DFT+DMFT及其在重费米子体系的应用

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Density Functional Theory

Condensed Matter Physics

Condensed matter physics is the science that studies microscopically the **structures**, laws of motion and macroscopic properties of condensed matter composed of **interacting many particles**.



✓存在大量不同的物质宏观状态✓微观上是复杂的相互作用多电子体系



自然界中的各种宏观物态

形形色色的实验探测手段



HOW TO DEAL WITH THE COMPLEX SYSTEMS FOR THEORETICAL EXPLORING?

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Density Functional Theory

Quantum mechanism



利用薛定谔方程求解量子力学问题

A quantum many body problem

$$\hat{H} = \sum_{i} -\frac{\hbar^{2}}{2M_{i}} \nabla_{R_{i}}^{2} - \sum_{i} \frac{\hbar^{2}}{2m_{e}} \nabla_{r_{i}}^{2} - \frac{1}{4\pi\varepsilon_{0}} \sum_{i,j} \frac{e^{2}Z_{i}}{|R_{i} - r_{j}|} + \frac{1}{8\pi\varepsilon_{0}} \sum_{i \neq j} \frac{e^{2}}{|r_{i} - r_{j}|} + \frac{1}{8\pi\varepsilon_{0}} \sum_{i \neq j} \frac{e^{2}Z_{i}Z_{j}}{|R_{i} - R_{j}|}$$

Born-Oppenheimer approximation

$$\hat{H}(r_i, R_\alpha) = \hat{H}_{R_\alpha}^{el}(r_i)$$

Time-independent Schrödinger equation

$$\hat{H}^{el}\psi_n(x_1,x_2,\cdots,x_n) = E_n\psi_n(x_1,x_2,\cdots,x_n)$$

$$\hat{H}^{el} = -\frac{\hbar^2}{2m_e} \sum_{i=1}^n \nabla_{r_i}^2 - \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^n \sum_{\alpha} \frac{Z_{\alpha}e^2}{|R_{\alpha} - r_i|} + \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^n \sum_{j>i} \frac{e^2}{|r_i - r_j|}$$

Electronic energy: functional of Ψ_n

$$E_n[\boldsymbol{\psi}_n] = \frac{\left\langle \boldsymbol{\psi}_n(x_1,\cdots,x_n) \middle| \hat{H}^{el} \middle| \boldsymbol{\psi}_n(x_1,\cdots,x_n) \right\rangle}{\left\langle \boldsymbol{\psi}_n(x_1,\cdots,x_n) \middle| \boldsymbol{\psi}_n(x_1,\cdots,x_n) \right\rangle}$$

Density functional theory

Use the electron density $\rho(r)$ as the basic variable, instead of the n-electron wave-function $\Psi_n(x_1, \dots, x_n)$.

Hohenberg and Kohn (1964)

The Ground-State properties of any system of ninteracting particles are rigourously deduced from the electron density distribution $\rho(r)$.

<u>Theorem</u> HK1: There is a unique correspondence $v(r) \leftrightarrow \rho(r)$

$$E_{v}[\rho] = \int \rho(r)v(r)dr + F_{HK}[\rho]$$

<u>Theorem</u> HK2: ρ(r) minimizes E[ρ]

$$E_{\nu}[\rho] = \min_{\rho} \left\{ F_{HK}[\rho] + \int \rho(r) v(r) \right\}$$



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Kohn-Sham Equation

A quantum many body problem:

$$\hat{H} = \hat{T} + \hat{V}_{\scriptscriptstyle Ne} + \hat{V}_{\scriptscriptstyle ee}$$

 $T[\rho]$ unknown

 $V_{ne}[\rho]$ known

 $V_{ee}[\rho]$ unknown

We are still left with the many-body problem ... We need a trick to solve this equation

Using an auxiliary system of n non-interacting particles for which the kinetic energy is know:

$$\rho_{s}(r) = \sum_{i=1}^{n} |\phi_{i}(r)^{2}| \qquad E[\rho] = T_{0}[\rho] + V_{Ne}[\rho] + V_{H}[\rho] + V_{xc}[\rho]$$

Kohn-Sham equation:

$$\sum_{i=1}^{n} \left\{ -\frac{1}{2m_e} \nabla_i^2 + V_{eff}(r) \right\} \phi_i(r) = \sum_{i=1}^{n} \varepsilon_i \phi_i(r)$$

The exchange-correlation functional

$$E_{xc}[\rho] = \int \varepsilon_{xc}[\rho] \rho(r) dr$$

- •Local Density Approximation (LDA)
- Generalized Gradient Approximation (GGA)
- Meta Generalized Gradient Approximation (meta-GGA)

$$\epsilon_{xc}[\rho] = f(\rho, \nabla \rho, \Delta \rho, \ldots)$$

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Success of Density Functional Theory

传统的磁性与自旋态的研究

$CaCu_{3}Fe_{2}Os_{2}O_{12}$



复杂磁性(非共线磁性)与奇异 物态(超导)的研究















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J. Phys.: Condens. Matter **29,** 244001 (2017) Sci. Rep. **7**, 14178 (2017)



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Breakdown of Density Functional Theory

The role of electronic correlations: localization versus itinerancy



有趣的物理往往都发生在两者交界, 如 高温超导 等等 缺乏共识,需要进行严格的数值求解以进行裁决!

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Breakdown of Density Functional Theory



The Kondo problem

e_d+U

Kondo effect in quantum dot

Coulomb blockade



single quantum dot

Kondo effect





At low temperature, spin-flip scattering between conduction band and impurity site become important.

Anderson impurity model

$$H = \sum_{ks} \epsilon_k c_{ks}^{\dagger} c_{ks} + \sum_s \epsilon_d d_s^{\dagger} d_s + U n_{d\uparrow} n_{d\downarrow} + \sum_{ks} V c_{ks}^{\dagger} d_s + h.c$$





HOW TO SOLVE ANDERSON IMPURITY MODEL NUMERICALLY?

Renormalization group approach
 Continues-time quantum Monte Carlo
 One-crossing approximation

We usually call these numerical methods as "impurity solver", which is the most important in DMFT calculations.

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Historic development of Kondo

✓ (19th Century) Electron behavior @ $T \rightarrow 0$

InT	Kondo/2D WL
Т	Non-Fermi liquid
T ²	Fermi liquid
T ⁵	Phonon scattering
e -∆/T	Insulator/Semiconductor
е ^{т-1/n}	VRH (Semiconductor)







Historic development of Kondo

- ✓ (19th Century) Electron behavior @ $T \rightarrow 0$
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity EXP→THEO
- ✓ 1934 Resistivity minima



de Haas, de Boer and van den Berg, 1934

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Historic development of Kondo

✓ (19th Century) Electron behavior @ T→0
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Figure 2.6 Resistance minima for Fe in a series of Mo-Nb alloys (from Sarachik et al, 1964). Compare the depths of the minima with the corresponding moments in figure 1.8.



Figure 1.8. The magnetic moment in $\mu_{\rm B}$ of Fe in various Mo-Nb and Mo-Re alloys as a function of alloy composition (Clogston et al, 1962).

Smoking gun: Resistance minima **Magnetic impurity**

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- ✓ 1934 Resistivity minima
- ✓ 1964 Kondo effect (logarithmic resistivity)

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun Kondo

Electro-technical Laboratory Nagatacho, Chiyodaku, Tokyo

(Received March 19, 1964)





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Historic development of Kondo

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The renormalization group: Critical phenomena and the Kondo problem*[†]

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

This review covers several topics involving renormalization group ideas. The solution of the *s*-wave Kondo Hamiltonian, describing a single magnetic impurity in a nonmagnetic metal, is explained in detail. See Secs. VII–IX. "Block spin" methods, applied to the two dimensional Ising model, are explained in Sec. VI. The first three sections give a relatively short review of basic renormalization group ideas, mainly in the context of critical phenomena. The relationship of the modern renormalization group to the older problems of divergences in statistical mechanics and field theory and field theoretic renormalization is discussed in Sec. IV. In Sec. V the special case of "marginal variables" is discussed in detail, along with the relationship of the modern renormalization by Gell-Mann and Low and others.



Numerical renormalization group (NRG)

Reduce the Hilbert space by throwing away high energy states

$$H = \underbrace{V \quad t_{0} \quad t_{1} \quad t_{2} \quad t_{3} \quad t_{4}}_{\text{Imp} \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5} \dots \qquad \begin{aligned} H_{N+1} &= H_{N} + H_{N+1}^{hop} \\ H_{N+1}^{hop} &= \sum_{\sigma} t_{N} c_{N\sigma}^{\dagger} c_{(N+1)\sigma} + \text{H.c.} \end{aligned}$$



K. G. Wilson Nobel prize 1982

Iterative diagonalization & Truncation (keep fixed number of low energy states)





require separation of energy scales ($t_{N+1}/t_N \sim \Lambda < 1$)

这解决了强关联物理的一大问题,即单格点(0维)的Kondo问题!

Monte Carlo Simulation

Classical Monte Carlo



对自旋空间的所有位形按照权重取样

$$Z = \sum_{\{S_i\}} e^{-\beta H(\{S_i\})}, \quad \langle m \rangle = \sum_{\{S_i\}} \frac{1}{Z} e^{-\beta H(\{S_i\})} m(\{S_i\}) = \sum_{\{S_i\}} w(\{S_i\}) m(\{S_i\})$$

Metropolis algorithm
▶ 随机选择z,翻转自旋:
$$\{\overline{S_i}\} = (S_1^k, S_2^k, \dots, -S_z^k, \dots, S_N^k)$$

▶ 计算前后权重的比值: $\alpha = \frac{w(\{\overline{S_i}\})}{w(\{S_i\}_k)} = e^{-\beta(H(\{\overline{S_i}\})-H(\{S_i\}_k))} = e^{-\beta\Delta E}$

对任一位形,H({Si})为一数

Quantum Monte Carlo (QMC)

 $H = U \sum_{i} c_{i\uparrow}^{\dagger} c_{i\uparrow} c_{i\downarrow}^{\dagger} c_{i\downarrow} - t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^{\dagger} c_{j\sigma}$

U

对任一状态,H({Si})不是数,

需要构造新的位形空间!

多种处理方法

▶ 选择随机数 γ , 若 $\alpha \ge \gamma$, 接受新位形, 否则拒绝!

☑辅助场QMC:引入合适的经典辅助场,将费米子态积掉,得到辅 助场的有效作用量,在辅助场的位形空间进行取样。

Hubbard-Stratonovich变换:

$$\exp\left[-\Delta\tau U(n_{\uparrow}n_{\downarrow}-\frac{1}{2}(n_{\uparrow}+n_{\downarrow}))\right]=\frac{1}{2}\mathrm{Tr}_{\sigma}[\exp[\lambda\sigma(n_{\uparrow}-n_{\downarrow})]$$

 $Z = \sum_{\{\sigma_i\}} D(\{\sigma_i\})$ After integrate out the "free" fermions, sample $\{\sigma\}$ with the Metropolis algorithm

☑ 连续时间QMC:在配分函数中将动能t项或相互作用U项做微扰展 开,以自由电子(t)或孤立原子(U)为背景,对微扰展开项进行取

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From the single impurity problem to the lattice problem

Dynamical Mean Field Theory

U

Lattice model

Coordination number Z



The action is:

$$S = \int_0^\beta d\tau \left[\sum_{i,\sigma} c_{i\sigma}^*(\tau) (\frac{\partial}{\partial \tau} - \mu) c_{i\sigma}(\tau) - \sum_{\langle i,j \rangle \sigma} t_{i,j} c_{i\sigma}^*(\tau) c_{j\sigma}(\tau) + \sum_i H_i^{on-site}(\tau)\right]$$

When we integrating out of the other site:

$$S_{eff} = -\int_0^\beta d\tau \int_0^\beta d\tau' \sum_\sigma c_{o\sigma}^\dagger(\tau) g_0^{-1}(\tau - \tau') c_{o\sigma}(\tau') + U \int_0^\beta d\tau n_{o\uparrow}(\tau) n_{o\downarrow}(\tau)$$



$$G(\tau - \tau') = -\langle Tc_{o\sigma}(\tau)c_{o\sigma}^{\dagger}(\tau') \rangle_{S_{eff}}$$
$$\Sigma(iw_n) = G_0^{-1}(iw_n) - G^{-1}(iw_n)$$



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Dynamical Mean Field Theory

Mapping the lattice Hubbard model to an impurity Anderson model



Bath function:

 $G_{0\sigma}^{-1}(i\omega_n) \equiv i\omega_n + \varepsilon_d - \Delta_{\sigma}(i\omega_n)$

Institute of Physics, Chinese Academy of Sciences

Electron reservoi

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Dynamical Mean Field Theory

Dynamical mean field theory self-consistent equation



Dynamical mean-field theory

$$H = \sum_{ij,\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + \varepsilon_d \sum_{i\sigma} n_{i\sigma}^d + U \sum_i n_{i\uparrow}^d n_{i\downarrow}^d + \sum_{ij,\sigma} (V_{ij} c_{i\sigma}^{\dagger} d_{j\sigma} + h.c)$$

Effective impurity model defined by hybridization function is solved with an "impurity " solver, e.g. QMC, NRG,ED...

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Density Functional Theory + Dynamical Mean-Field Theory

Framework: DFT+DMFT

Density Functional Theory



A framework for materials calculations

- ✓ Kohn-Hohenberg theorem
- ✓ Kohn-Sham equation

WIEN2k package; WIEN2Wannier

- ✓ Electronic/magnetic/structural/orbital
- ✓ Ground state tuning; Materials design

Dynamical Mean-Field THeory



Strongly correlated electron systems

- ✓ Kondo and Mott physics beyond DFT
- ✓ Non-perturbative; time-consuming

NRG/QMC/CTQMC/NCA/OCA/IPT

- ✓ Quantum phase transitions
- ✓ Quantum criticality & Non-Fermi liquid

(c)



LDA L DA+DMF T L DA+U 0 U/W ~

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Framework: DFT+DMFT

(1) Calculate LDA band structure

$$\varepsilon_{lml'm'}(k) \rightarrow \hat{H}_{LDA}$$

(2) Supplement LDA band by local Coulomb interaction (only for correlated bands)



(3) Solve model by DMFT with impurity solver

$$\begin{split} \boldsymbol{G} &= -\frac{1}{Z} \int \mathcal{D}[\psi\psi^*] \psi\psi^* e^{\psi^*} [\boldsymbol{G}^{-1} + \boldsymbol{\Sigma}] \psi - U\psi^* \psi\psi^* \psi + \boldsymbol{J}\psi^* \psi\psi^* \psi \\ & \boldsymbol{G}_{mm'}^{\sigma}(\omega) = \frac{1}{V_B} \int d^3k \left[(\omega - \boldsymbol{\Sigma}^{\sigma}(\omega)) \delta_{m,m'} - \left(H_{LDA}^{0\ eff}(\mathbf{k}) \right)_{m,m'} \right]^{-1} \end{split}$$

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Application to the Kondo lattice

Heavy Fermion Materials



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Kondo lattice CeColnGa3



6

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Kondo lattice CeColnGa₃



4

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Kondo lattice CeColnGa3

(a) (C) A(k,ω) 2 -T=1 K total 0.1 0 0.1 -0.1 -T = 200Ce Co In Ga Energy (eV) 200 K -0.2 0 -2 -4 0 2 4 6 ΖU ΧГ Г γ Energy (eV) (b) ₁₆ (d) 0.2 A(k,ω) DOS (state/eV f.u.) 8 8 7 -lm Σ(ω = 0) 2 e) (c -1 peak height $-\text{Im }\Sigma(\omega = 0)$ -d (Im Σ)/d(In T)/_{ω =} (eV) -0.1 -2 ΧГ 50 Г ΖU Y 5 20 80 6 DOS (state/eV f.u.) 200 ΖU 50 100 150 ΧГ 0 Temperature (K) The failure of DFT: There is no flat band and large Anisotropic hybridization density of states at the Fermi level ! (a) 0.010 0.008 The success of DFT+DMFT: H⊥a X (emu/mol) H = 5 T20 Typical heavy fermion evolution with temperature 0.006 30 can be well captured. 4// (0 2 30 0.004

•The anisotropic hybridization gap agree well with experiments.



DFT results

DFT+DMFT calculations

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Summary

有趣的物理往往都发生在局域和巡游的边界:高温超导、庞磁阻等等



Correlation in Condensed Matter Physics

Mott insulator





Correlations & long-range order





Exotic quantum state



Quasiparticles

Electron doping



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