmetry

Experiments

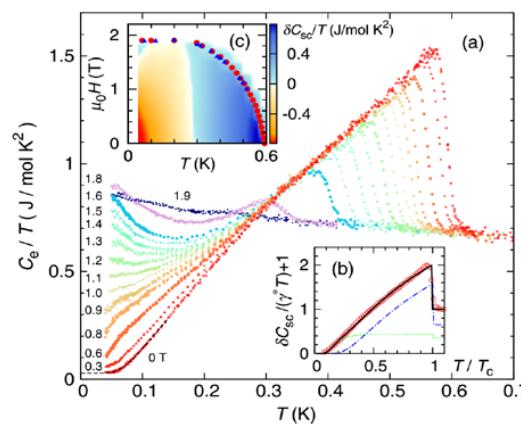


Steglich et al., PRL 1979

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two nodeless gaps

$s^\pm, d+d, s^{++}, \dots$



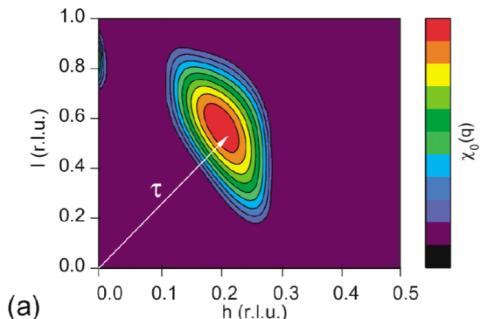
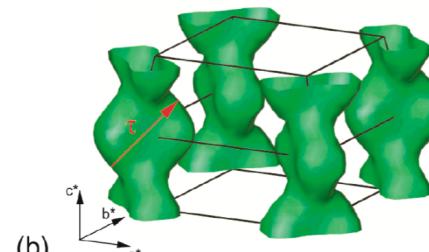
Kittaka et al., PRL 2014 & many others (penetration depth, STM, ...)

What is missing in theory?

- ✓ Band structures
- ✓ Pairing glues



Gap equation

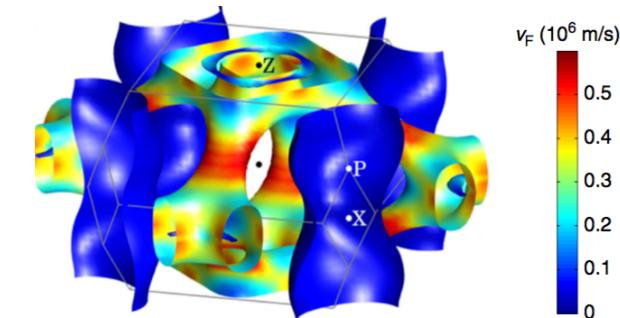


Zwicknagl & Stockert et al., PRL 2004; 2008 ...

Nested heavy band + QC magnetic fluctuations



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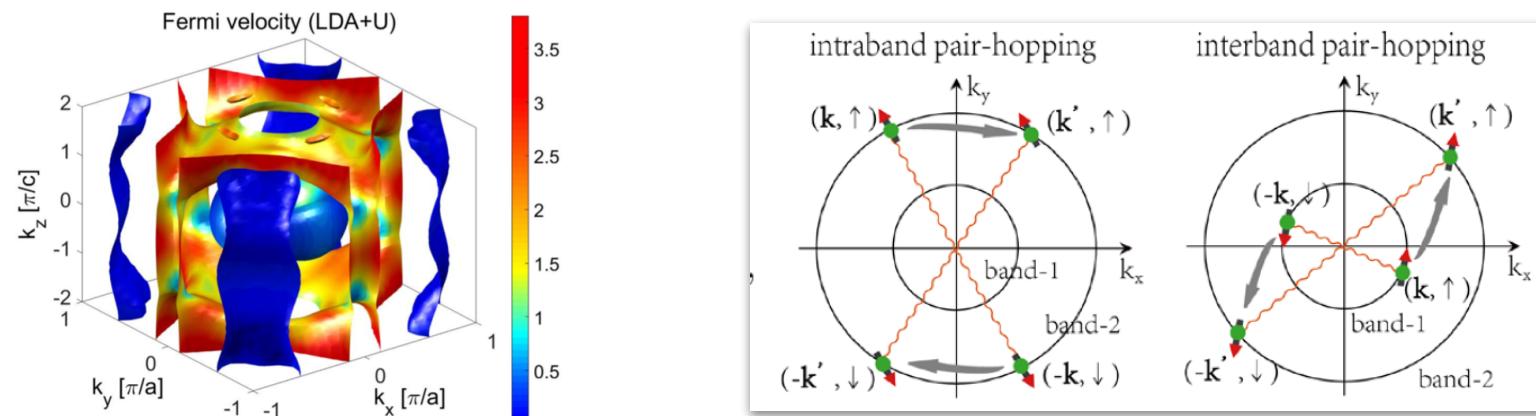
Ikeda et al., PRL 2015

Multi-bands + magnetic octupole fluctuations

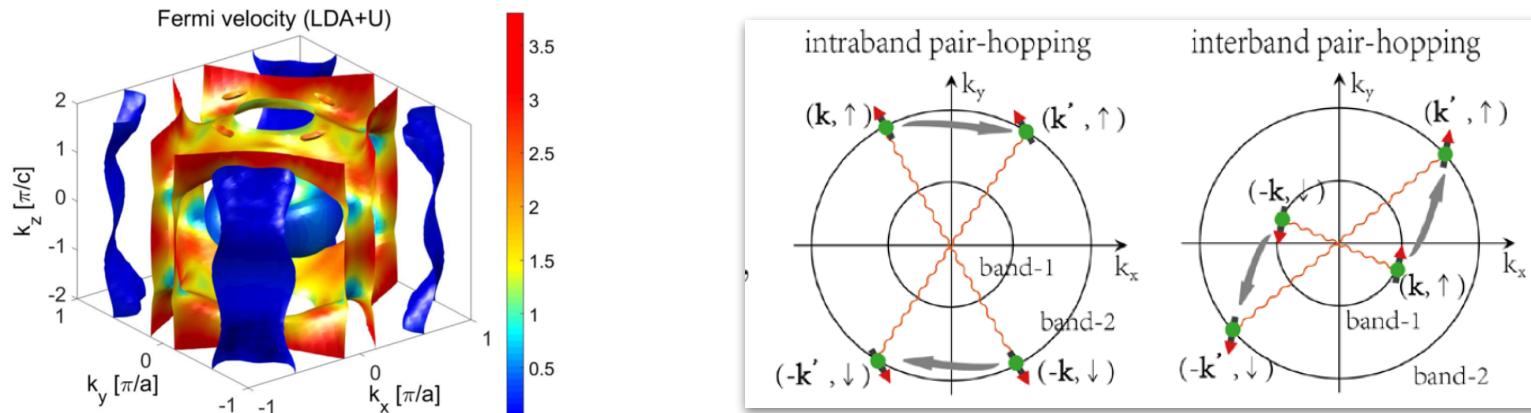


loop-nodal s-wave gap

How to reconcile the conflict ? multi-bands + intra/inter-band pair interactions



How to reconcile the conflict ? multi-bands + intra/inter-band pair interactions



Eliashberg Equations

$$Z_\mu(\mathbf{k}, i\omega_n) = 1 + \frac{\pi T}{\omega_n} \sum_\nu \oint_{FS_\nu} \frac{dk'_\parallel}{(2\pi)^3 v_{k'_F}} \sum_{i\omega_m} \text{sgn}(\omega_m) g_{eff}^2 \chi^{\mu\nu}(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_m)$$

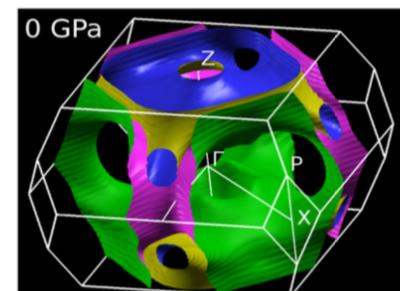
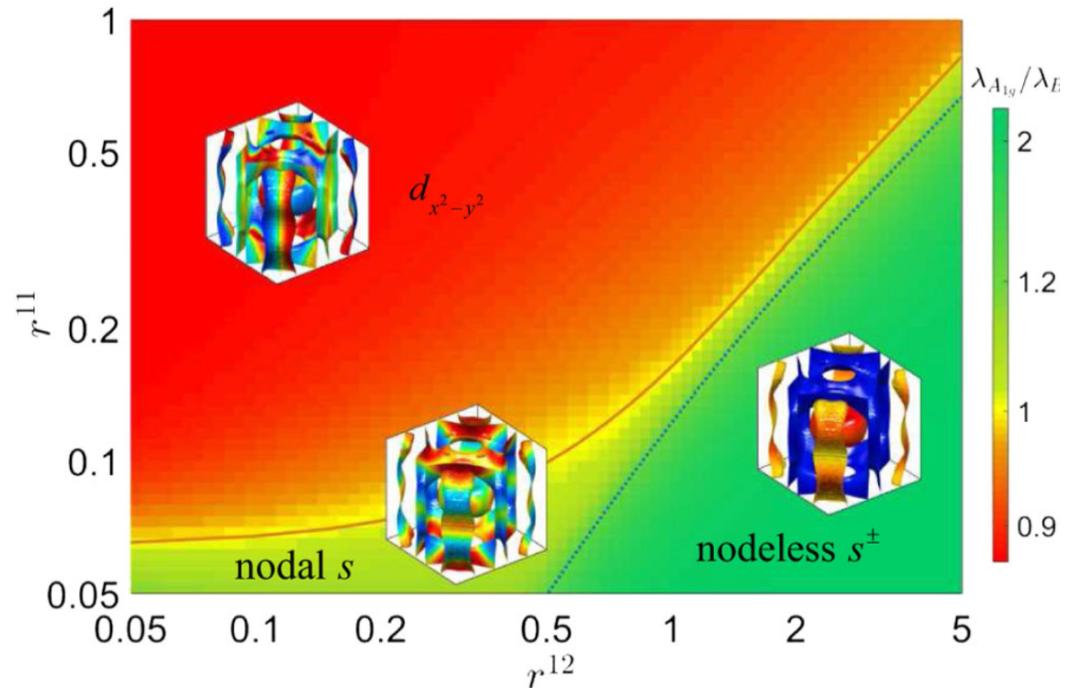
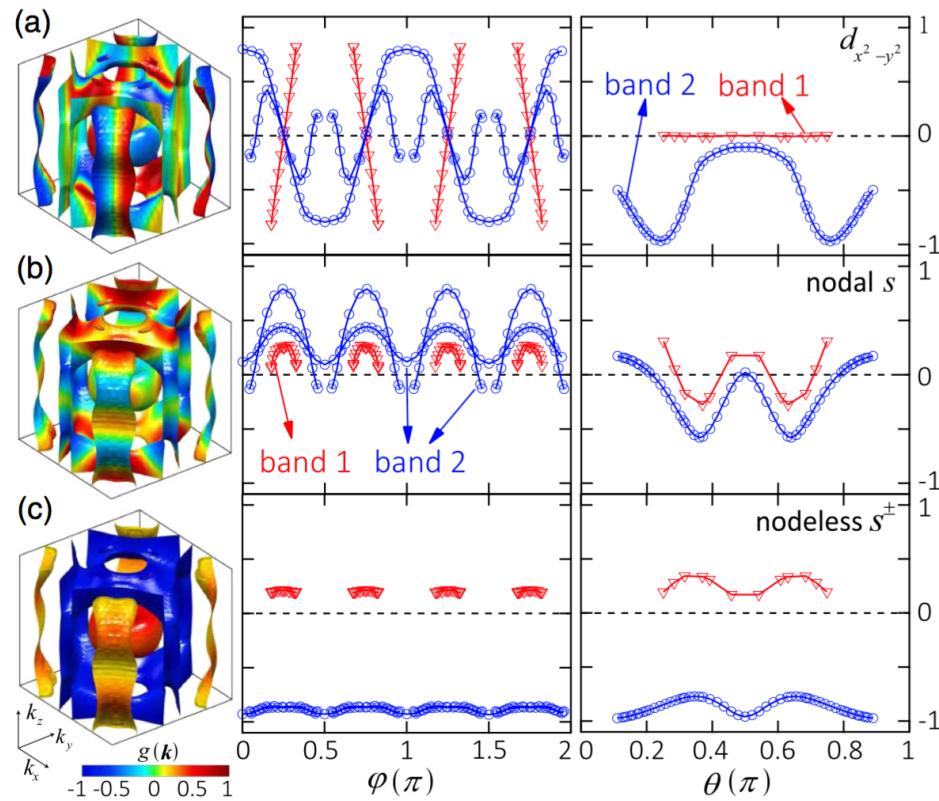
$$\lambda\phi_\mu(\mathbf{k}, i\omega_n) = - \sum_\nu \oint_{FS_\nu} \frac{dk'_\parallel \pi T}{(2\pi)^3 v_{k'_F}} \sum_{i\omega_m} \phi_\nu(\mathbf{k}', i\omega_m) \frac{g_{eff}^2 \chi^{\mu\nu}(\mathbf{k} - \mathbf{k}', i\omega_n - i\omega_m)}{|\omega_m| Z_\nu(\mathbf{k}', i\omega_m)}$$

A phenomenological form of the pairing force

$$\chi^{\mu\nu}(\mathbf{q}, i\omega_l) = \frac{\chi_0^{\mu\nu}}{1 + \xi^2(\mathbf{q} - \mathbf{Q})^2 + \frac{|\omega_l|}{\omega_{sf}}}$$

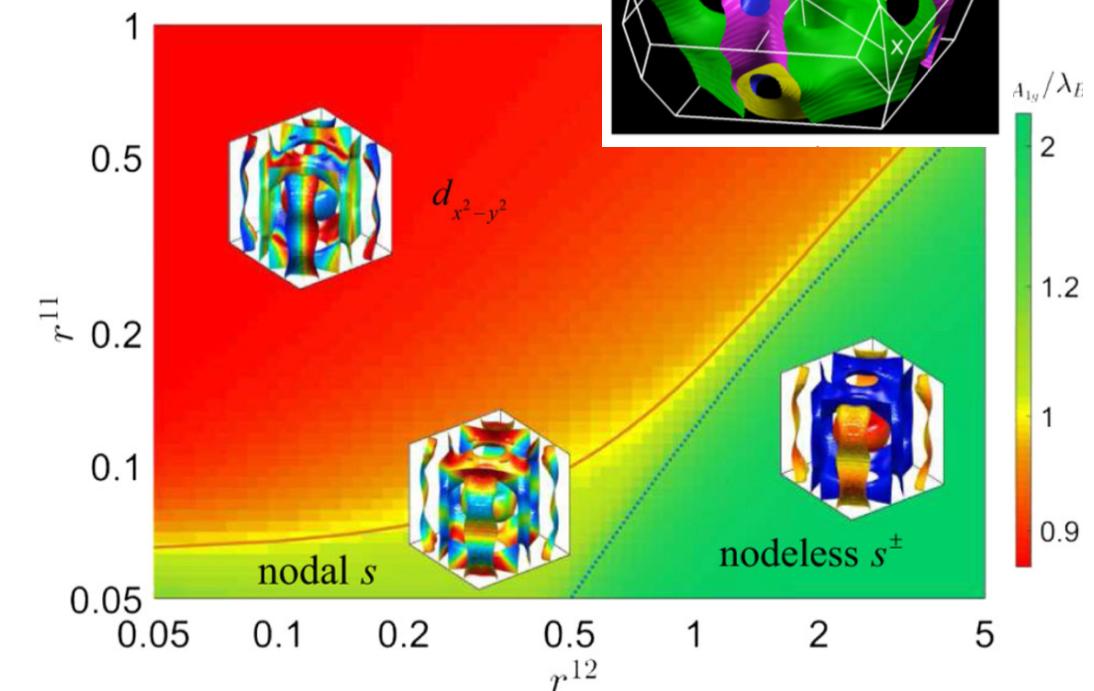
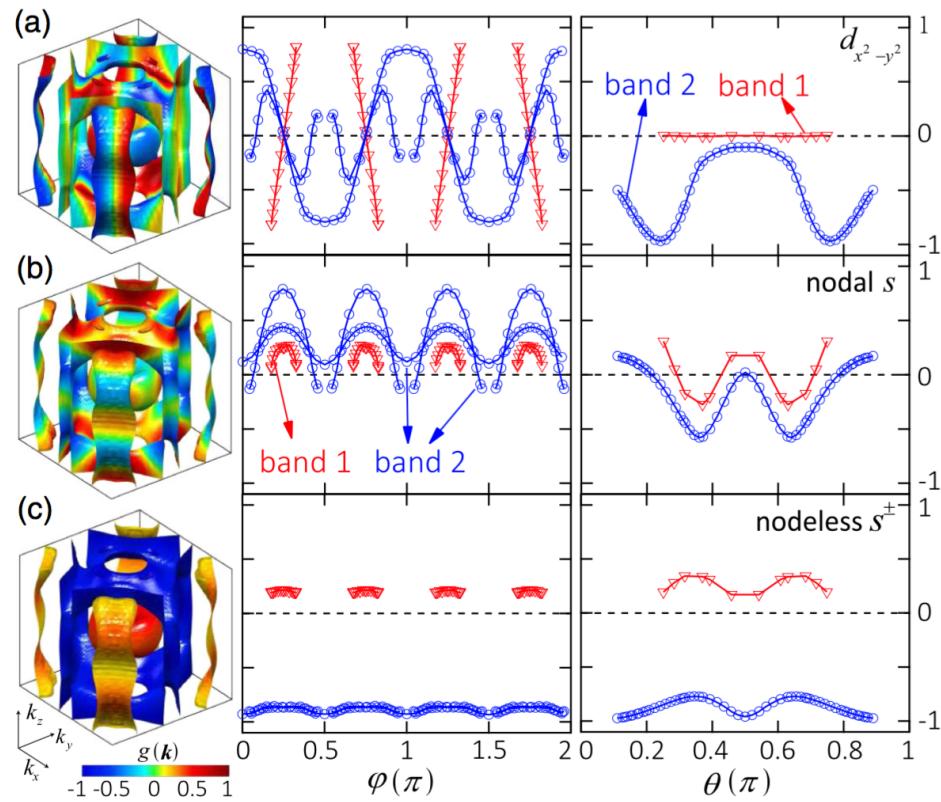
- ✓ Ignore inter-band pairing (momentum mismatch)
- ✓ Phenomenological MMP susceptibility (Millis et al., PRB 1990)
- ✓ Intra & inter-band scattering strength as free parameters

The theoretical phase diagram



DMFT Fermi surface

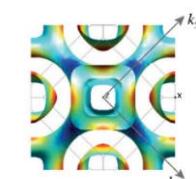
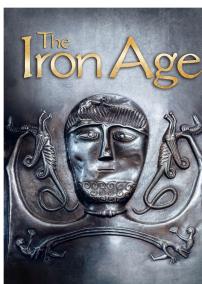
The theoretical phase diagram



The gap symmetry depends on relative strength of the inter- and intra-band pair interactions

- ✓ Inter-band scattering favors nodeless gap
- ✓ Parameters to be extracted

Similar to pnictides superconductivity !!!



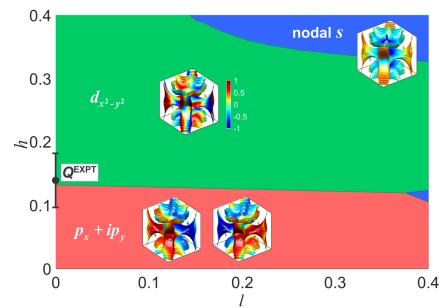
loop nodal s^\pm -wave
(multi-orbital RPA)

H. Ikeda *et al*, PRL 114, 147003 (2015)

Fermi-surface nesting
on the single-heavy
band (single-band RPA)

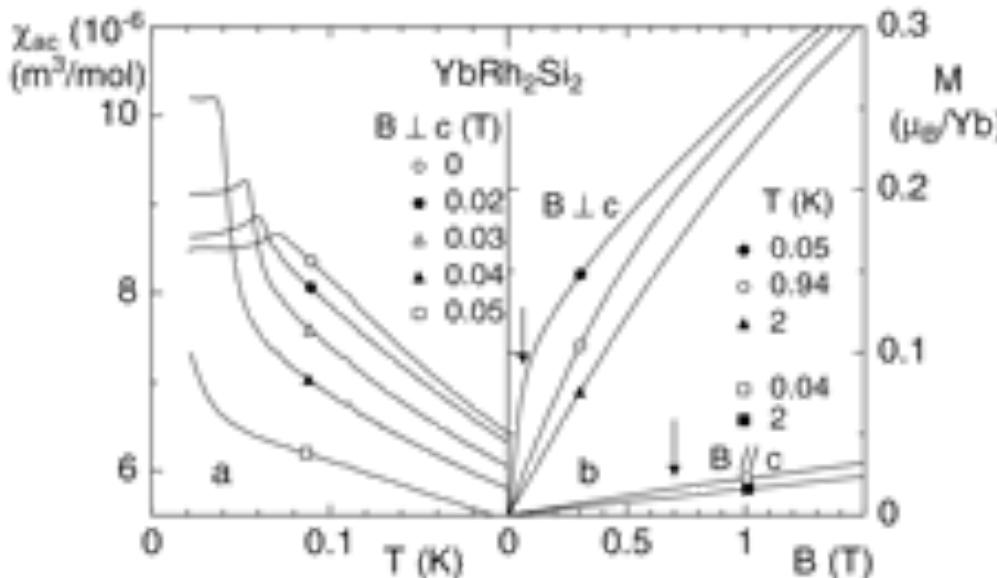
I. Eremin *et al*, PRL 101, 187001 (2008)

Nearly degenerate $d_{x^2-y^2}$ and p_x+ip_y pairing in YbRh_2Si_2

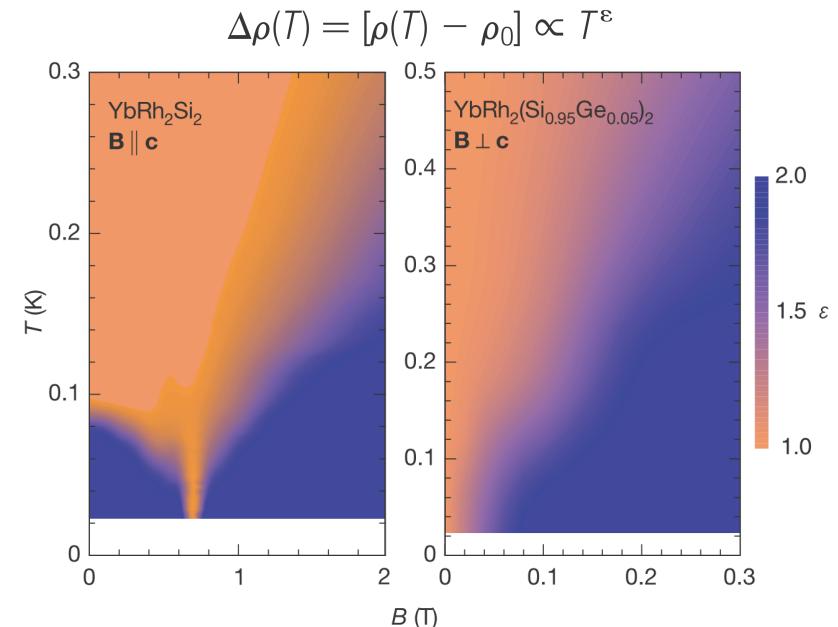


Y. Li & YY et al., arXiv:1901.09196 (2019)

YbRh₂Si₂: ultralow-temperature superconductivity

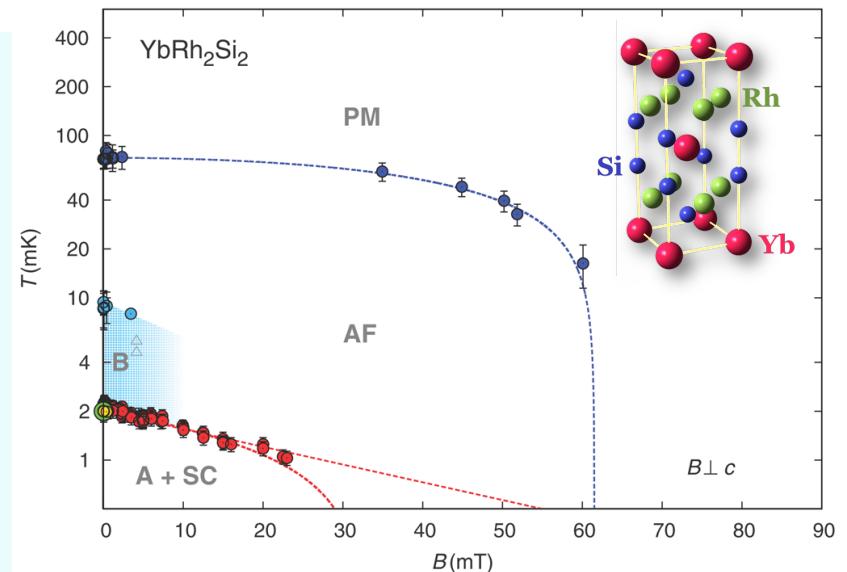


P. Gegenwart et al, Phys. Rev. Lett. **89**, 056402 (2002)



J. Custer et al, nature **424**, 524 (2003)

- ThCr₂Si₂ structure: $I4/mmm$ space group (D_{4h}).
- Crystal constants: $a=4.010 \text{ \AA}$, $c=9.841 \text{ \AA}$.
- $T_K=25 \text{ K}$, $T^*=70 \pm 20 \text{ K}$, $T_N \approx 70 \text{ mK}$.
- Field-induced QCP: $B_{\perp c} \approx 60 \text{ mT}$, $B_{\parallel c} \approx 0.7 \text{ T}$.
- Above T_N : $\Delta\rho \propto T$ — Non-Fermi liquid.
- Ultra-low SC: $T_c=2 \text{ mK}$.

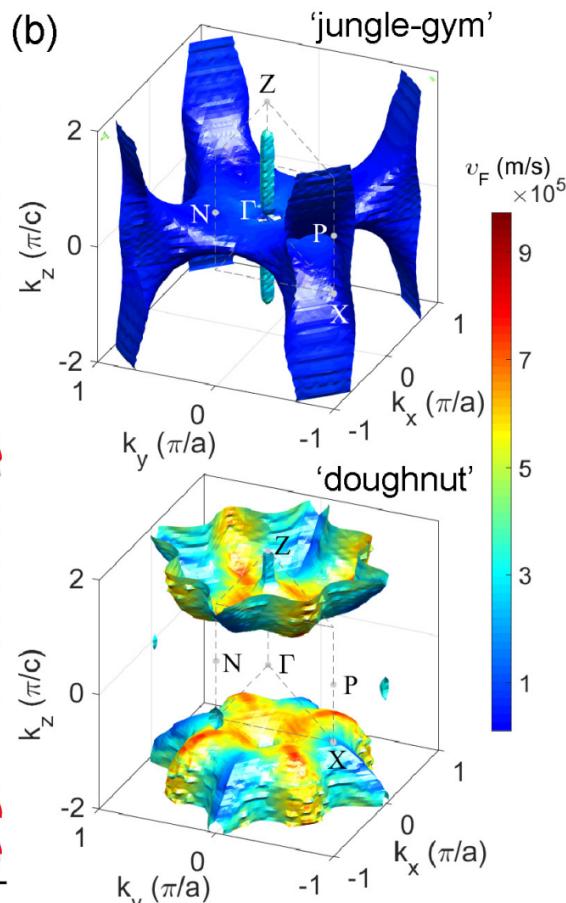
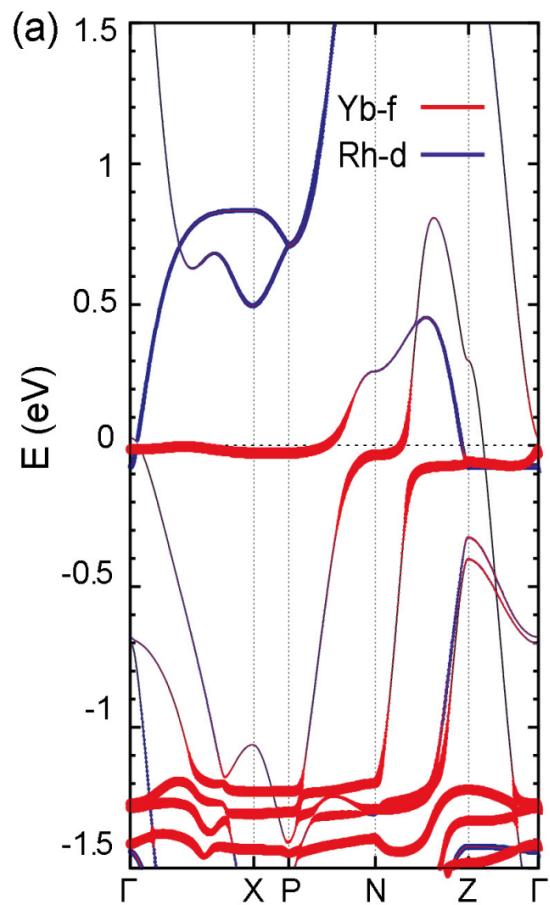


E. Schuberth et al, Science **351**, 485 (2016)

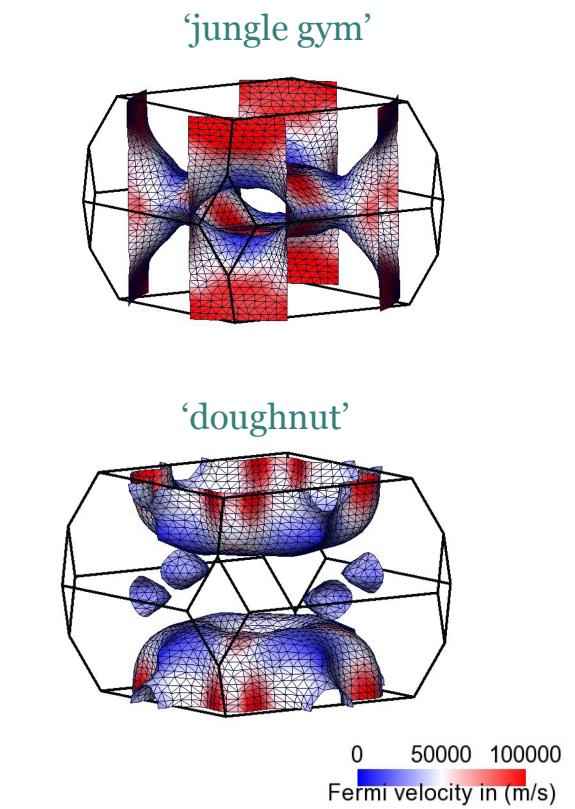
How shall we understand its superconductivity?

Band structures: first-principles calculations

DFT+U



LDA+ Renormalized band
(G. Zwicknagl's group)



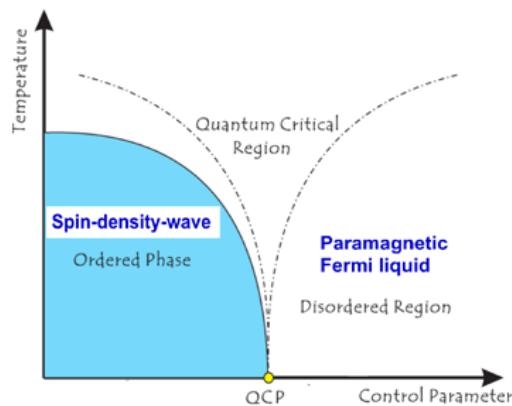
G. Zwicknagl, Rep. Prog. Phys.
79, 124501 (2016).

- Near the Fermi level, there are **strong hybridization** between Yb-4f electrons and Rh-4d electrons.
- The obtained '**jungle-gym**' and '**donut**' Fermi surfaces are qualitatively consistent with previous calculations.

Quantum criticality under debate

SDW

(Hertz-Millis-Moriya)



- Itinerant AFM order parameter fluctuations.
- Supported by the neutron scattering experiment.

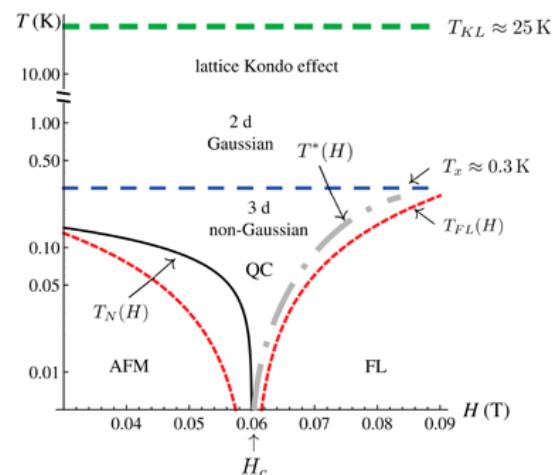
$$\chi(\mathbf{q}, \omega) = \frac{\chi_0}{r + \xi^2(\mathbf{q} - \mathbf{Q})^2 - i\omega/\Omega}$$



$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi_0}{r + \xi^2(\mathbf{q} - \mathbf{Q})^2 + |\omega_n/\Omega|^{1/2}}$$

Critical quasiparticle

(P. Wölfle, E. Abrahams)



- Extending quasiparticles into the quantum critical region.
- Can explain various quantities, such as: $\gamma(T)$, $\rho(T)$, $M(T)$, $\chi(T)$, ...

$$\chi(\mathbf{q}, \omega) = \frac{N_0}{r + \xi_0^2(\mathbf{q} - \mathbf{Q})^2 - i\Lambda_Q^2(\omega) \frac{\omega}{v_F Q}}$$

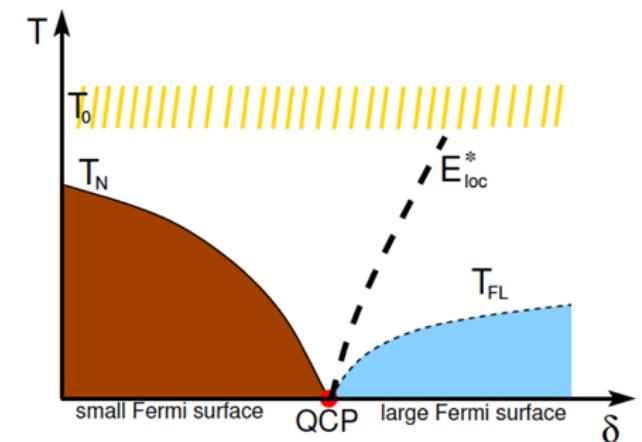
where $\Lambda_Q(\omega) \approx Z^{-1}(\omega) = C\omega^{-1/4}$.



$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi_0}{r + \xi^2(\mathbf{q} - \mathbf{Q})^2 + |\omega_n/\Omega|^{1/2}}$$

Local quantum criticality

(Q. Si, et al)



- Local AFM QCP.
- a small-to-large FS jump is predicted when across the QCP.
- Supported by the anomalous hall evolution,...

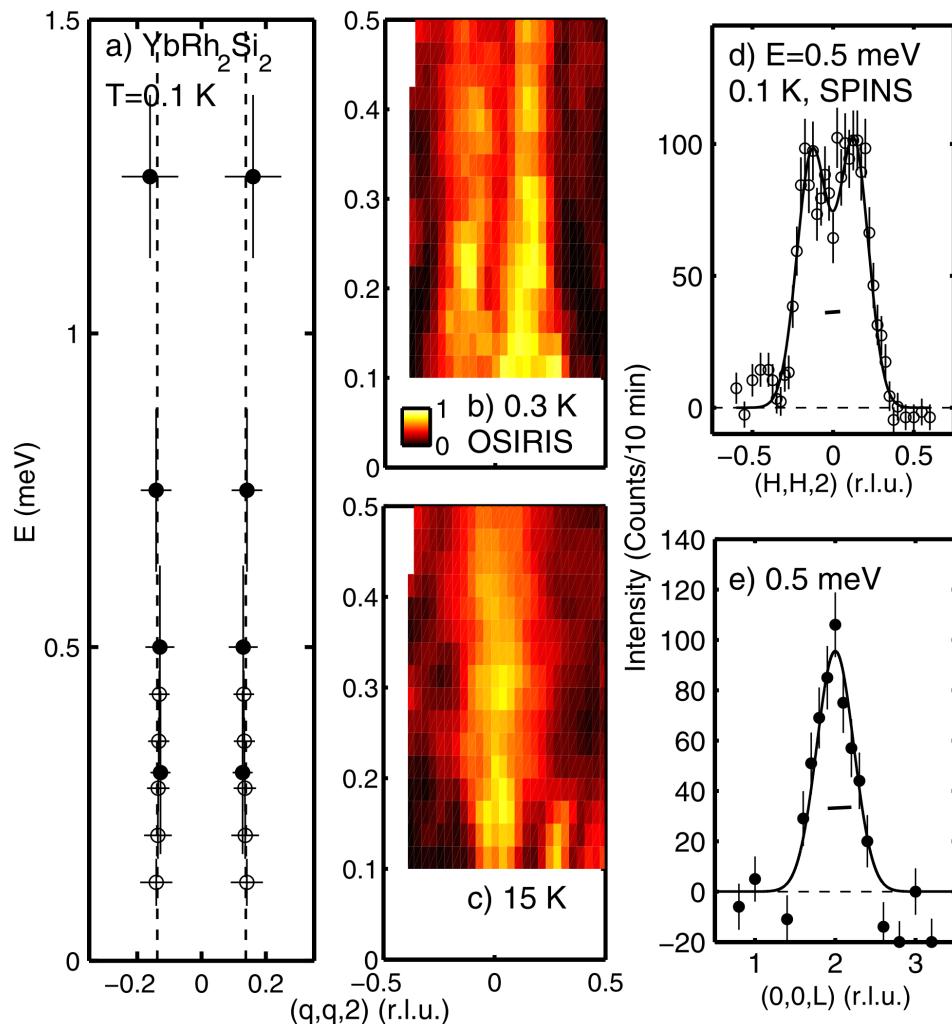
$$\chi(\mathbf{q}, \omega, T=0) = \frac{1}{I_{\mathbf{q}} - I_{\mathbf{Q}} + \Lambda_0(-i\omega/\Lambda)^{\alpha}},$$

where $\alpha = 0.75$.

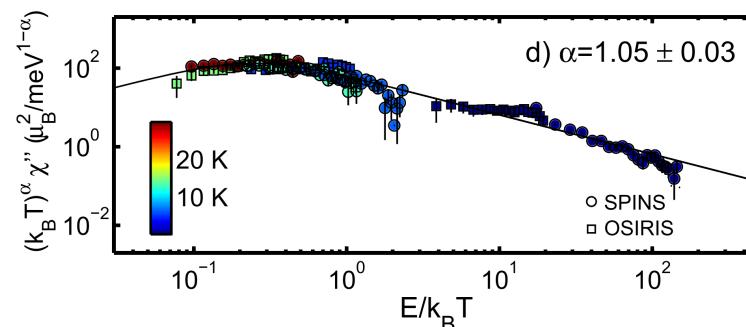


$$\chi(\mathbf{q}, i\omega_n) = \frac{\chi_0}{r + \xi^2(\mathbf{q} - \mathbf{Q})^2 + |\omega_n/\Omega|^{3/4}}$$

Neutron scattering experiment on Q-vector



With lowering T to 0.1K, FM fluctuations evolved into a incommensurate AFM correlations located at $\mathbf{q}_0 = (0.14 \pm 0.04, 0.14 \pm 0.04, 0)$.



$$\chi''(Q, \omega) = \chi'_Q \frac{\omega \Gamma(T)}{\Gamma(T)^2 + \omega^2}$$

ω/T -scaling is observed for $T\chi''$ from FM to AFM fluctuations within 0.1~30K.

C. Stock et al, Phys. Rev. Lett. **109**, 127201 (2012)

But the characteristic wave vectors below T_{AFM} (70mK) is still unknown.

Phenomenological pairing interactions

PHYSICAL REVIEW B

VOLUME 42, NUMBER 1

1 JULY 1990

Phenomenological model of nuclear relaxation in the normal state of $\text{YBa}_2\text{Cu}_3\text{O}_7$

A. J. Millis

AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974

Hartmut Monien and David Pines

*Physics Department, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois 61801
(Received 27 November 1989; revised manuscript received 7 March 1990)*

A. J. Millis, H. Monien, and D. Pines, Phys. Rev. B **42**, 167 (1990)

$$\chi(\mathbf{q}, \omega) = \frac{\chi_0}{1 + \xi^2 (\mathbf{q} - \mathbf{Q})^2 - i\omega/\omega_{sf}},$$

Static susceptibility

↑ Magnetic correlation length ↑ Magnetic characteristic wave vectors ↑ Characteristic frequency for spin fluctuations

Spin fluctuation mechanism



$$V^{\mu\nu} \propto g^2 \chi^{\mu\nu}$$

We choose this phenomenological interaction to describe the quantum critical fluctuations.

We take \mathbf{Q} and α both as tuning parameters !

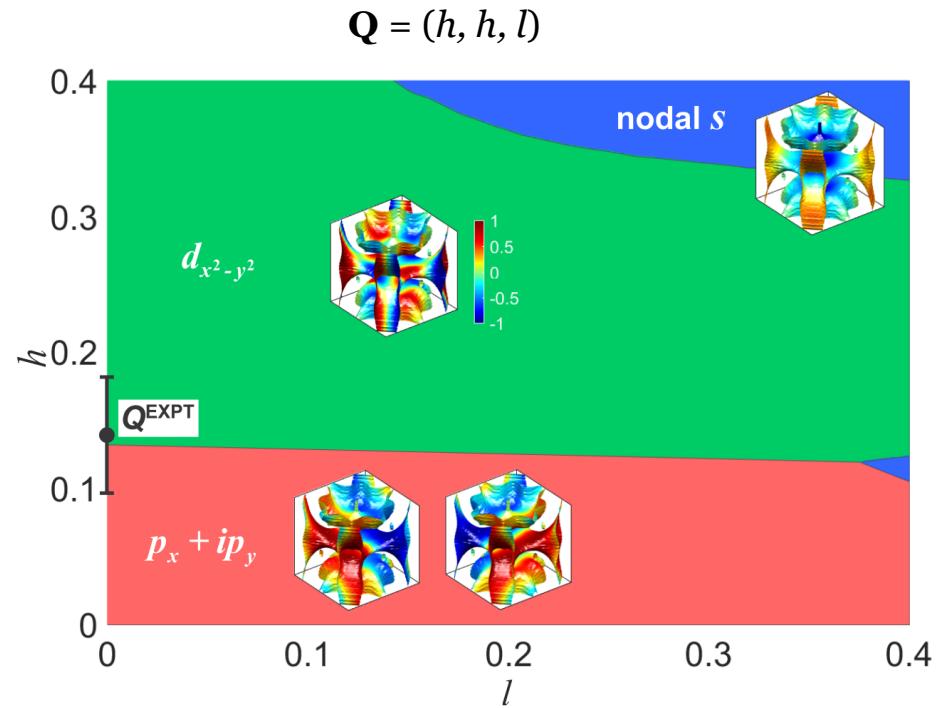
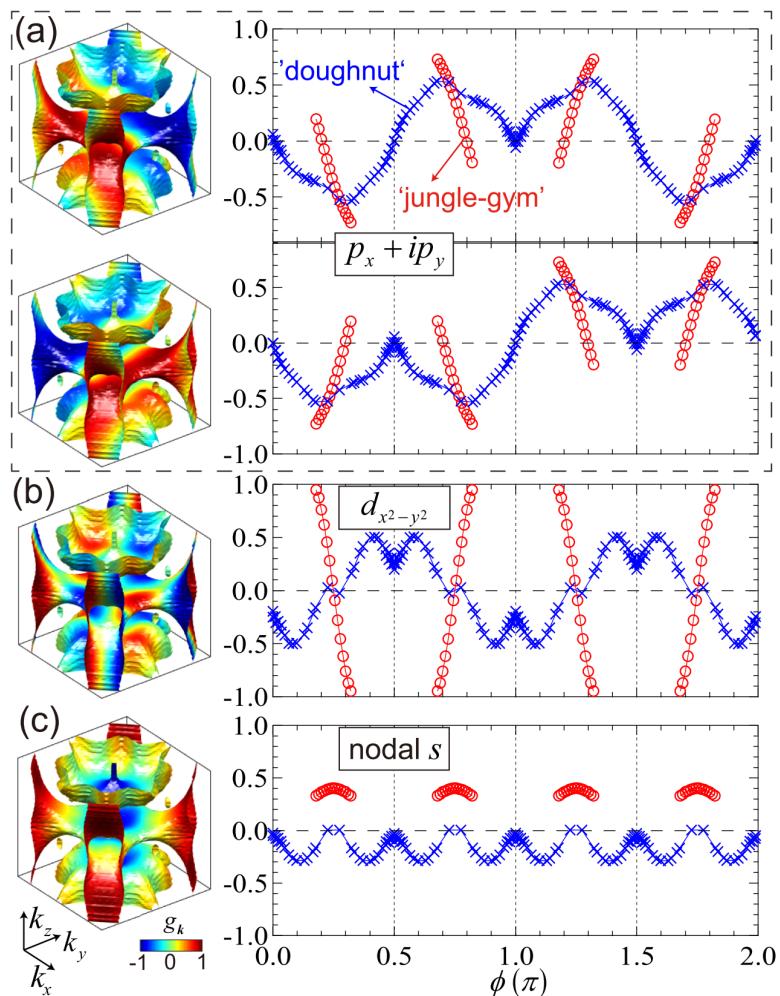
A general phenomenological quantum critical interactions

$$V^{\mu\nu}(\mathbf{q}, v_n) = \frac{V_0^{\mu\nu}}{1 + \xi^2 (\mathbf{q} - \mathbf{Q})^2 + |v_n/\omega_{sf}|^\alpha}$$

Spin-density wave scenario: $\alpha = 1$
 Local quantum criticality: $\alpha \approx 0.75$
 Critical quasiparticle scenario: $\alpha \approx 0.5$

(However, in our calculations, pairing symmetry is nearly unchanged with choosing different α .)

A theoretical phase diagram

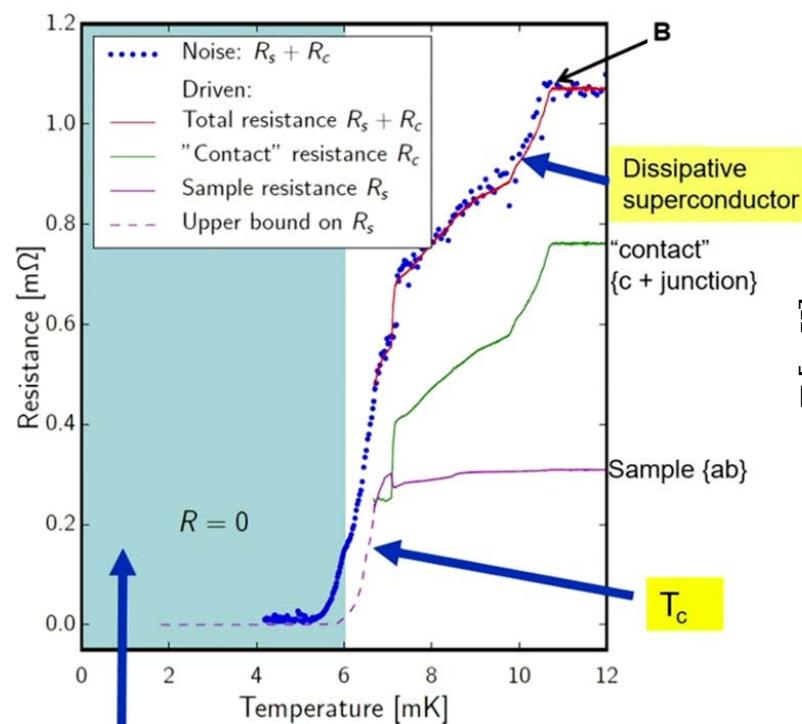


- Experimental $\mathbf{Q}^{\text{EXP}} = (0.14 \pm 0.04, 0.14 \pm 0.04, 0)$

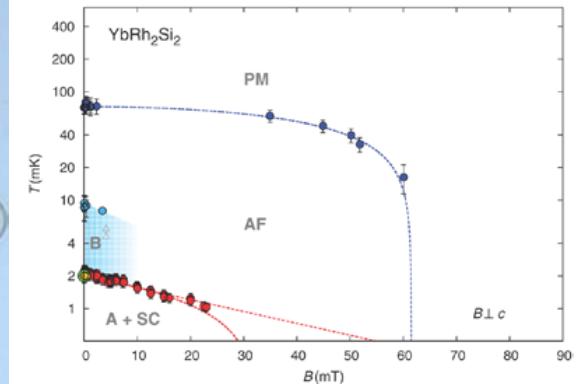
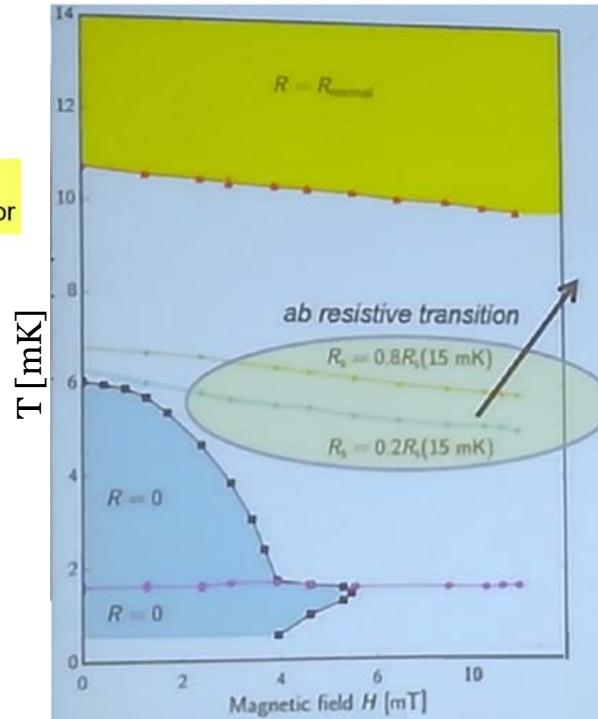
→ *Nearly degenerate $d_{x^2-y^2}$ and $p_x + ip_y$ wave pairing*

Updated H-T phase diagram

Recent experiments (J. Saunders's group)



Macroscopic superconducting phase coherence in $\text{YbRh}_2\text{Si}_2 + \text{Al} + \text{Nb} + \text{NbTi}$ loop



E. Schuberth et al, Science **351**, 485 (2016)

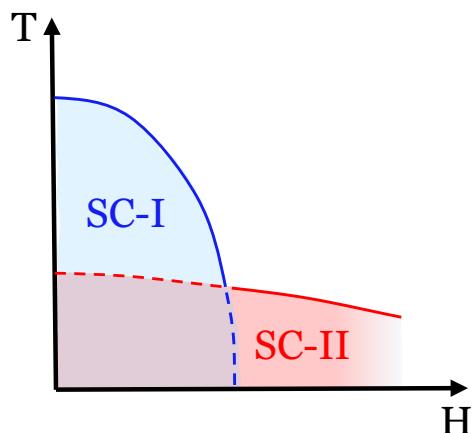
$$T_c = 6 \text{ mK}$$

Multiple superconducting phase?

The field-induced phase is an intrinsic electronic property of YbRh_2Si_2 , not induced by nuclear order

Hints from the updated H-T phase diagram

Schematic H-T phase diagram
(low-T) for YbRh_2Si_2



Pauli-limiting field: $H_{c2,P}(0) \approx 1.84T_c$

(Clogston-Chandrasekhar relation)

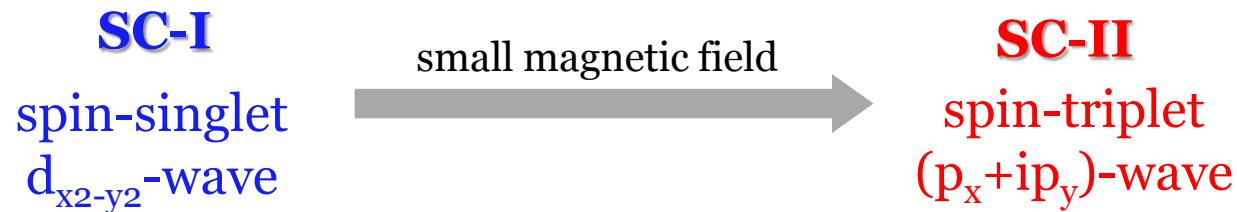
A. M. Clogston, Phys. Rev. Lett. **9**, 266 (1962).
B. S. Chandrasekhar, Appl. Phys. Lett. **1**, 2 (1962).

Orbital-limiting field: $H_{c2,\text{orb}}(0) \approx 0.7 \left| \frac{dH_{c2}}{dT} \right|_{T_c} T_c$

(Werthamer-Helfand-Hohenberg relation)

N. R. Werthamer, E. Helfand, and P. C. Hohenberg,
Phys. Rev. **147**, 295 (1966).

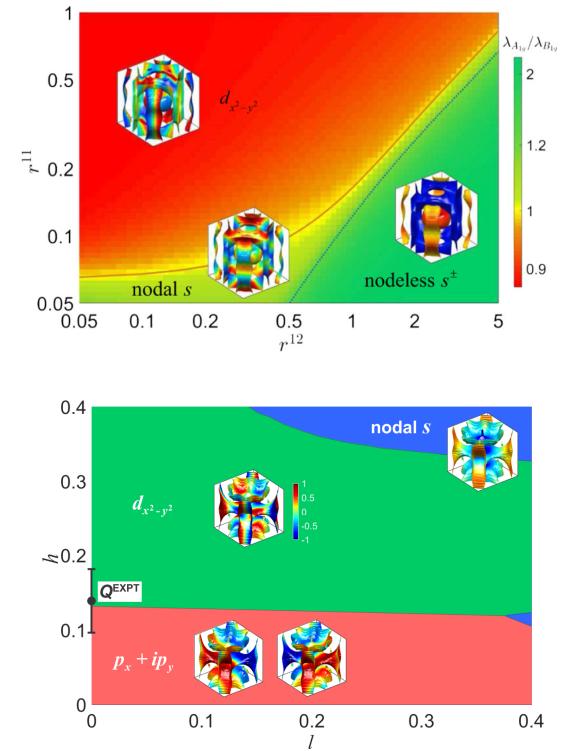
SC phase	T_c	$-\frac{dH_{c2}}{dT} _{T_c}$	H_{c2}	$H_{c2,P}(0)$	$H_{c2,\text{orb}}(0)$	Conclusion
SC-I	6 mK	$\approx 6.4 \text{ T/K}$	$\approx 4 \text{ mT}$	11.0mT	26.9mT	$H_{c2} < H_{c2,P}(0) < H_{c2,\text{orb}}(0)$: singlet pairing
SC-II	2 mK	$\approx 25 \text{ T/K}$	30-50mT	3.7mT	35mT	$H_{c2,P}(0) \ll H_{c2} \approx H_{c2,\text{orb}}(0)$: triplet pairing



Summary

• A general framework for quantum critical SC

Realistic band structures from first-principles calculations
+ phenomenological quantum critical pairing interactions

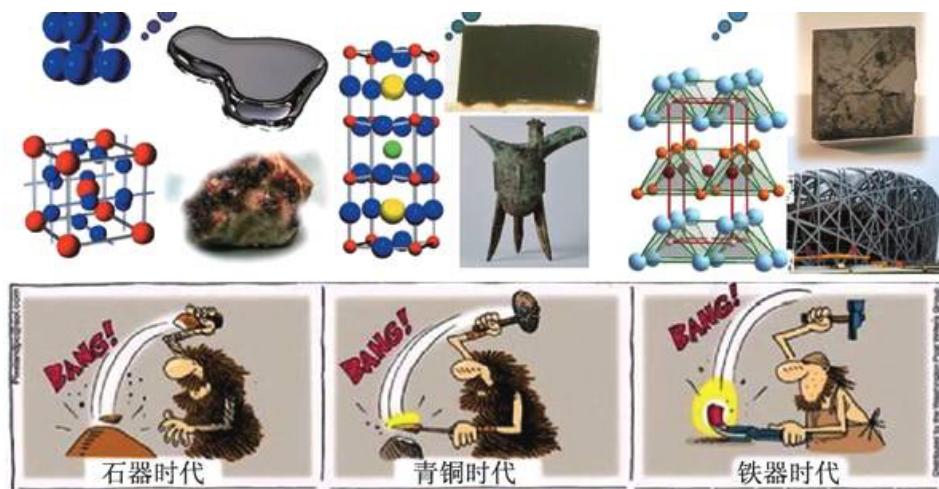


• Possible nodeless s^\pm superconductivity in CeCu_2Si_2

Possibly induced by inter-band orbital fluctuations
Multiband may be important for HF SC

• Nearly-degenerate $d_{x^2-y^2}$ or p_x+ip_y -wave in YbRh_2Si_2

Sensitively depending on the Q-vector of the pairing force
Easily tuned by weak external magnetic field



s-wave

d-wave

s^\pm -wave



topological ?

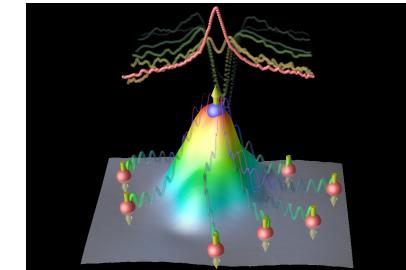
*Unusual richness of
heavy fermion
superconductivity !!!*



Summary

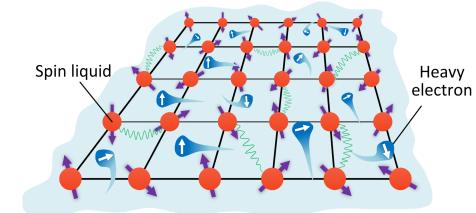
The Kondo problem

Incoherence Kondo scattering ($-\ln T$) above T_K
vs. Kondo singlet formation (resonance) below T_K



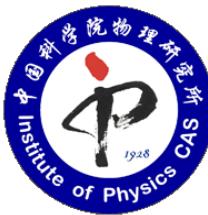
The heavy fermion problem

Localized-itinerant transition (Coherence)
Hybridization picture (mean-field approach)
Two-fluid theory (scaling and T^*)
Quantum criticality (magnetic+fermionic excitations)



The heavy fermion superconductivity

Pairing glues + multiband scattering + dual nature of f-electrons



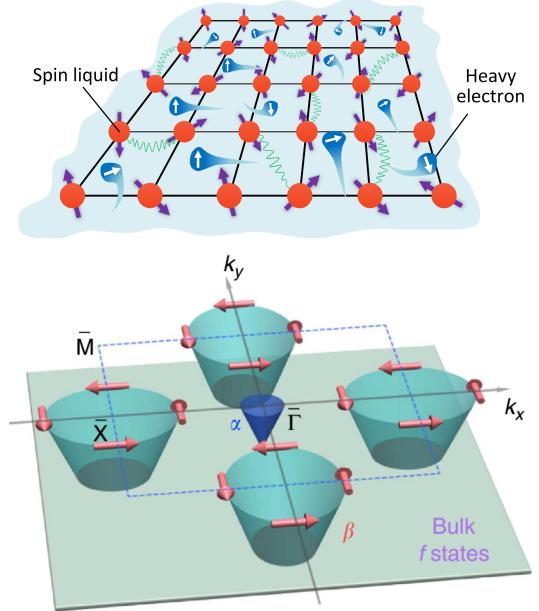
<http://hf.iphy.ac.cn>

2019年凝聚态理论前沿暑期讲习班 @青岛大学

Future directions of heavy fermion research

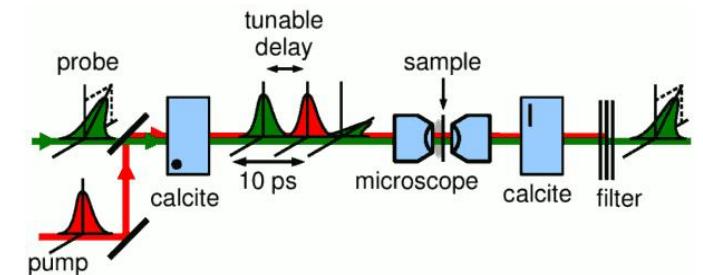
Explore the basic nature of heavy fermion physics

We are approaching the final answer of the questions:
local/itinerant duality & interplay of various QC fluctuations



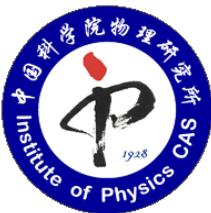
Find new frontiers: “Heavy fermion +”

Topological, Weyl, frustration, new exotic phases ...
Thin film, superstructure, cold atoms, d-electron systems ...



Implement state-of-the-art techniques

DMFT/CTQMC, tensor network, dual approach ...
ARPES, STM, MBE, pump probe, ultrasound ...



<http://hf.iphy.ac.cn>

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