Discussions on Conceptual Difficulties of Heavy Fermion Physics

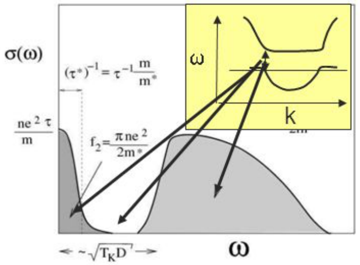
Yi-feng Yang

Institute of Physics,
Chinese Academy of Sciences

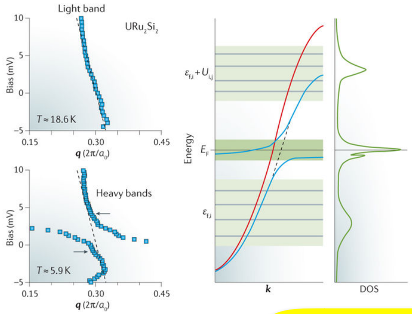
http://hf.iphy.ac.cn

Schematic Understanding of Heavy Fermion Concepts

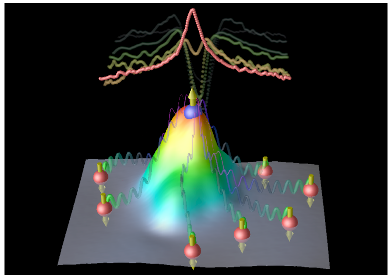
Optical



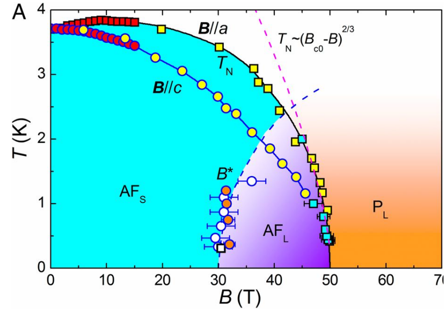
Mean-field

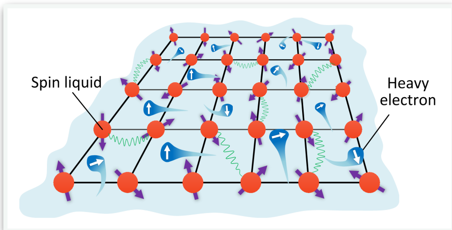


Kondo



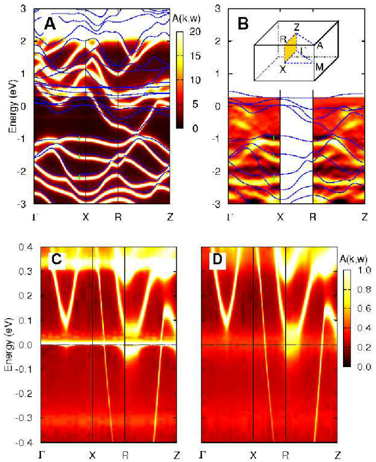
Kondo lattice



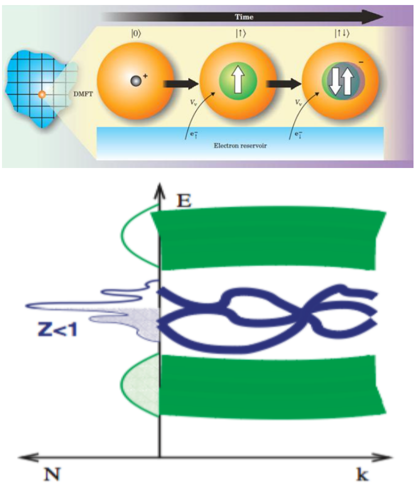


Local/Itinerant crossover

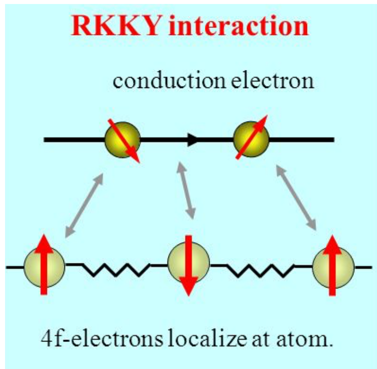
ARPES



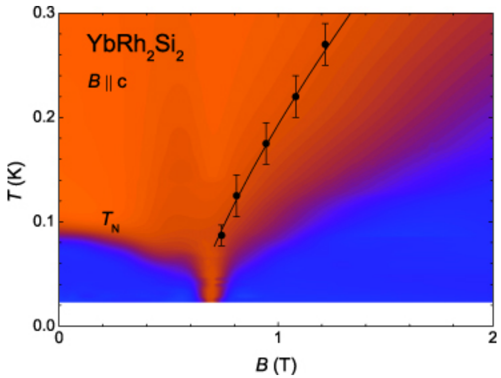
DMFT



Magnetism

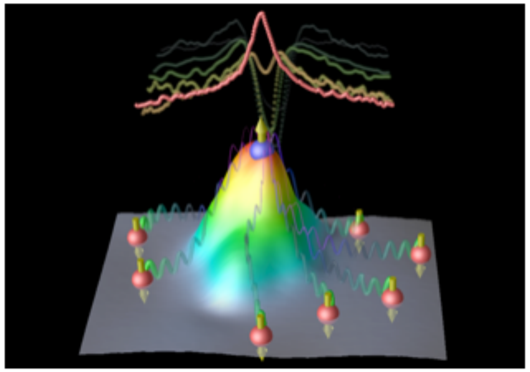


Quantum criticality



一、重费米子研究的历史发展

Kondo



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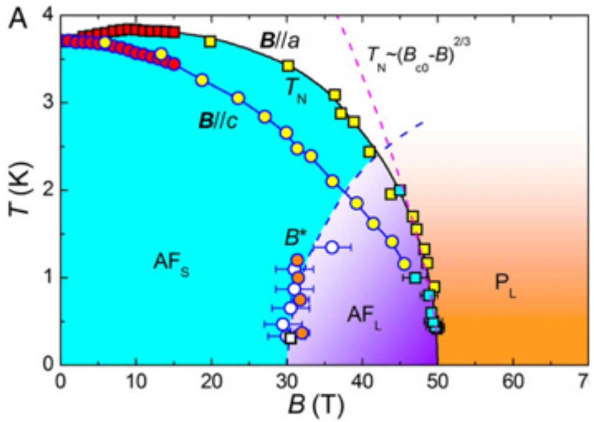
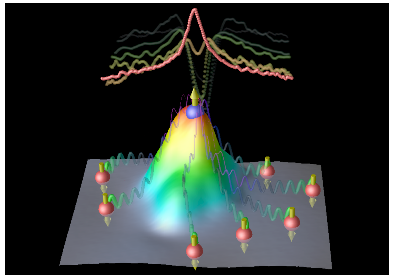
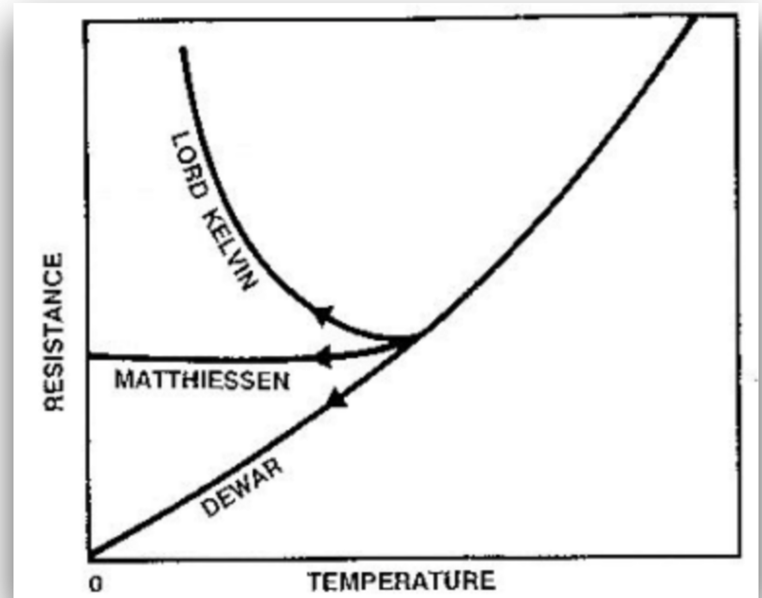


Kondo & Kondo Lattice



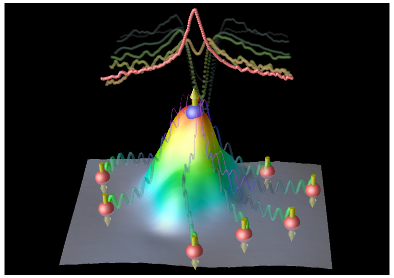
- **Historic development of Kondo**
  - ✓ (19th Century) Electron behavior @ $T \rightarrow 0$

$\ln T$	Kondo/2D WL
T	Non-Fermi liquid
T^2	Fermi liquid
T^5	Phonon scattering
$e^{-\Delta/T}$	Insulator/Semiconductor
$e^{T^{-1/n}}$	VRH (Semiconductor)
...	...



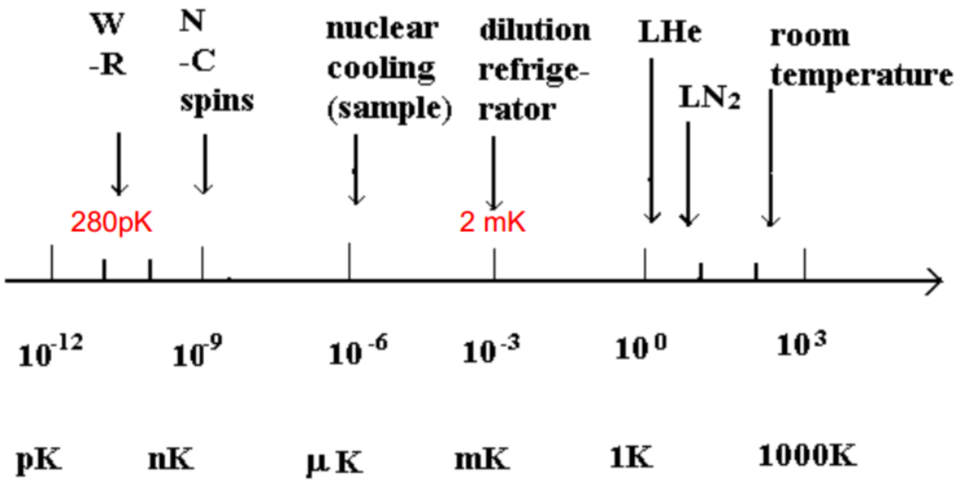
Kelvin Dewar

Kondo & Kondo Lattice

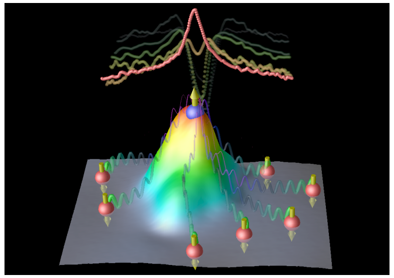


- **Historic development of Kondo**
  - ✓ (19th Century) Electron behavior @ T→0
  - ✓ Liquification & Resistance measurement

☺1895：空气被液化 -192℃ （81K）
☺1898：氢气被液化 -253℃ （20K）
☺1908：氦气被液化 4.25K



Kondo & Kondo Lattice



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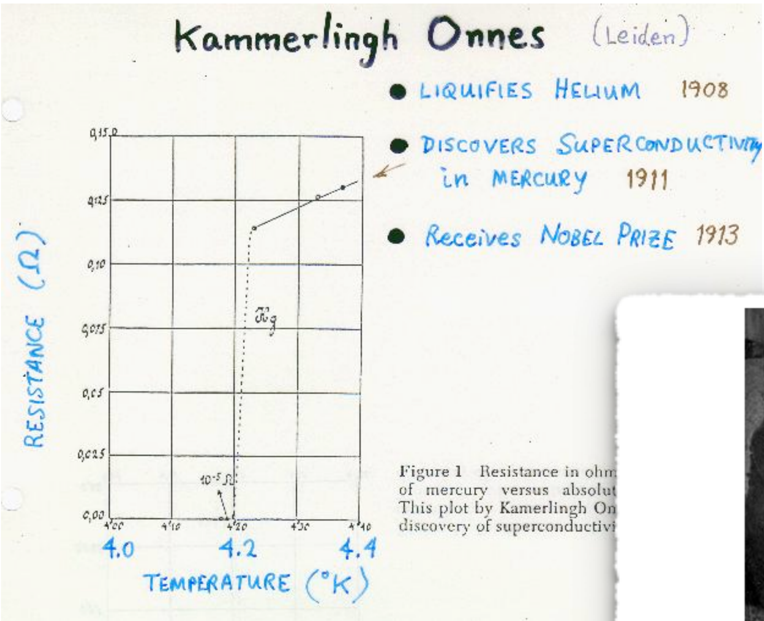
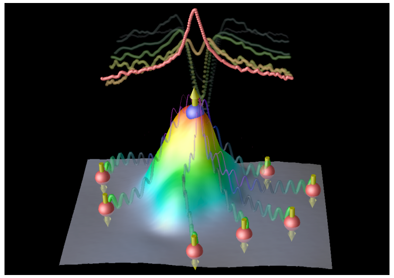


Fig. 1. Albert Einstein, Paul Ehrenfest, Paul Langevin, Heike Kamerlingh Onnes, and Pierre Weiss discussing superconductivity during the "Magnet-Woche" in Leiden in November 1920 (Photo: AIP)

Kondo & Kondo Lattice



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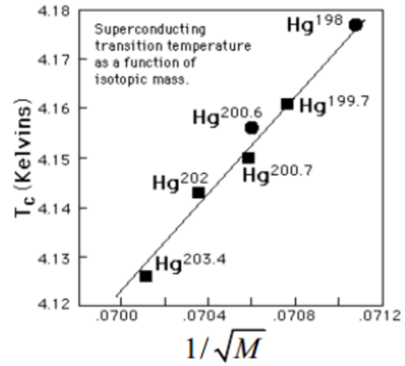
  - ✓ (19th Century) Electron behavior @ T→0
  - ✓ Liquification & Resistance measurement
  - ✓ 1911 Superconductivity in Hg
  - ✓ 1957 BCS Theory of Superconductivity

Smoking gun: Isotope effect



E. Maxwell, Phys. Rev. (1950)

C.A. Reynolds et al., Phys. Rev. 78 (1950).

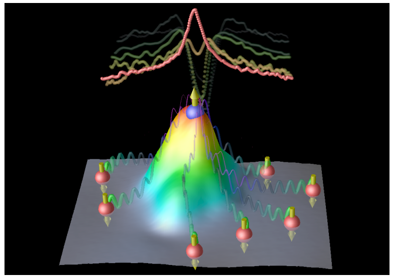
PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. Bardeen, L. N. Cooper,† and J. R. Schrieffer‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

A theory of superconductivity is presented, based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch states and by using the rest to form a linear combination of virtual pair con-

Kondo & Kondo Lattice



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  - ✓ (19th Century) Electron behavior @ T➔0
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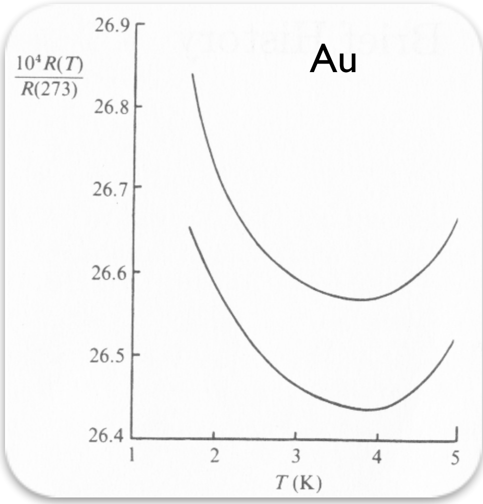
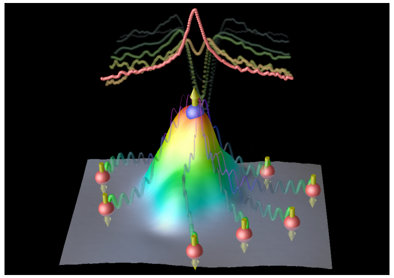
  - ✓ 1911-1957 Superconductivity EXP➔THEO
  - ✓ 1934 Resistivity minima



de Haas, de Boer and van den Berg, 1934

Kondo & Kondo Lattice



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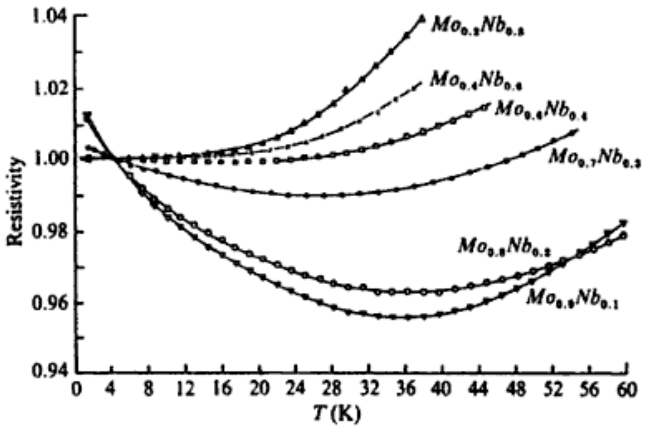
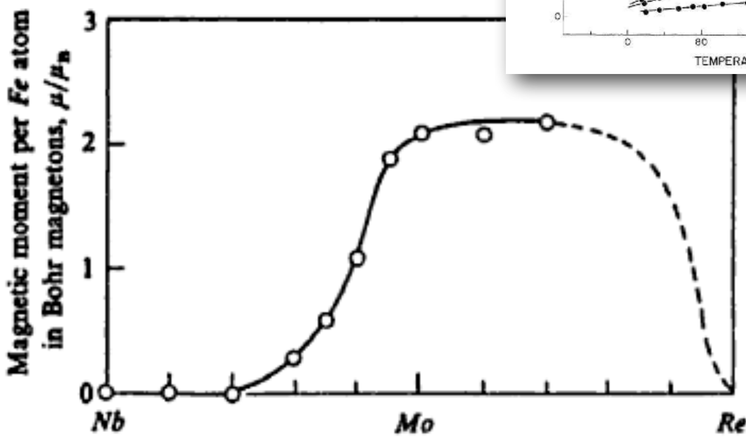


Figure 2.6 Resistance minima for *Fe* in a series of *Mo–Nb* alloys (from Sarachik et al, 1964). Compare the depths of the minima with the corresponding moments in figure 1.8.



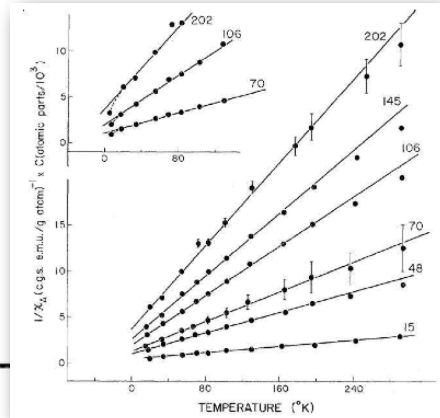


Figure 1.8. The magnetic moment in μ_B of *Fe* in various *Mo–Nb* and *Mo–Re* alloys as a function of alloy composition (Clogston et al, 1962).

Smoking gun:* Resistance minima ☛ *Magnetic impurity

Kondo & Kondo Lattice

Journal of the Physical Society of Japan
Vol. 74, No. 1, January, 2005, pp. 4–7

SPECIAL TOPICS

Kondo Effect — 40 Years after the Discovery

Kondo Lattices and the Mott Metal–Insulator Transition

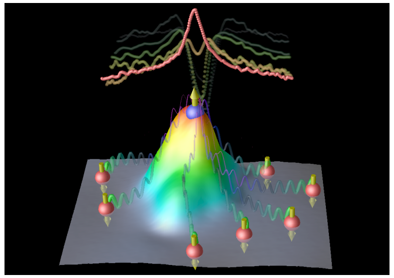
Ph. NOZIÈRES

Laboratoire d'Etude des Propriétés Electroniques des Solides, Centre National de la Recherche Scientifique, BP166, 38042 Grenoble Cedex 9, France

(Received August 29, 2004)

In the Summer of 1964, I visited Urbana in order to work with my thesis advisor David Pines. The first person I met when entering the building was John Bardeen, just out of the seminar room. He was beaming when he told me "the long standing puzzle of the resistance minimum in metals is gone: we just heard a young Japanese theorist who has a beautiful explanation that is obviously the good one". The Kondo effect was born—and John was right! But it was only the

Kondo & Kondo Lattice



Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

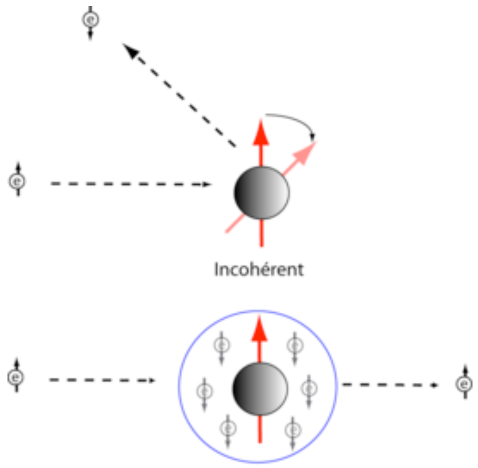
Jun Kondo

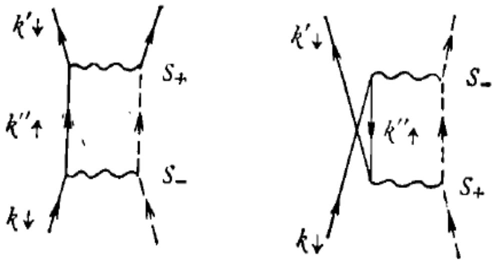
Electro-technical Laboratory
Nagatacho, Chiyodaku, Tokyo

(Received March 19, 1964)

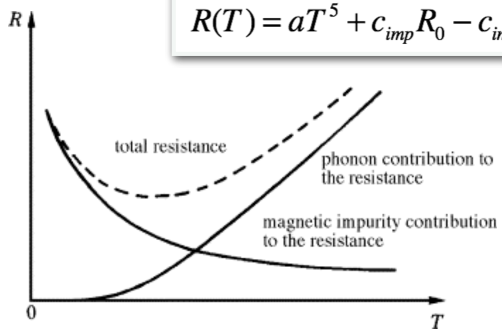
Historic development of Kondo

- ✓ (19th Century) Electron behavior @ T→0
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity EXP→THEO
- ✓ 1934 Resistivity minima
- ✓ 1964 Kondo effect (logarithmic resistivity)



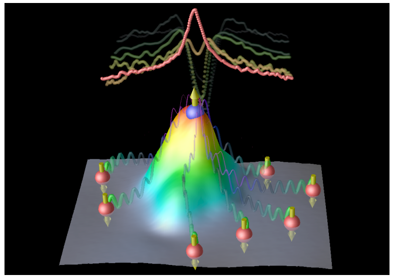


$$R(T) = aT^5 + c_{imp}R_0 - c_{imp}R_1 \ln(k_B T / D)$$



$$H_{sd} = \sum_{\mathbf{k},\mathbf{k}'} J_{\mathbf{k},\mathbf{k}'}(S^+ c^\dagger_{\mathbf{k},\downarrow} c_{\mathbf{k}',\uparrow} + S^- c^\dagger_{\mathbf{k},\uparrow} c_{\mathbf{k}',\downarrow} + S_z(c^\dagger_{\mathbf{k},\uparrow} c_{\mathbf{k}',\uparrow} - c^\dagger_{\mathbf{k},\downarrow} c_{\mathbf{k}',\downarrow}))$$

Kondo & Kondo Lattice



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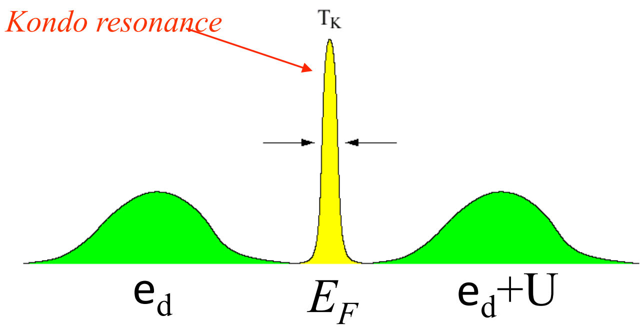
- ✓ 1934-1964 Kondo effect EXP➔THEO
- ✓ Many approaches: EOM, NRG, QMC …

The renormalization group: Critical phenomena and the Kondo problem*†

Kenneth G. Wilson

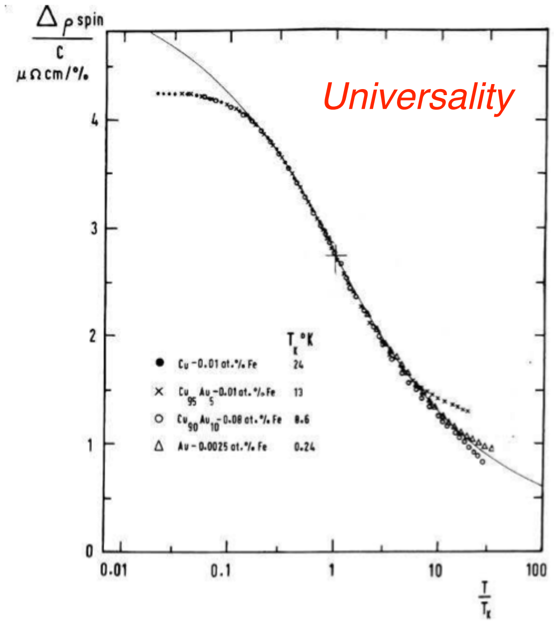
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

This review covers several topics involving renormalization group ideas. The solution of the s-wave Kondo Hamiltonian, describing a single magnetic impurity in a nonmagnetic metal, is explained in detail. See Secs. VII–IX. "Block spin" methods, applied to the two dimensional Ising model, are explained in Sec. VI. The first three sections give a relatively short review of basic renormalization group ideas, mainly in the context of critical phenomena. The relationship of the modern renormalization group to the older problems of divergences in statistical mechanics and field theory and field theoretic renormalization is discussed in Sec. IV. In Sec. V the special case of "marginal variables" is discussed in detail, along with the relationship of the modern renormalization group to its original formulation by Gell-Mann and Low and others.

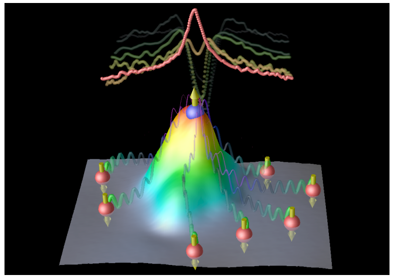


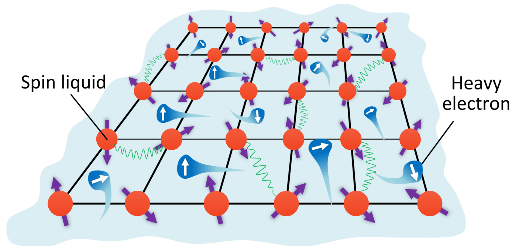
- A sharp resonance of width T_K develops at E_F for $T<T_K$
- Unitary scattering for $T=0$ and $<n>=1$

$$A(\varepsilon=0,T=0)=\frac{1}{\pi\Gamma}\sin^2(\delta)\quad,\quad\delta=\frac{\pi\langle n\rangle}{2}$$



Kondo & Kondo Lattice





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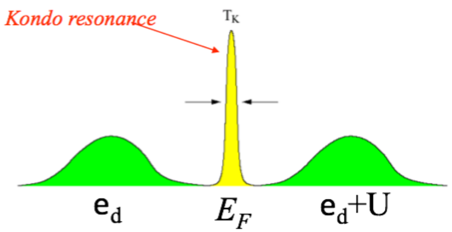
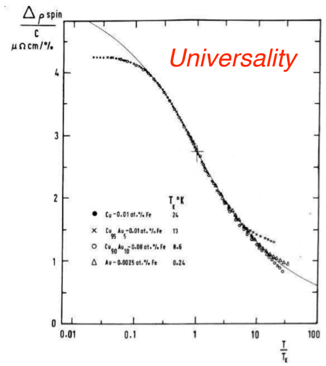




Historic development of heavy fermions

- ✓ $CeAl_3$ @ 1975 @ γ=1.62 J/mol K^2

VOLUME 35, NUMBER 26 PHYSICAL REVIEW LETTERS 29 DECEMBER 1975

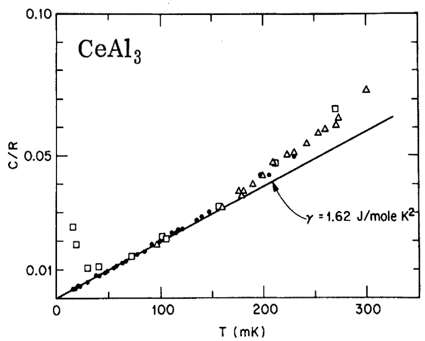
4f-Virtual-Bound-State Formation in $CeAl_3$ at Low Temperatures

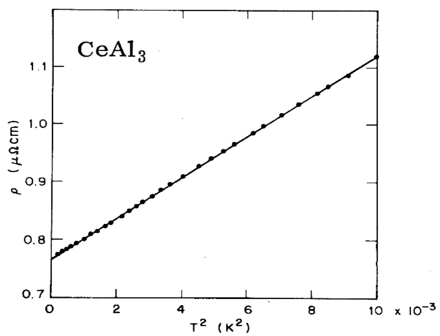
K. Andres and J. E. Graebner
Bell Laboratories, Murray Hill, New Jersey 07974

and

H. R. Ott
Laboratorium für Festkörperphysik, Eidgenössische Technische Hochschule, Hönggerberg, Zürich, Switzerland

(Received 25 August 1975)

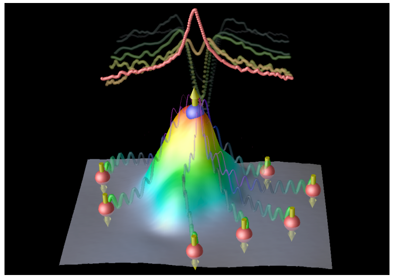


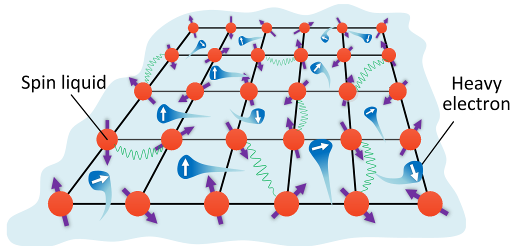


Cu

γ_{Cu} ; 0.7mJ mol^{-1} K^{-2}

Kondo & Kondo Lattice





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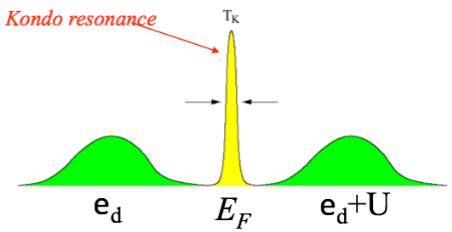

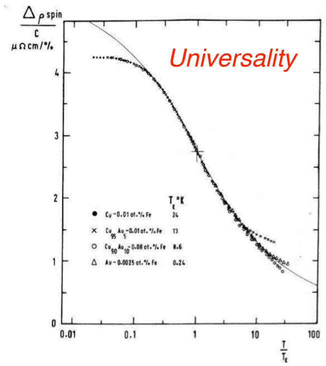

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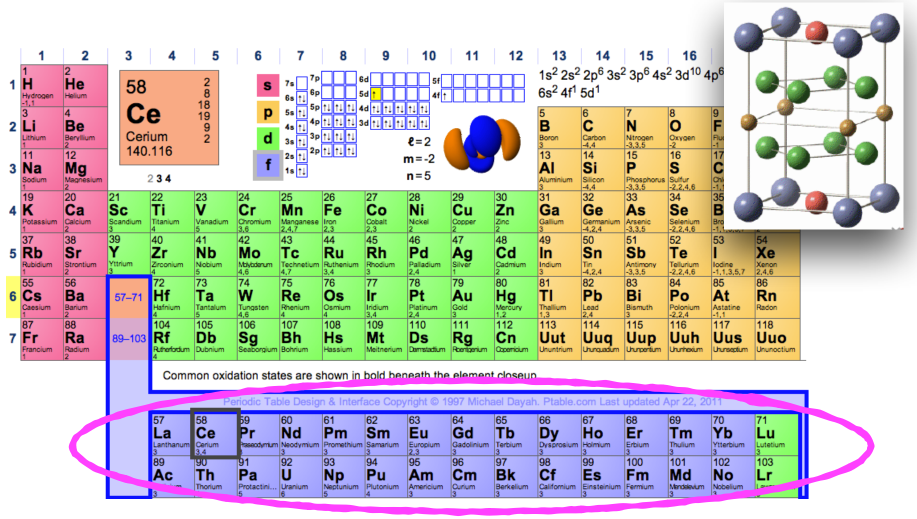
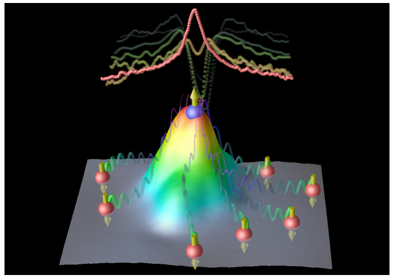
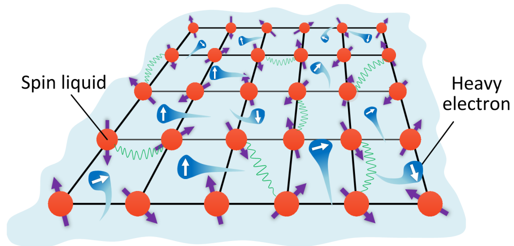
  - ✓ Ce, Yb, U, …, 4f/5f intermetallics



Some d-electron systems: LiV_2O_4, $CaCu_3Ir_4O_{12}$ …

Kondo & Kondo Lattice





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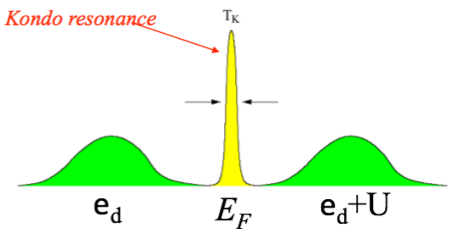

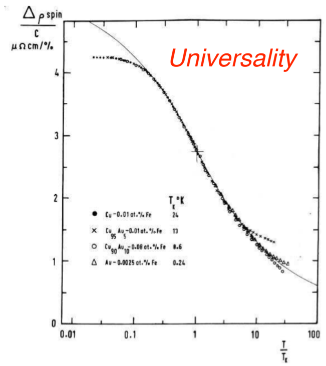

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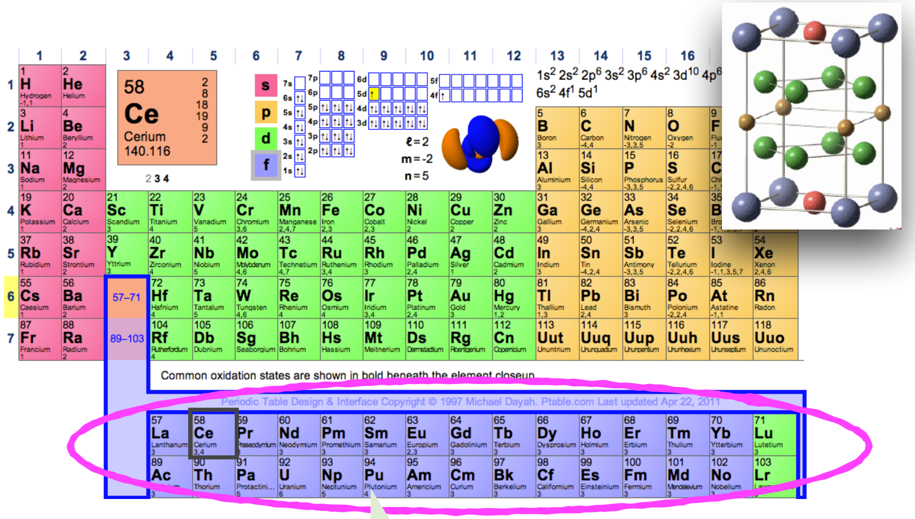


金/铜等

> 50

$CeAl_3$ ~ 2000

1 100 1000 m^*/m_e

Kondo & Kondo Lattice

PHYSICAL REVIEW B VOLUME 11, NUMBER 1 1 JANUARY 1975

Electronic properties of beryllides of the rare earth and some actinides

E. Bucher,* J. P. Maita, G. W. Hull, R. C. Fulton, and A. S. Cooper
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 14 March 1974)

UBe_{13}

We tried to detect any possible magnetic ordering below 1 °K. Instead we found a sharp superconducting transition at 0.97 °K, which was reduced by about 0.3 °K only in a field of 60 kOe. This suggests that the superconductivity is not an intrinsic property of UBe_{13}, but probably linked with precipitated filaments. Subsequent powdering did not shift nor reduce the superconducting signal, although calibration with a Pb cylinder showed that the signal of UBe_{13} was only about 50% of the expected full signal. From the fact that none of the other MBe_{13} phases showed superconductivity down to 0.45 °K, one is tempted to conclude that the superconductivity and perhaps also the susceptibility tail at low temperature is due to precipitated U filaments.

Z. Physik B 31, 7 – 17 (1978)
Zeitschrift für Physik B
© by Springer-Verlag 1978

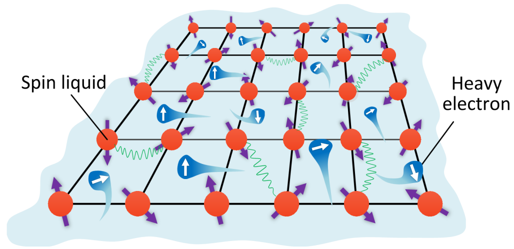
Transport Properties of $LaCu_2Si_2$ and $CeCu_2Si_2$ Between 1.5 K and 300 K***

W. Franz, A. Grießel, F. Steglich, and D. Wohlleben
II. Physikalisches Institut der Universität zu Köln, Köln, Fed. Rep. Germany

Received May 23, 1978

$CeCu_2Si_2$

* The resistivity measurement of W. Lieke below 1.5 K showed a continuous drop down to about 0.6 K and then a superconducting transition. The superconductivity could be suppressed at 0.3 K by application of a magnetic field of 3 T. At this field the resistivity was 41 μΩcm and it did not change upon further increase of the field. We therefore consider 41 μΩcm as representative of the residual resistivity of this sample. In order to check whether the superconductivity was a bulk property of $CeCu_2Si_2$ or due to a second phase forming a network through grain boundaries the specific heat was measured down to 0.3 K by C.D. Bredl and the static Meissner effect (on bulk and powdered samples) in a SQUID magnetometer by R.F. Hoyt and A.C. Mota down to 30 mK. Both measurements indicate the absence of bulk superconductivity. According to the static Meissner effect less than 0.1 % of the sample volume is superconducting.



Historic development of heavy fermions

✓ $CeAl_3$ γ=1.62 J/mol K² @1975

✓ $CeCu_2Si_2$ T_c=0.5 K @1979

PHYSICAL REVIEW LETTERS 17 December 1979

…tivity in the Presence of Strong Pauli Paramagnetism: $CeCu_2Si_2$

F. Steglich
Institut für Festkörperphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany

and

J. Aarts, C. D. Bredl, W. Lieke, D. Meschede, and W. Franz
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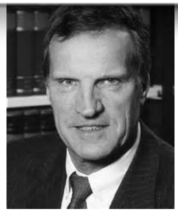
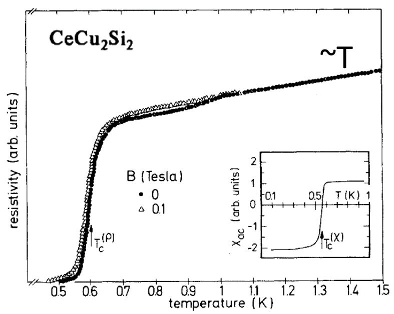
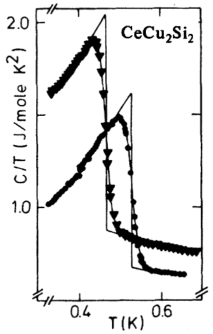
H. Schäfer
Eduard-Zintl-Institut, Technische Hochschule Darmstadt, D-6100 Darmstadt, West Germany
(Received 10 August 1979; revised manuscript received 7 November 1979)



(F. Steglich)

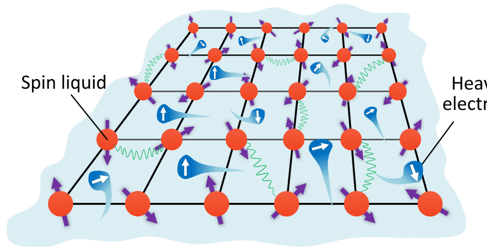




The size of the specific-heat jump at T_c, in proportion to γT_c, suggests that Cooper-pair states are formed by these heavy fermions.

Since the Debye temperature, Θ, is of the order of 200 K,[5] we find $T_c < T_F < \Theta$ with $T_c/T_F \simeq T_F/\Theta \simeq 0.05$. This suggests that $CeCu_2Si$ (i) behaves as a "high-temperature superconductor" and (ii) cannot be described by conventional theory of superconductivity which assumes a typical phonon frequency $k_B\Theta/h \ll k_BT_F/h$, the characteristic frequency of the fermions.

Kondo & Kondo Lattice



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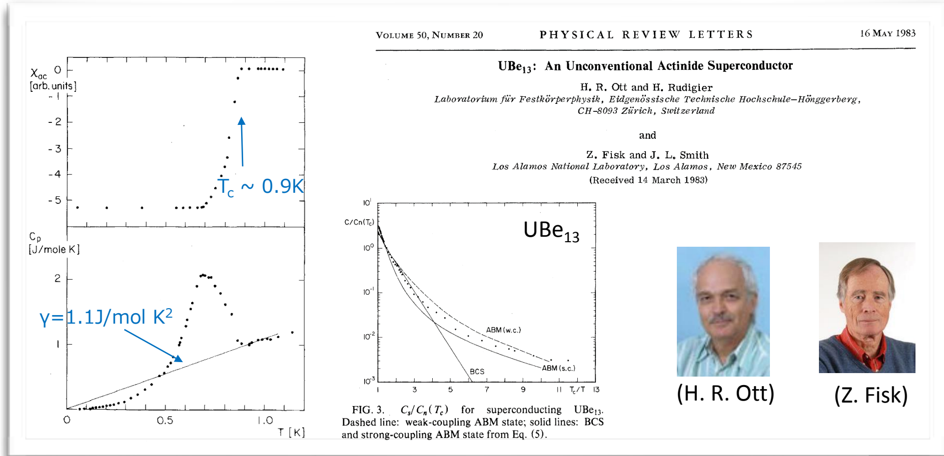

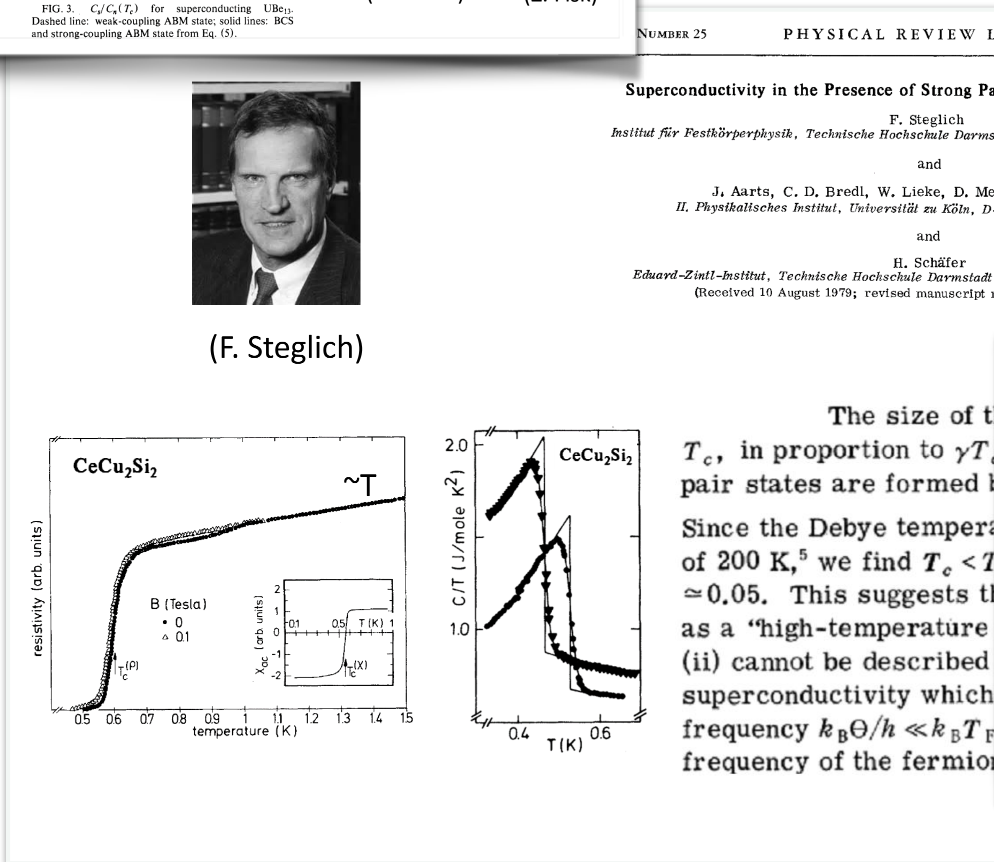

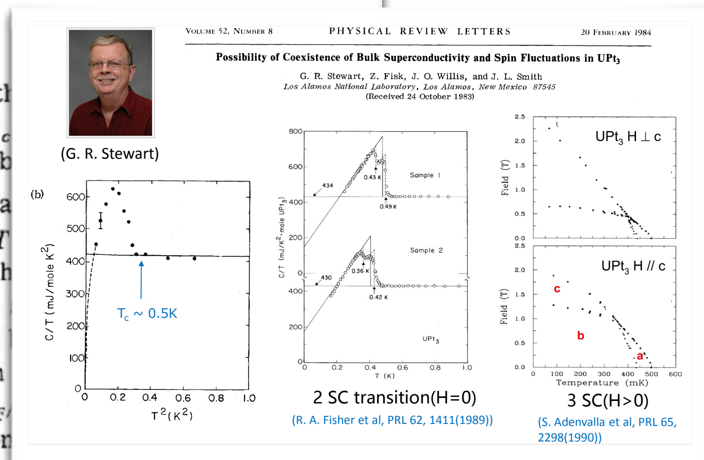

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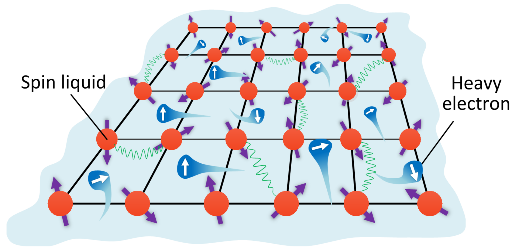

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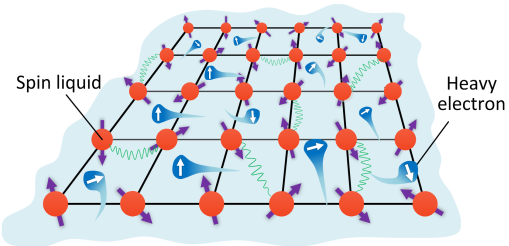
Serials	Compounds	T_c (K)	γ	nodes
CeM_2X_2 (AFM SC)	$CeCu_2Si_2$	0.6-0.7	1000	No
	$CeCu_2Ge_2$	0.64(10.1GPa)	200	–
	$CePd_2Si_2$	0.43 (3GPa)	65	–
	$CeRh_2Si_2$	0.42 (1.06GPa)	23	–
	$CeAu_2Si_2$	2.5K(22.5GPa)	-	-
	$CeNi_2Ge_2$	0.3	350	–
$Ce_nM_mIn_{3n+2m}$ (AFM SC)	$CeIn_3$	0.23 (2.46GPa)	140	Line
	$CeIrIn_5$	0.4	750	Line
	$CeCoIn_5$	2.3	250	Line
	$CeRhIn_5$	2.4 (2.3GPa)	430	–
	$CePt_2In_7$	2.3 (3.1GPa)	340	–
	Ce_2RhIn_8	2.0 (2.3GPa)	400	–
	Ce_2PdIn_8	0.68	550	Line
	Ce_2CoIn_8	0.4	500	-
	Ce_3PdIn_{11}	0.42	290	–
Ce-based non-centresymmetric SC	$CePt_3Si$	0.75	390	Line
	$CeIrSi_3$	1.65 (2.5GPa)	120	–
	$CeRhSi_3$	1.0 (2.6GPa)	120	–
	$CeCoGe_3$	0.69 (6.5GPa)	32	–
Other Ce-based AFM SC	$CeNiGe_3$	0.43 (6.8GPa)	45	–
	$Ce_2Ni_3Ge_5$	0.26 (4.0GPa)	90	–
	$CePd_5Al_2$	0.57 (10.8GPa)	56	–

Serials	Compounds	T_c (K)	γ	nodes
U-based AFM SC	UPd_2Al_3	2.0	210	Line
	UNi_2Al_3	1.06	120	–
	UBe_{13}	0.95	1000	No
	U_6Fe	3.8	157	–
	UPt_3	0.53, 0.48	440	Line, Point
U-based FM SC	UGe_2	0.8 (~1.2GPa)	34	Line
	URhGe	0.3	164	–
	UCoGe	0.6	57	Point
	UIr	0.15 (~2.6GPa)	49	–
	U_2PtC_2	1.47	150	–
Hidden order SC	URu_2Si_2	1.5	70	Line
Pr-based SC	$PrOs_4Sb_{12}$	1.85	500	点
	$PrTi_2Al_{20}$	0.2, 1.1(8.7GPa)	100	–
	PrV_2Al_{20}	0.05	90	–
Pu-based SC (Pu-115)	$PuCoGa_5$	18.5	77	Line
	$PuCoIn_5$	2.5	200	Line
	$PuRhGa_5$	8.7	70	Line
	$PuRhIn_5$	1.6	350	Line
Np-based SC	$NpPd_5Al_2$	4.9	200	Point
Yb-based SC	β-$YbAlB_4$	0.08	150	-
	$YbRh_2Si_2$	0.002	-	–

Kondo & Kondo Lattice



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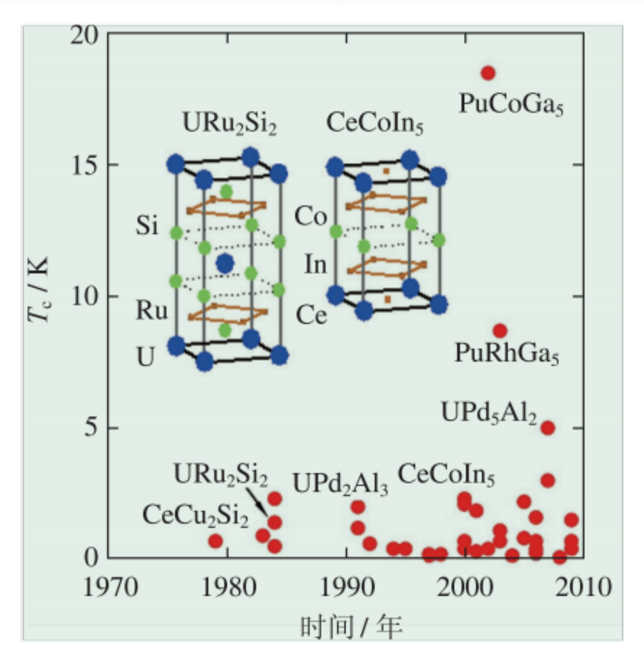

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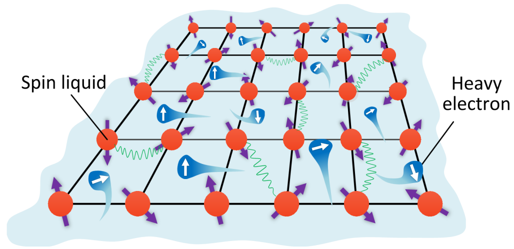

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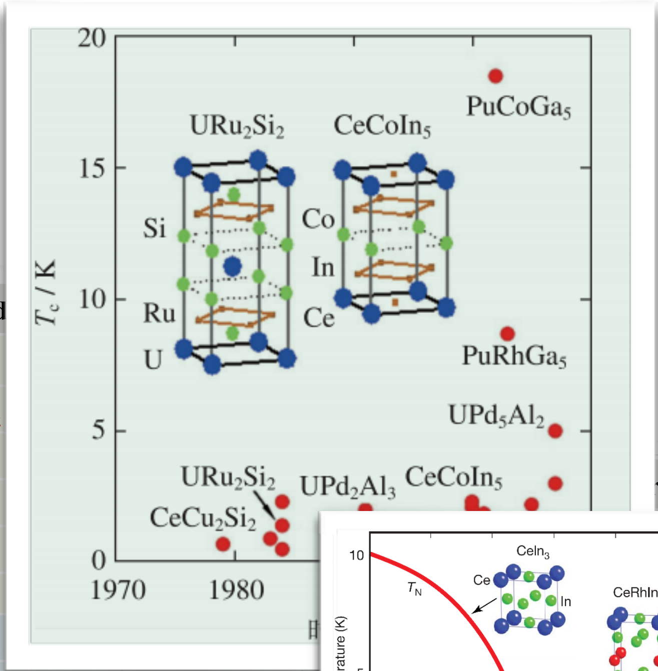

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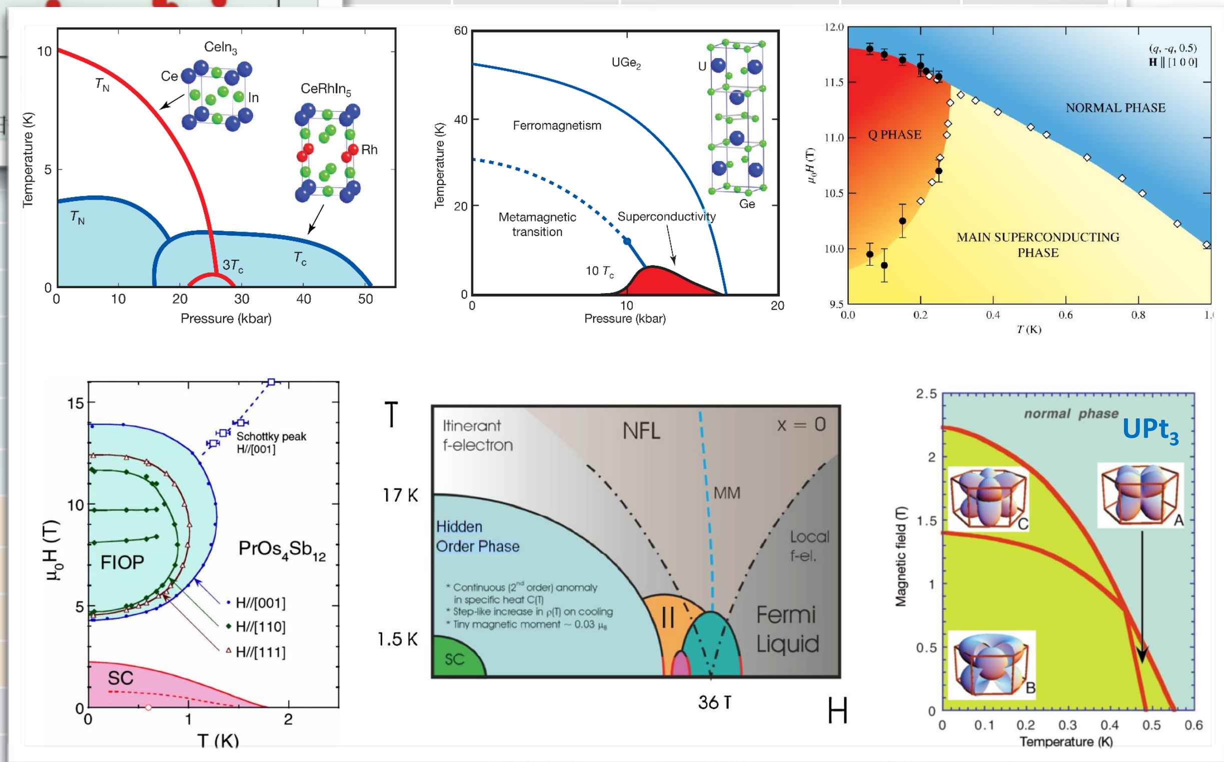

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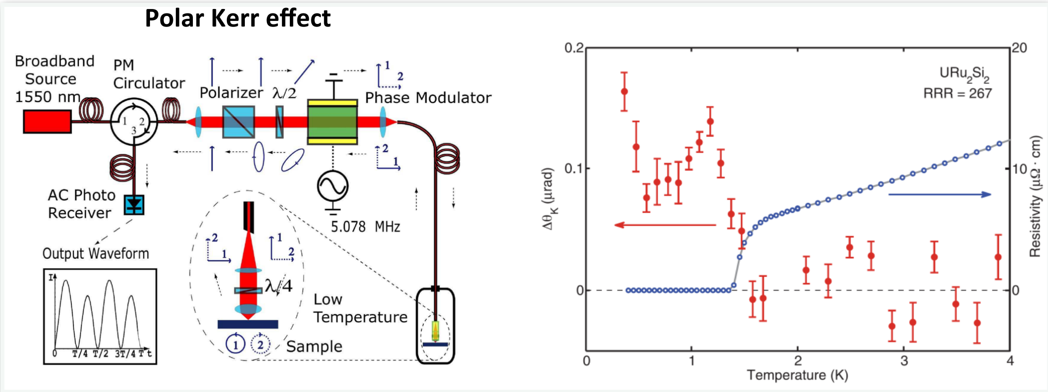

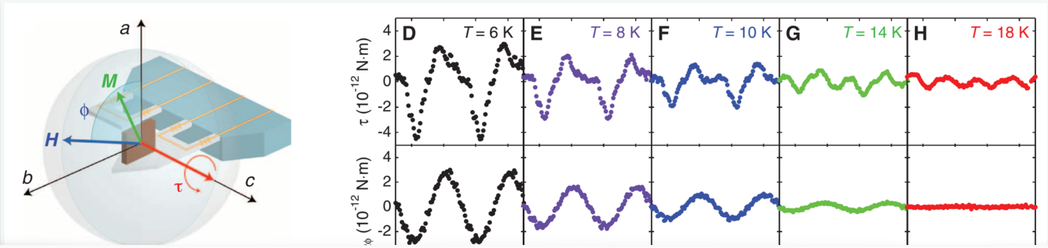

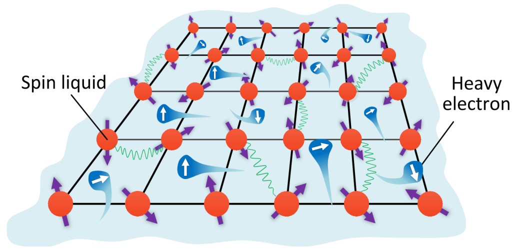

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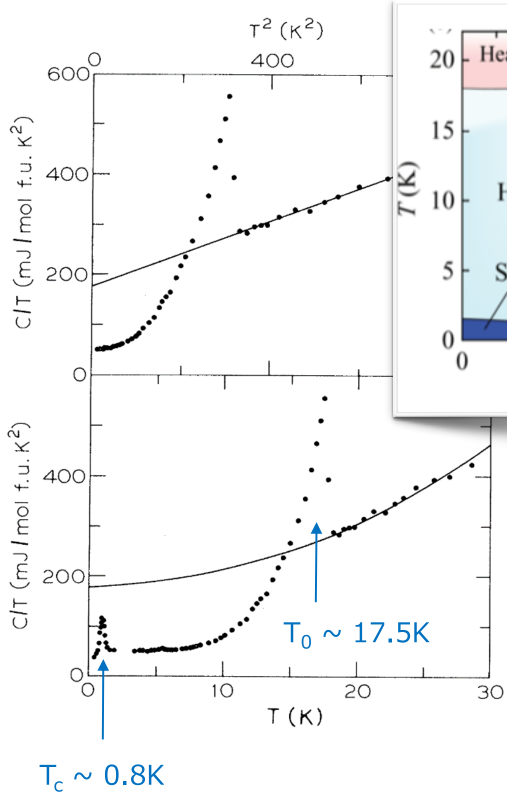
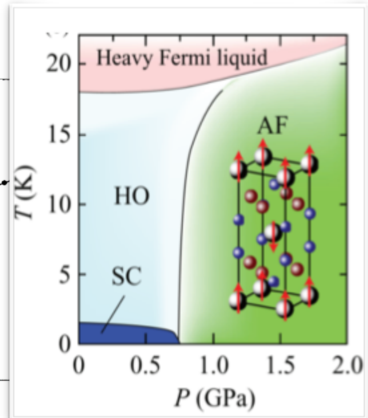
Theories (recent contribution)

Barzykin and Gorkov (1995)	three-spin correlations [45]
Kasuya (1997)	uranium dimerisation [46]
Ikeda and Ohashi (1998)	*d*-spin density wave [47]
Okuno and Miyake (1998)	CEF and quantum fluctuations [48]
Chandra et al. (2002)	orbital currents [49]
Viroszek et al. (2002)	unconv. spin density wave [50]
Mineev and Zhitomirsky (2005)	staggered spin density wave [51]
Varma and Zhu (2006)	helicity (Pomeranchuk) order [52]
Elgazzar et al. (2009)	dynamical symmetry breaking [53]
Kotetes et al. (2010)	chiral *d*-density wave [54]
Dubi and Balatsky (2011)	hybridization wave [55]
Pepin et al. (2011)	modulated spin liquid [56]
Fujimoto (2011)	spin nematic order [57]
Riseborough et al. (2012)	unconv. spin-orbital density wave [58]
Das (2012)	spin-orbital density wave [59]
Chandra et al. (2013)	hastatic order [60]
Hsu and Chakravarty (2013)	singlet-triplet *d*-density wave [61]

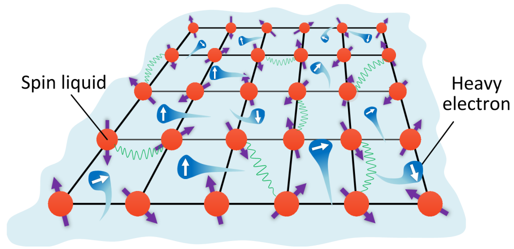
Proposals of multipole magnetic ordering for HO

Nieuwenhuys (1987)	dipole (2^1) order [62]
Santini and Amoretti (1994)	quadrupolar (2^2) order [63]
Kiss and Fazekas (2005)	octupolar (2^3) order [64]
Hanzawa and Watanabe (2005)	octupolar order [65]
Hanzawa (2007)	incommensurate octupole [66]
Haule and Kotliar (2009)	hexadecapolar (2^4) order [67]
Cricchio et al. (2009)	dotriacontapolar (2^5) order [68]
Harima et al. (2010)	antiferro quadrupolar order [69]
Thalmeier and Takimoto (2011)	$E(1, 1)$-type quadrupole [70]
Kusunose and Harima (2011)	antiferro hexadecapole [71]
Ikeda et al. (2012)	E^--type dotriacontapole [72]
Rau and Kee (2012)	E-type dotriacontapole [73]
Ressouche et al. (2012)	dotriacontapolar order [16]





Kondo & Kondo Lattice



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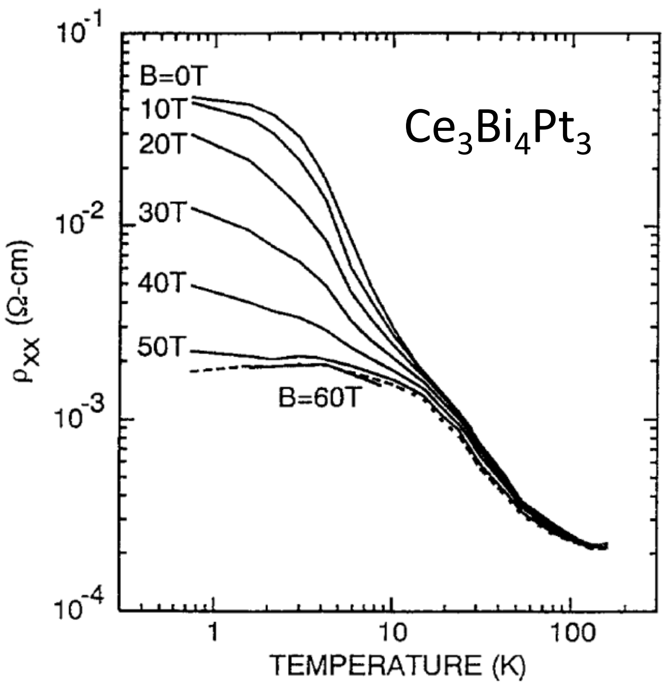

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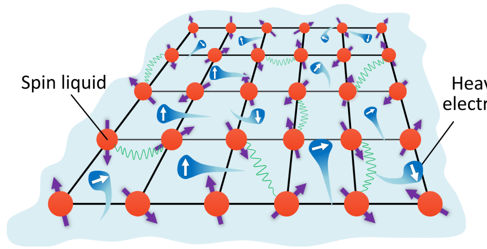

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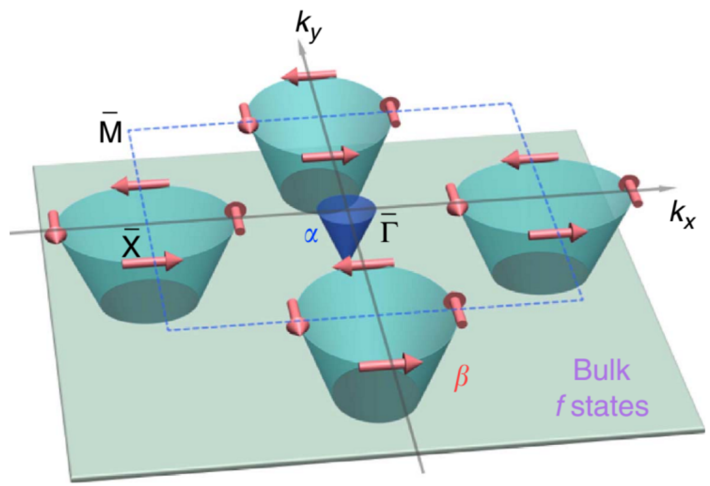

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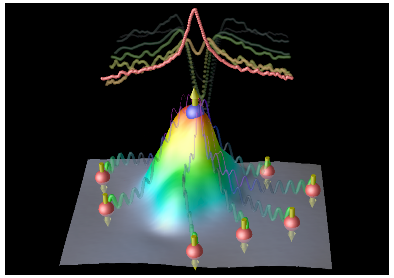

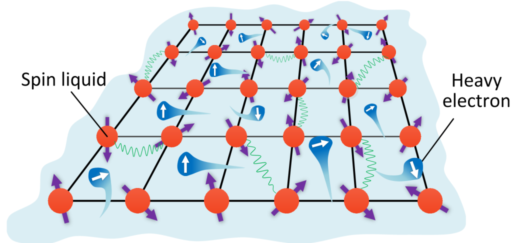

Historic development of Kondo

- ✓ (19th Century) Electron behavior @ T➔0
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity EXP➔THEO
- ✓ 1934-1964 Kondo effect EXP➔THEO
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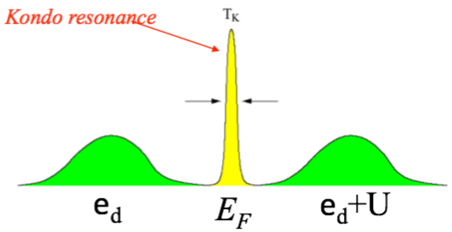

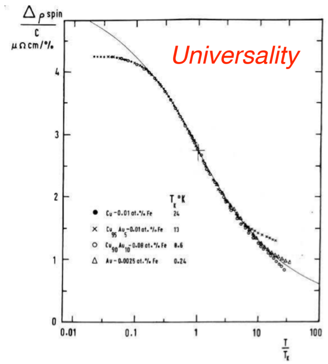

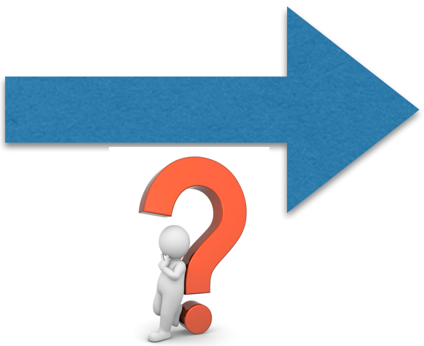

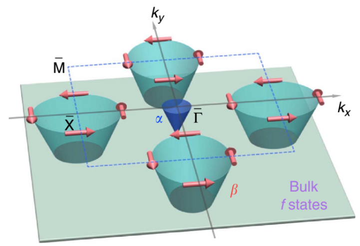

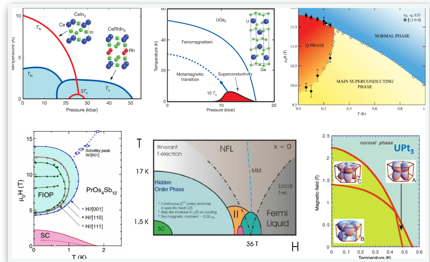

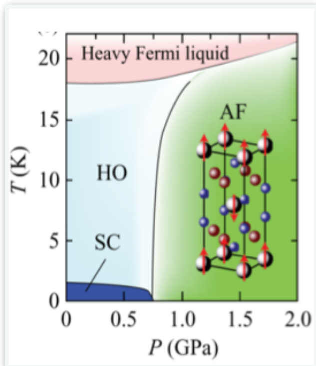

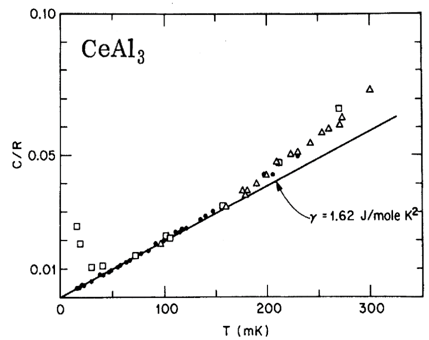

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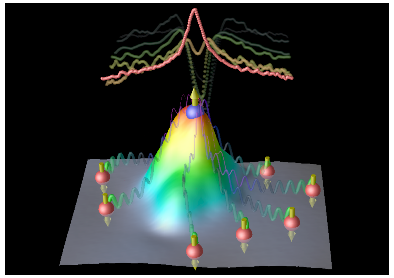

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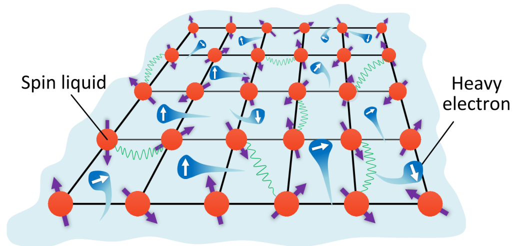
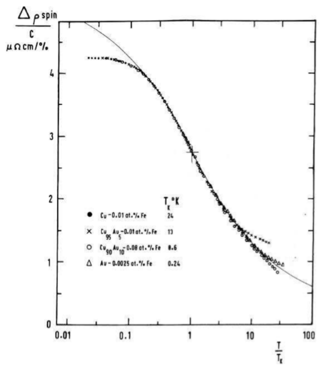
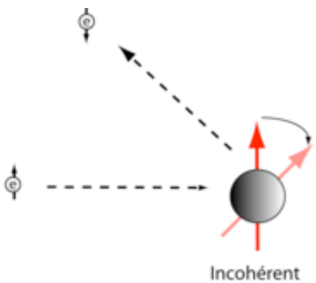


Historic development of heavy fermions

- ✓ $CeAl_3$ γ**=1.62 J/mol K²** @1975
- ✓ $CeCu_2Si_2$ **T_c=0.5 K** @1979
- ✓ URu_2Si_2 **Hidden order** @1985
- ✓ $Ce_3Bi_4Pt_3$ **Kondo insulator** @1992
- ✓ SmB_6 **Topological KI** @2008

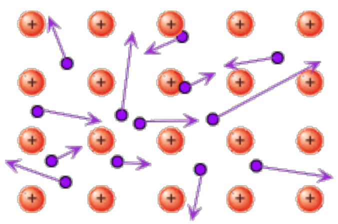
Incoherent Kondo Scattering above T_K

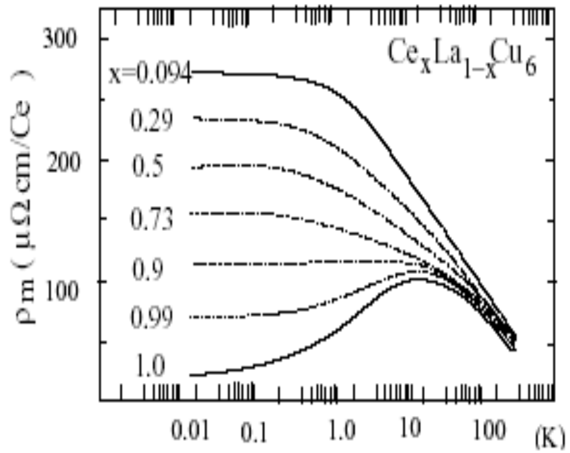




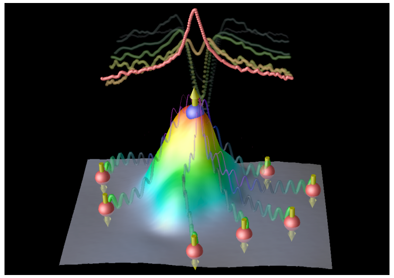
Ce: Ground state doublet

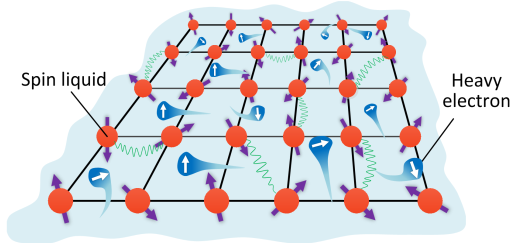
Incoherent Kondo Scattering above T*





Kondo & Kondo Lattice





Historic development of Kondo

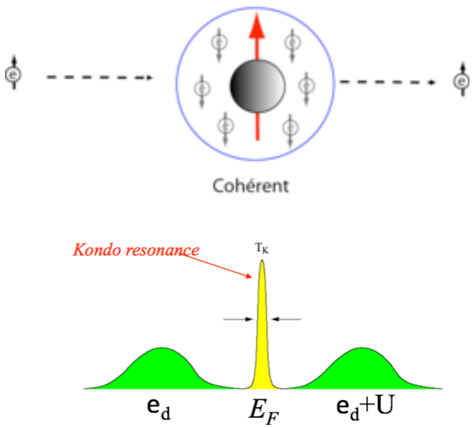
- ✓ (19th Century) Electron behavior @ T➔0
- ✓ Liquification & Resistance measurement
- ✓ 1911-1957 Superconductivity EXP➔THEO
- ✓ 1934-1964 Kondo effect EXP➔THEO
- ✓ Many approaches: EOM, NRG, QMC …

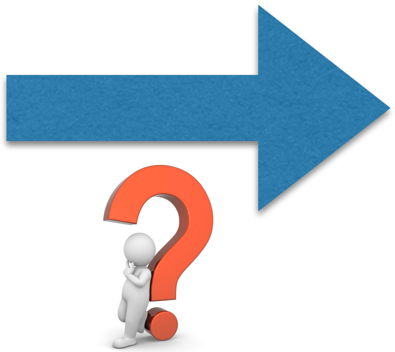
Historic development of heavy fermions

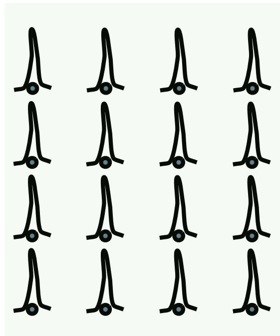
- ✓ $CeAl_3$ **γ=1.62 J/mol K²** @1975
- ✓ $CeCu_2Si_2$ **T_c=0.5 K** @1979
- ✓ URu_2Si_2 **Hidden order** @1985
- ✓ $Ce_3Bi_4Pt_3$ **Kondo insulator** @1992
- ✓ SmB_6 **Topological KI** @2008

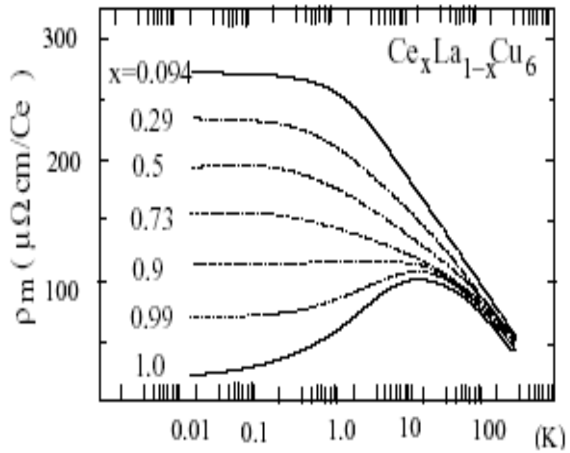
Coherent Kondo Screening below T_K

Coherent behavior below T*



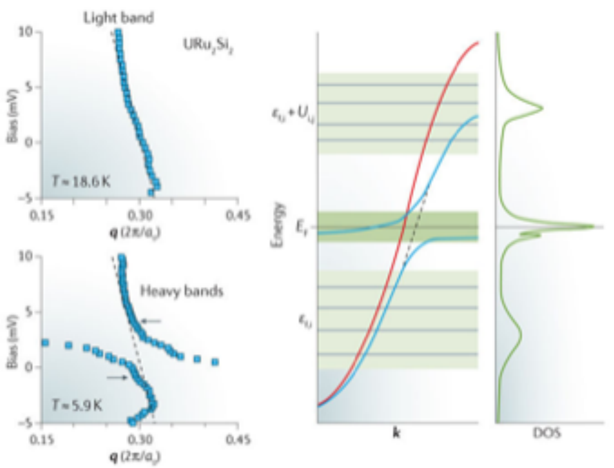




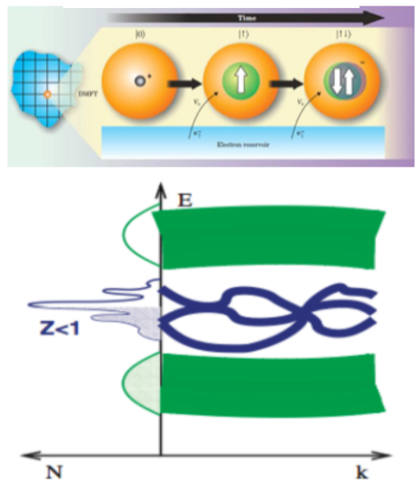


二、重费米子物理的概念问题

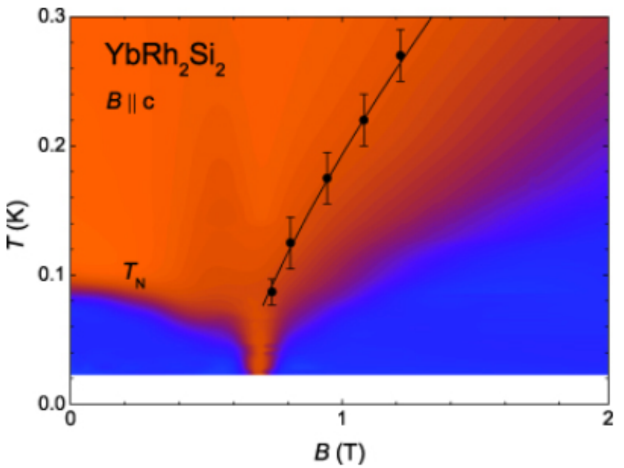
Hybridization



Coherence

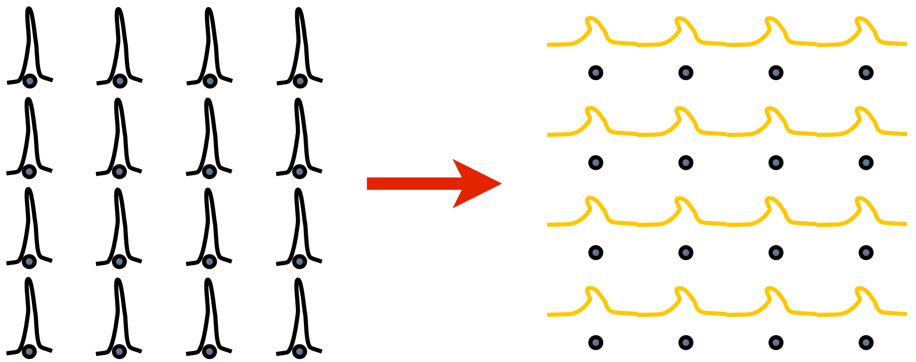


Quantum criticality



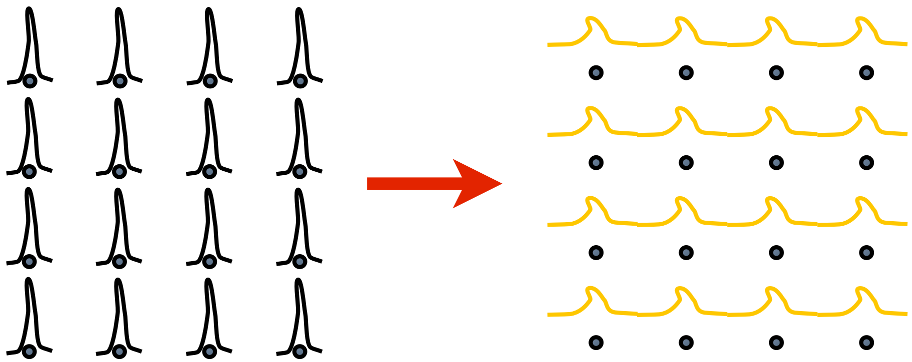
Mean-field theory & DMFT

- **Mean-field theory & Hybridization**
  - ✓ Lattice coherence @ T<T*

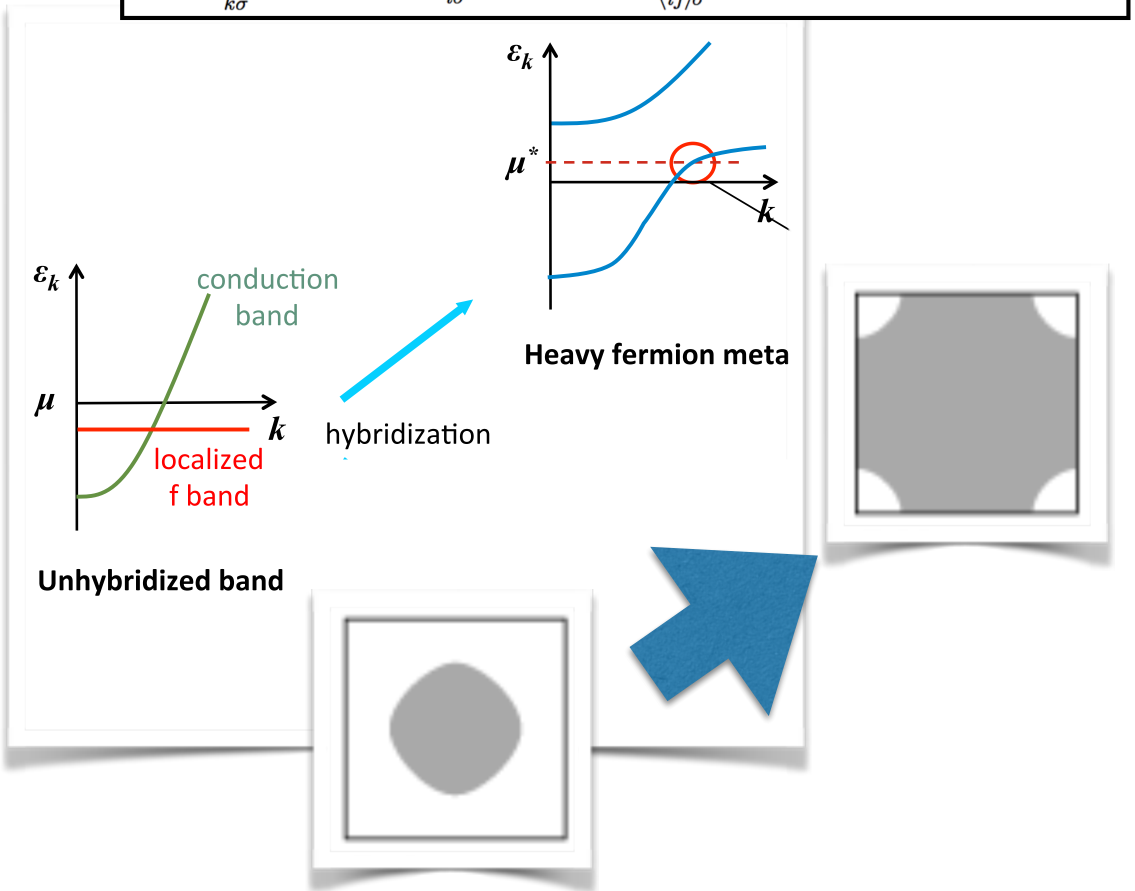


Mean-field theory & DMFT

- **Mean-field theory & Hybridization**
  - ✓ Lattice coherence @ T<T*
  - ✓ Hybridization & FS change



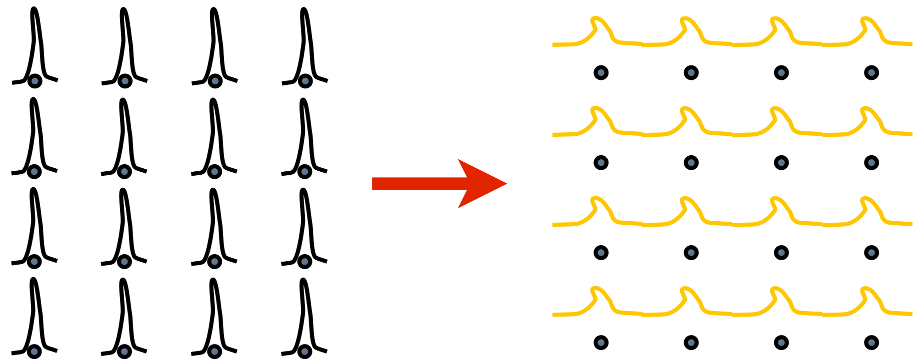
$$H=\sum_{\vec{k}\sigma}\epsilon_{c\vec{k}}c^{\dagger}_{\vec{k}\sigma}c_{\vec{k}\sigma}+\lambda\sum_{i\sigma}f^{\dagger}_{i\sigma}f_{i\sigma}+\frac{J_H}{2}\chi\sum_{\langle ij\rangle\sigma}(f^{\dagger}_{i\sigma}f_{j\sigma}+f^{\dagger}_{j\sigma}f_{i\sigma})-N\lambda+NJ_H\chi^2$$



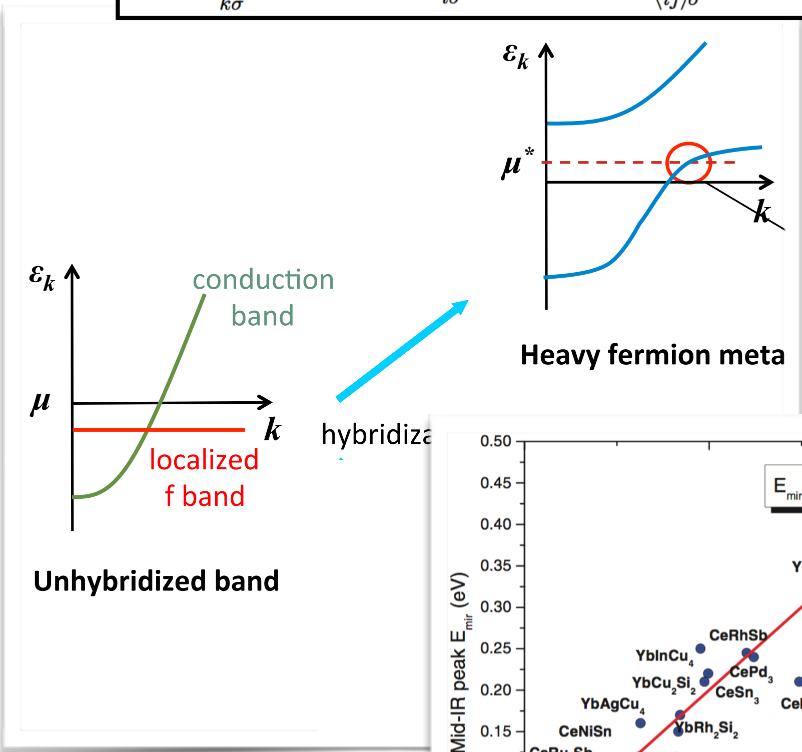
Mean-field theory & DMFT

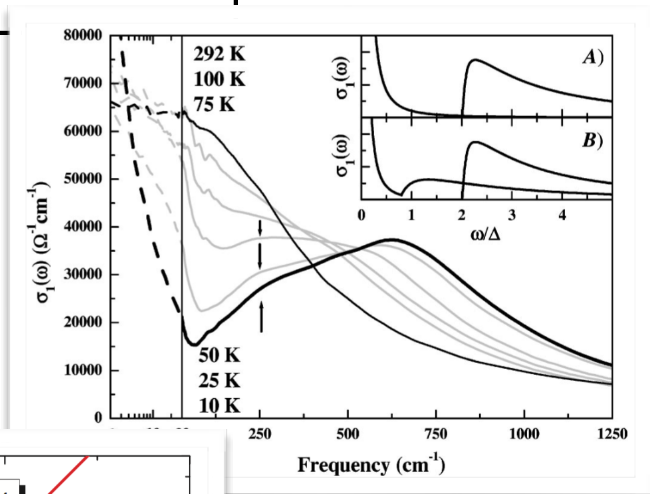
Mean-field theory & Hybridization

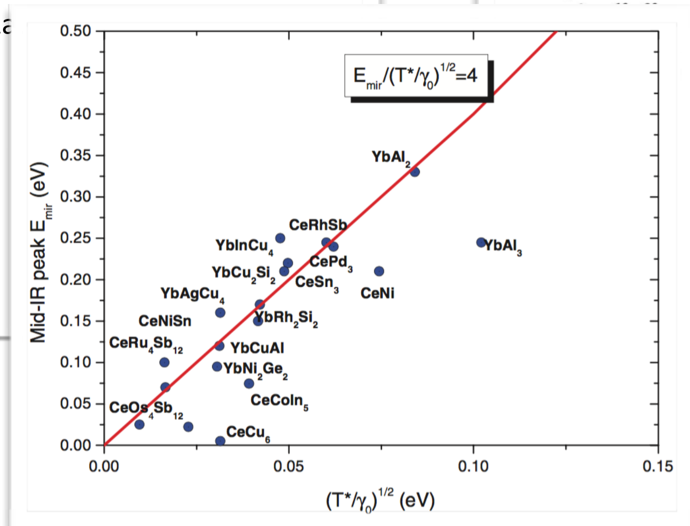
- ✓ Lattice coherence @ T<T*
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap



$$H = \sum_{\vec{k}\sigma} \epsilon_{c\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + \lambda \sum_{i\sigma} f^\dagger_{i\sigma} f_{i\sigma} + \frac{J_H}{2} \chi \sum_{\langle ij \rangle \sigma} (f^\dagger_{i\sigma} f_{j\sigma} + f^\dagger_{j\sigma} f_{i\sigma}) - N\lambda + N J_H \chi^2$$

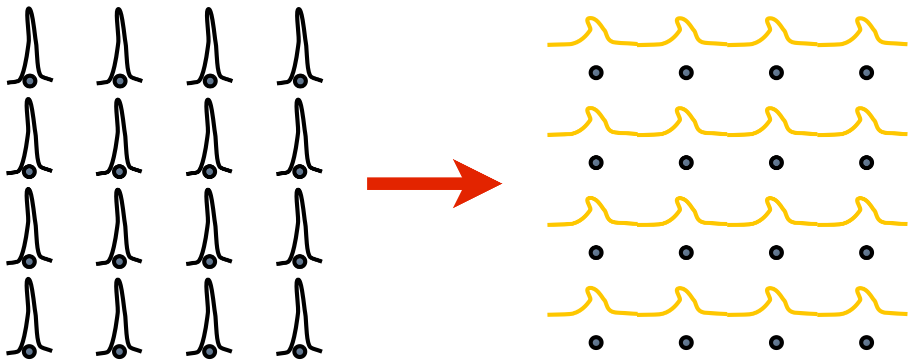




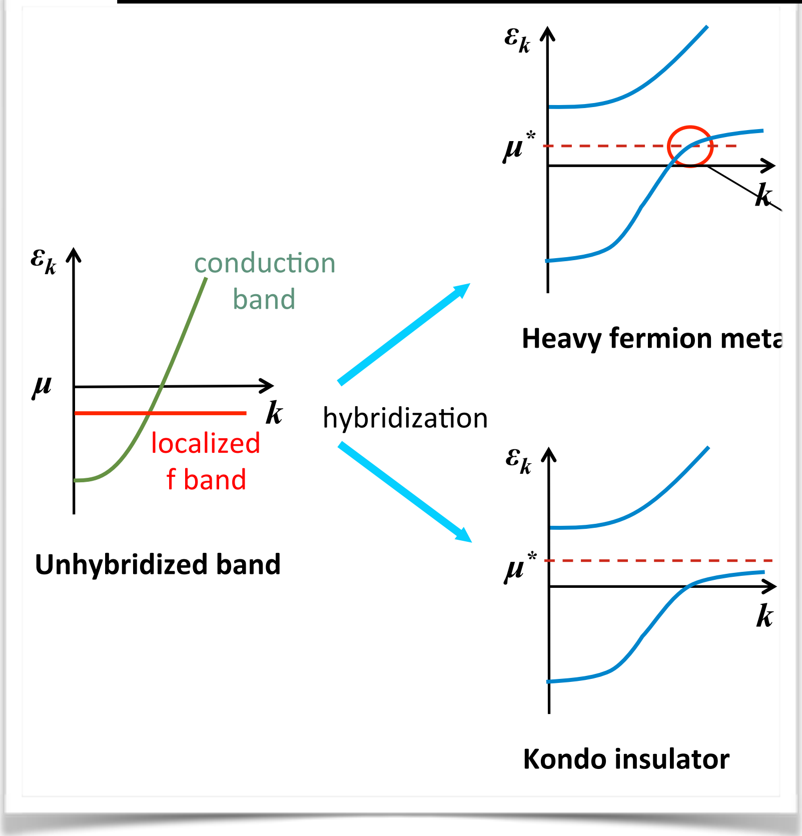


Mean-field theory & DMFT

- **Mean-field theory & Hybridization**
  - ✓ Lattice coherence @ T<T*
  - ✓ Hybridization & FS change
  - ✓ Optical & Hybridization gap
  - ✓ Kondo insulator



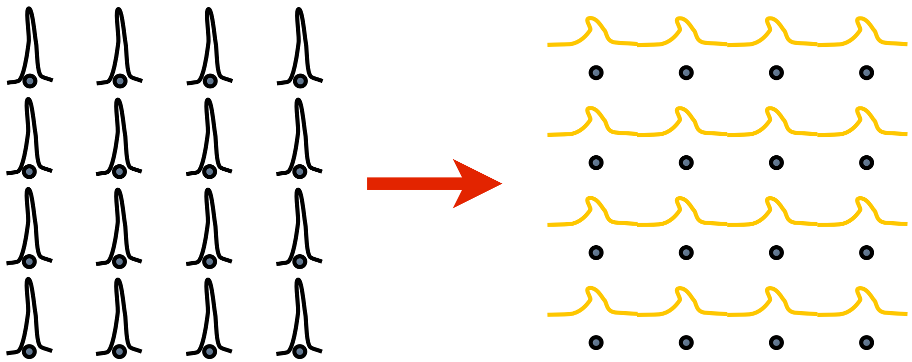
$$H = \sum_{\vec{k}\sigma} \epsilon_{c\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + \lambda \sum_{i\sigma} f^\dagger_{i\sigma} f_{i\sigma} + \frac{J_H}{2}\chi \sum_{\langle ij\rangle\sigma} (f^\dagger_{i\sigma} f_{j\sigma} + f^\dagger_{j\sigma} f_{i\sigma}) - N\lambda + N J_H \chi^2$$



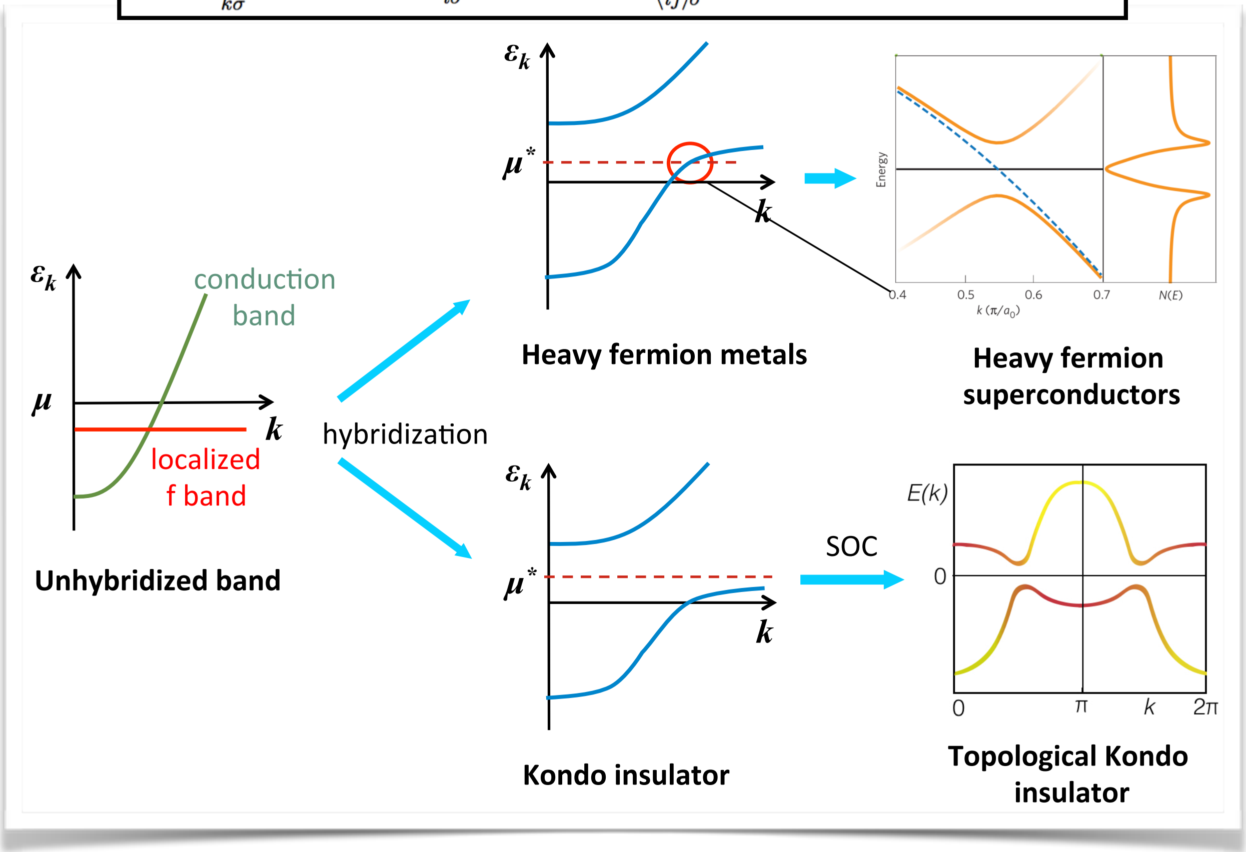
Mean-field theory & DMFT

Mean-field theory & Hybridization

- ✓ Lattice coherence @ T<T*
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap
- ✓ Ground state: KI & SC & TKI & SL …



$$H = \sum_{\vec{k}\sigma} \epsilon_{c\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} + \lambda \sum_{i\sigma} f^\dagger_{i\sigma} f_{i\sigma} + \frac{J_H}{2}\chi \sum_{\langle ij\rangle\sigma}(f^\dagger_{i\sigma}f_{j\sigma}+f^\dagger_{j\sigma}f_{i\sigma}) - N\lambda + NJ_H\chi^2$$



Static Hybridization, only below T*

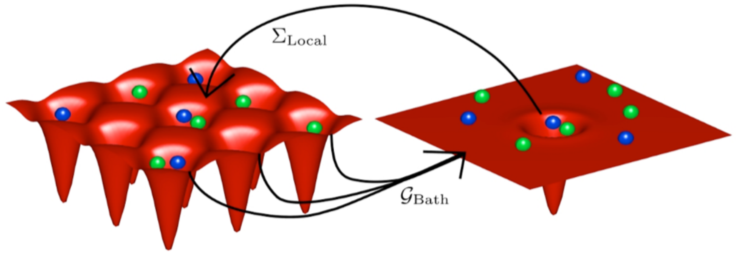
Mean-field theory & DMFT

Mean-field theory & Hybridization

- ✓ Lattice coherence @ T<T*
- ✓ Hybridization & FS change
- ✓ Optical & Hybridization gap
- ✓ Ground state: KI & SC & TKI & SL …

DMFT & Coherence/Duality

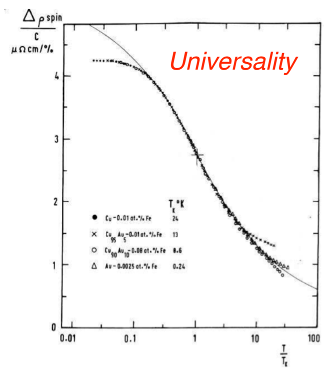
- ✓ Dynamic fluctuations @ T>T*
- ✓ Local/Itinerant crossover @ T<T*
- ✓ Development of coherence @ T<T*
- ✓ Short range correlation (*Extended*)

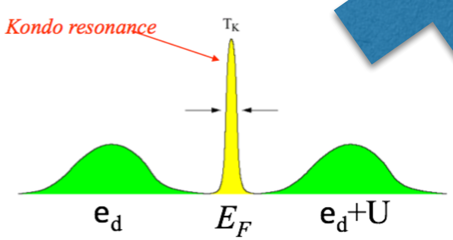


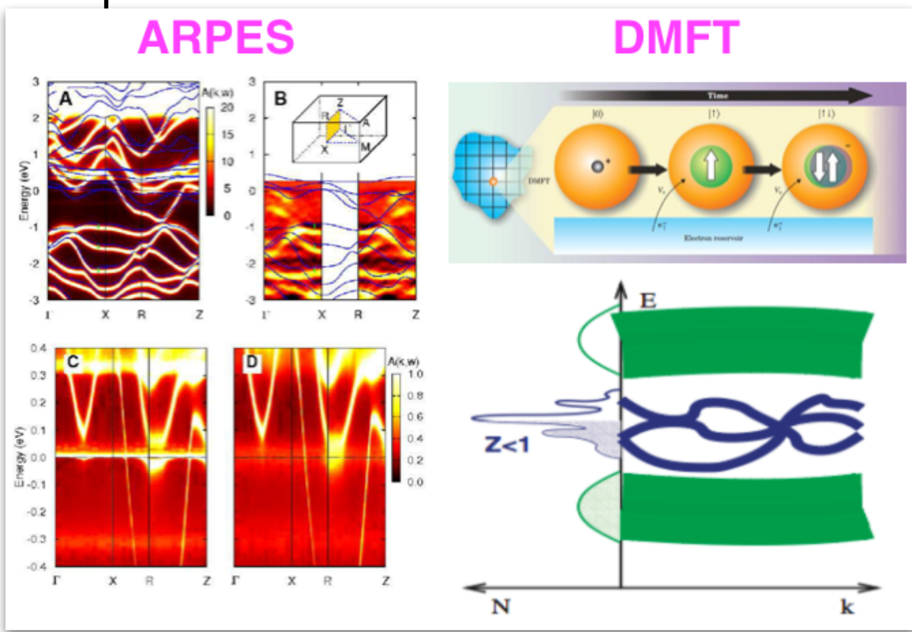
Dynamical mean-field theory

No long range magnetic correlations

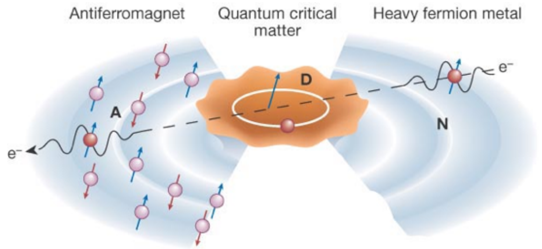
Hard to describe quantum criticality





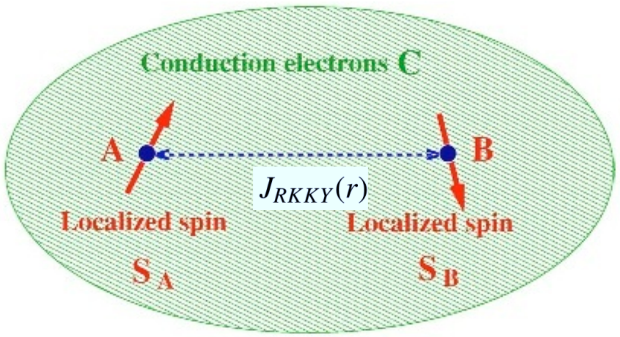


Quantum criticality & Non-Fermi liquid



- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism

RKKY interaction



$$H_{RKKY} = \frac{1}{2}\sum_{\mathbf{x},\mathbf{x}'} \overbrace{-J^2\chi(\mathbf{x}-\mathbf{x}')}^{J_{RKKY}(\mathbf{x}-\mathbf{x}')}\vec{S}(\mathbf{x})\cdot\vec{S}(\mathbf{x}'),$$

$$J_{RKKY}(r) \sim J^2\rho\frac{\cos 2k_F r}{|r|^3}$$

(S. Doniach)

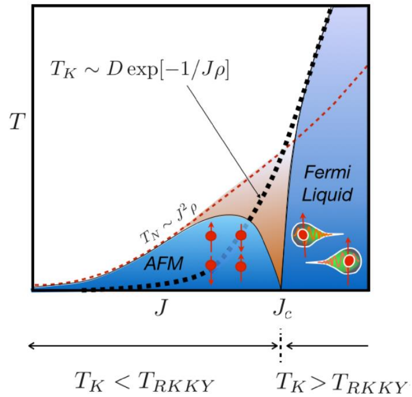
Physica 91B (1977) 231–234 © *North-Holland*

THE KONDO LATTICE AND WEAK ANTIFERROMAGNETISM

S. DONIACH*

Department of Applied Physics, Stanford University, California 94305, USA

By considering a one-dimensional analog of a system of conduction electrons exchange coupled to a localized spin in each cell of a lattice, it is suggested that a second-order transition from an antiferromagnetic to a Kondo spin-compensated ground state will occur as the exchange coupling constant J is increased to a critical value J_c. For systems in which $J \lesssim J_c$, a very weak sublattice magnetization may occur as a result of nearly complete spin-compensation.



RKKY

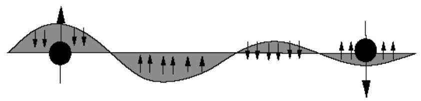
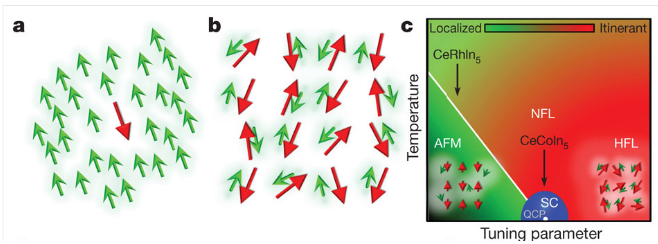
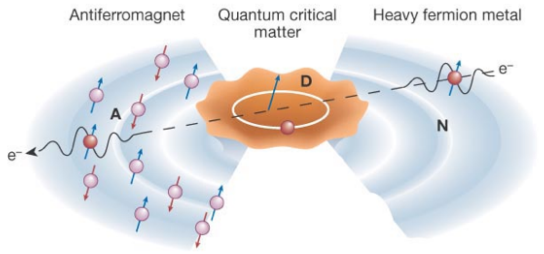


FIGURE 12. Illustrating how the polarization of spin around a magnetic impurity gives rise to Friedel oscillations and induces an RKKY interaction between the spins



$$H_{KL} = J_H\sum_{\langle ij\rangle}\mathbf{S}_i\cdot\mathbf{S}_j + J_K\sum_i \mathbf{s}_i\cdot\mathbf{S}_i + \sum_{k\sigma}\epsilon_{k\sigma}c^\dagger_{k\sigma}c_{k\sigma}$$

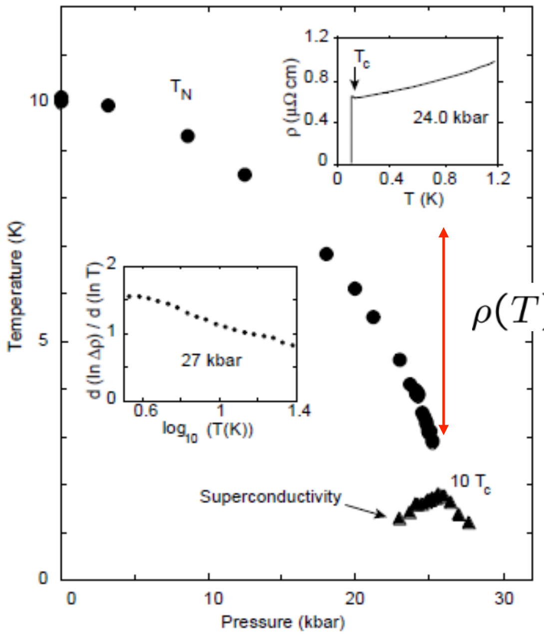
Quantum criticality & Non-Fermi liquid



Quantum criticality @ low T

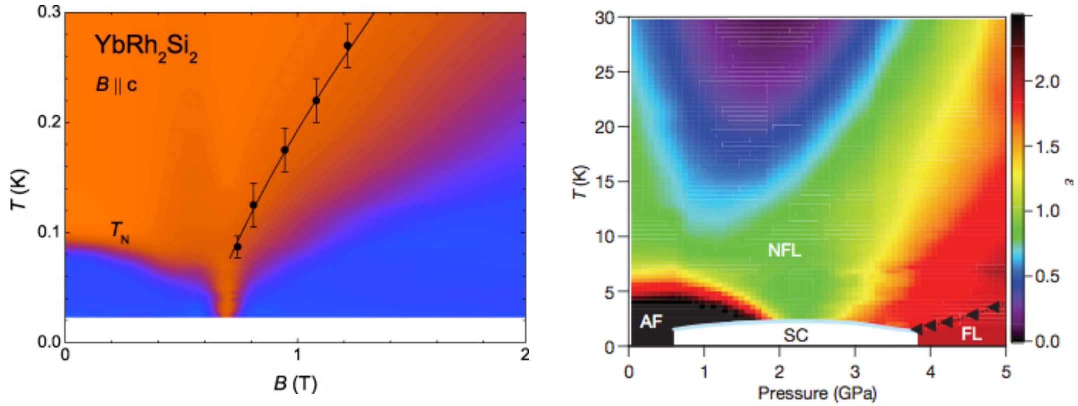
✓ RKKY & Magnetism
✓ SDW scenario (Hertz/Millis/Moriya)

- Power-law scaling in resistivity, specific heat, susceptibility
- Peak at finite wave vector in dynamic susceptibility
- No ω/T scaling …



$\rho(T) \propto T^{3/2}$

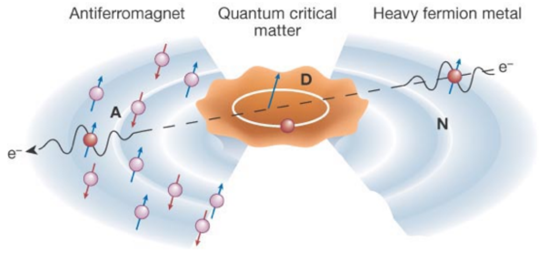
Consistent with SDW quantum critical theory



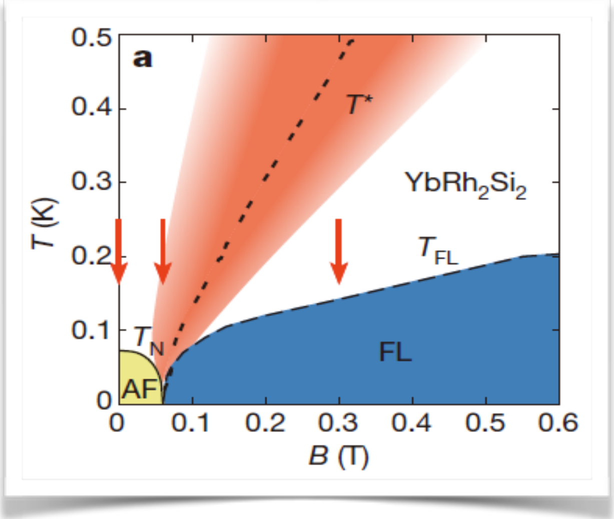
The SDW predictions are violated

Long-wavelength Spin density wave (SDW) fluctuations

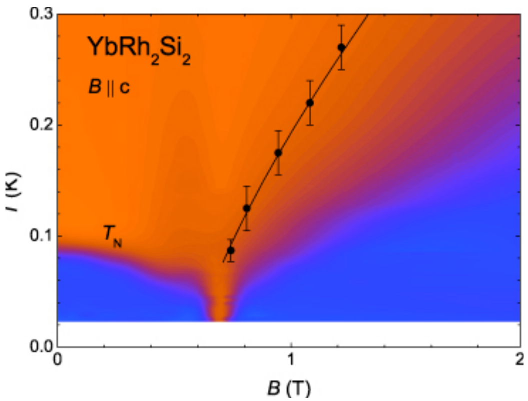
Quantum criticality & Non-Fermi liquid

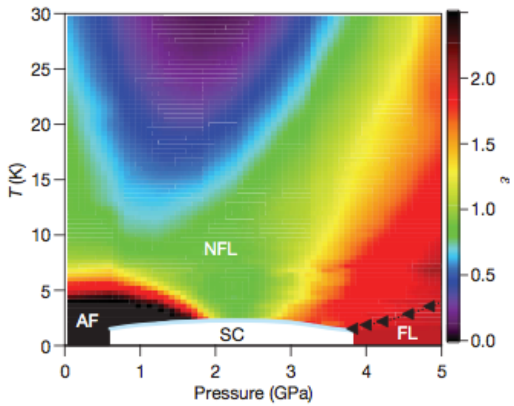


AFM quantum phase transition accompanied with suppression of Kondo screening (Fermi surface change)



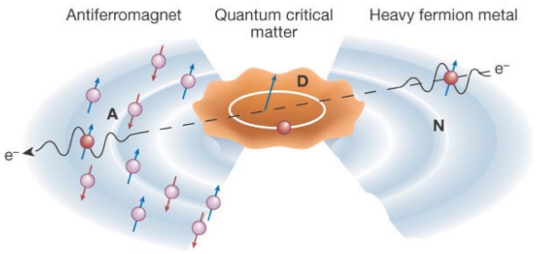
- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
  - ✓ Kondo breakdown @2001





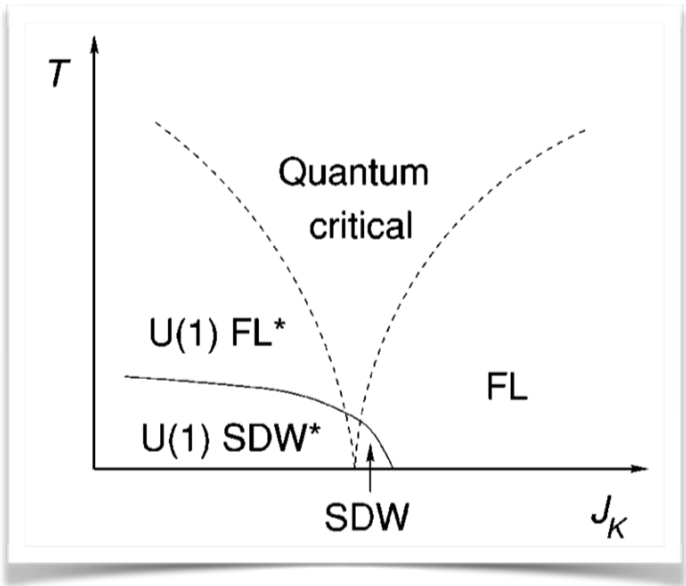
The SDW predictions are violated

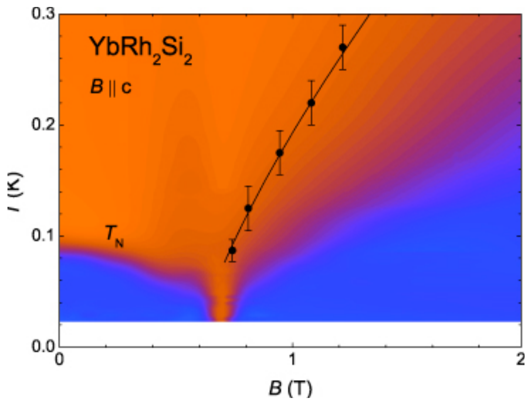
Quantum criticality & Non-Fermi liquid

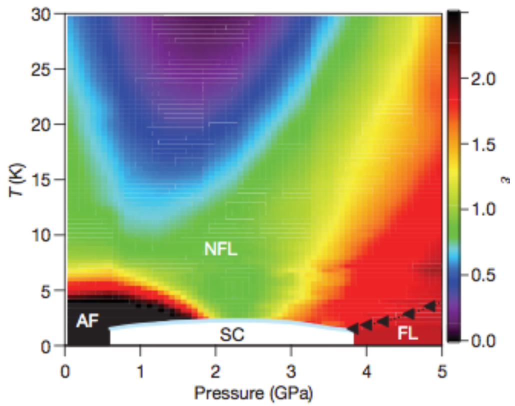


- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
  - ✓ Kondo breakdown @2001
  - ✓ Fractionalization @2004

Fractionalized excitations near QCP
spinon/spinon SDW …

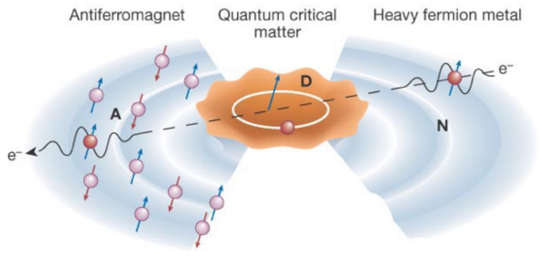




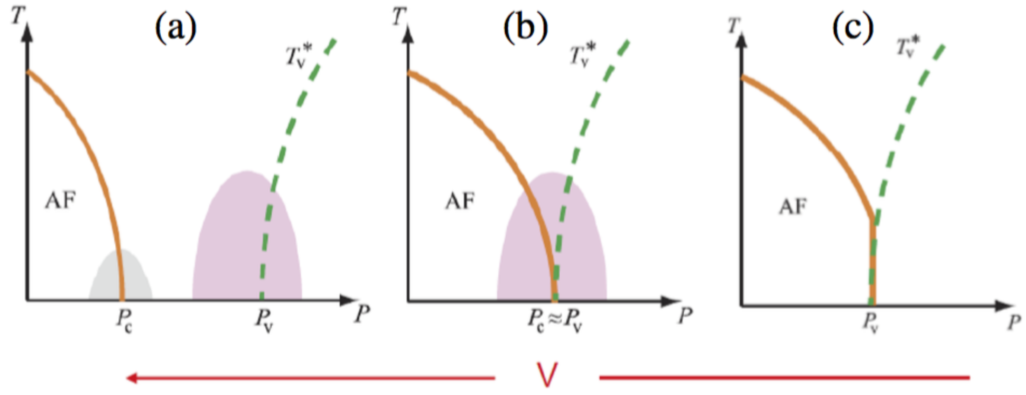


The SDW predictions are violated

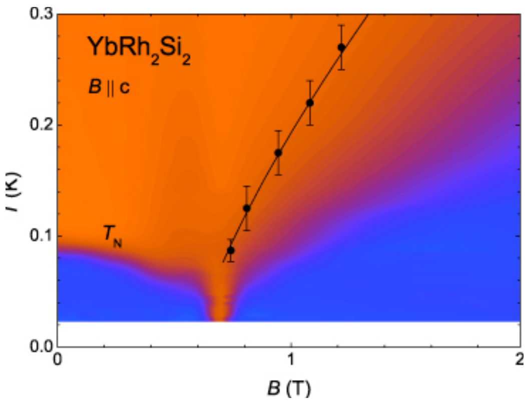
Quantum criticality & Non-Fermi liquid

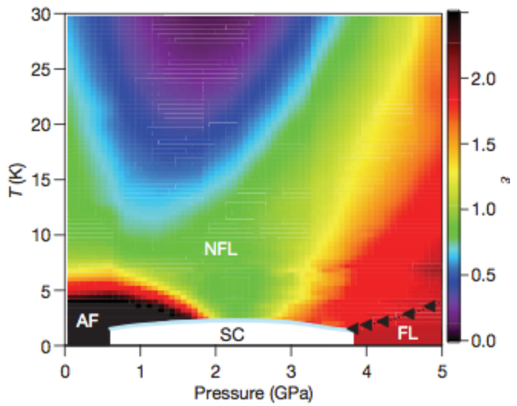


Valence transition & abrupt FS change
with hybridization on both sides



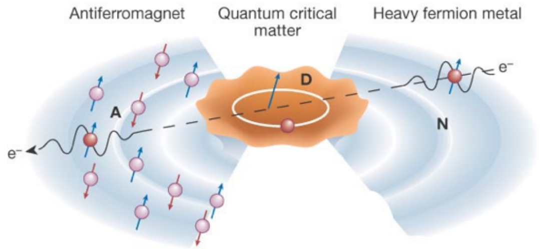
- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
  - ✓ Kondo breakdown @2001
  - ✓ Fractionalization @2004
  - ✓ Critical valence fluctuations @2010



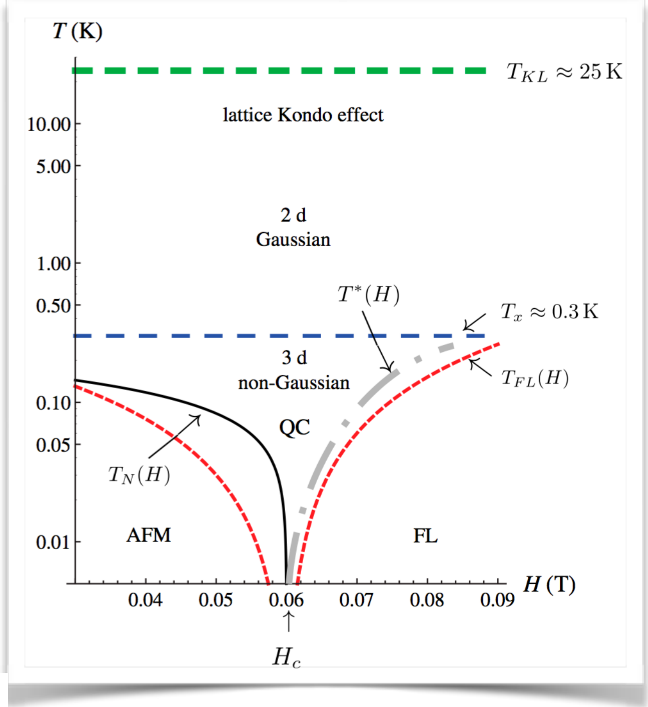


The SDW predictions are violated

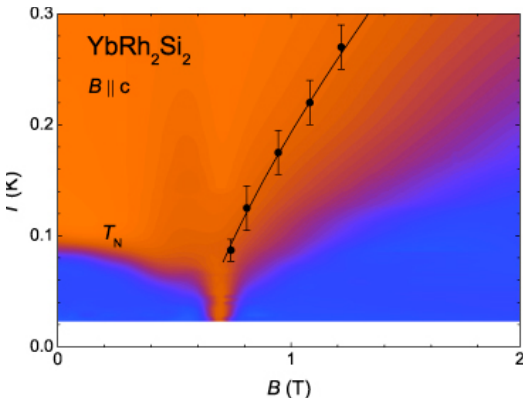
Quantum criticality & Non-Fermi liquid

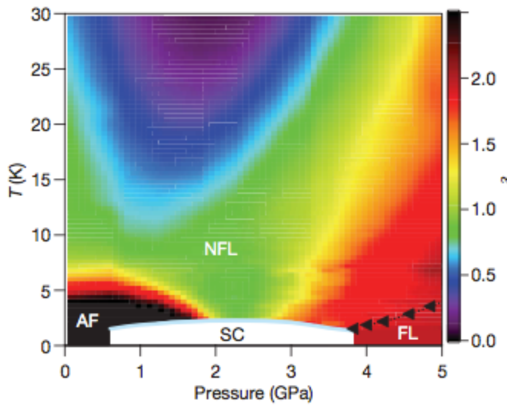


Heavy quasiparticles still exist
but become critical: N(E) ~ |E|$^{\alpha}$



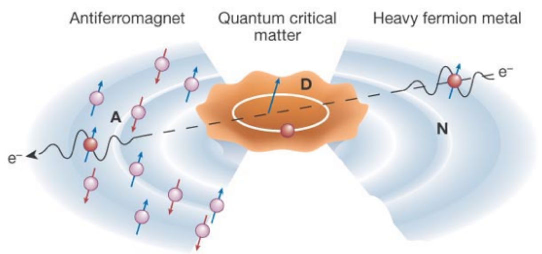
- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
  - ✓ Kondo breakdown @2001
  - ✓ Fractionalization @2004
  - ✓ Critical valence fluctuations @2010
  - ✓ Critical quasiparticle @2011





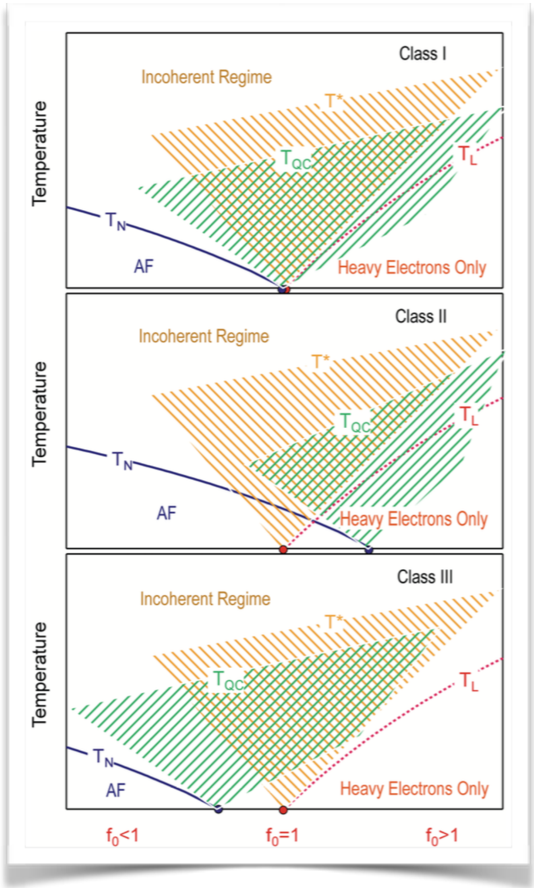
The SDW predictions are violated

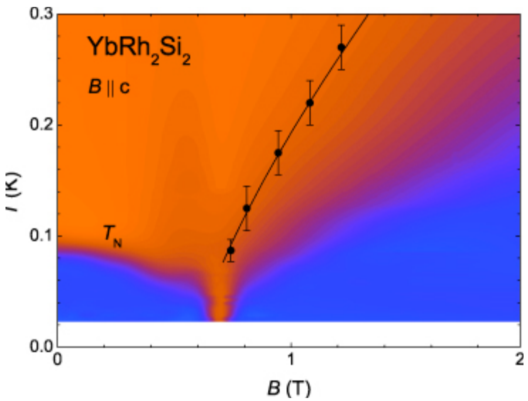
Quantum criticality & Non-Fermi liquid

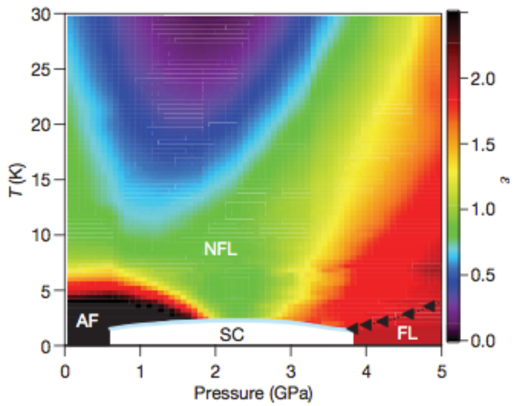


- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
  - ✓ Kondo breakdown @2001
  - ✓ Fractionalization @2004
  - ✓ Critical valence fluctuations @2010
  - ✓ Critical quasiparticle @2011
  - ✓ Magnetic & hybridization fluc. @2017

Interplay of two types of quantum critical fluctuations

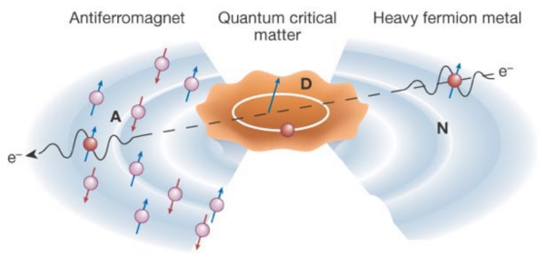






The SDW predictions are violated

Quantum criticality & Non-Fermi liquid



- **Quantum criticality @ low T**
  - ✓ RKKY & Magnetism
  - ✓ SDW scenario (Hertz/Millis/Moriya)
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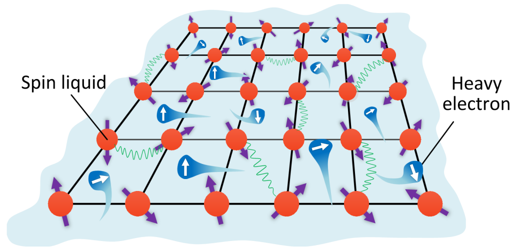

- **Non-Fermi liquid @ high T**
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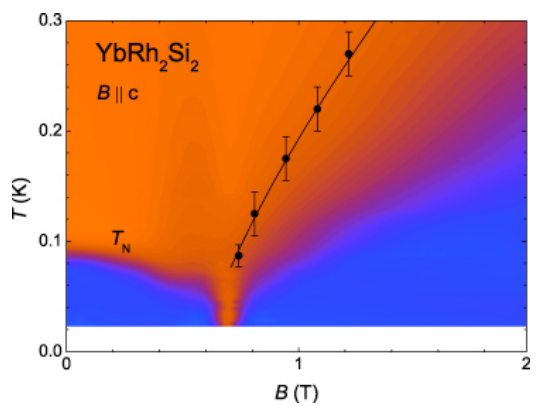

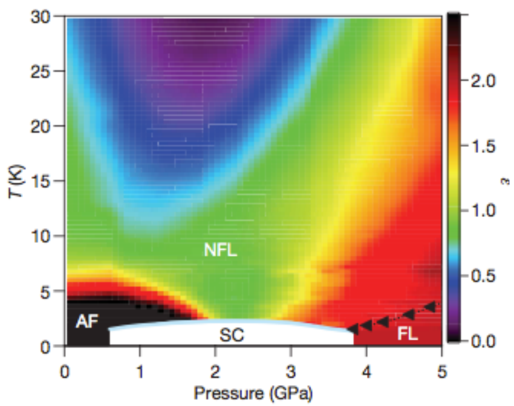

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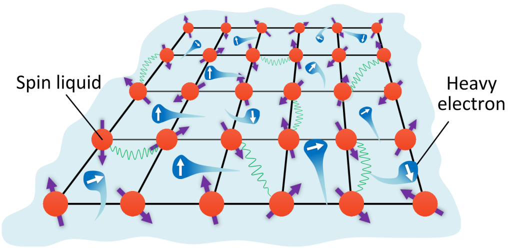


$$T^* = cJ^2\rho$$

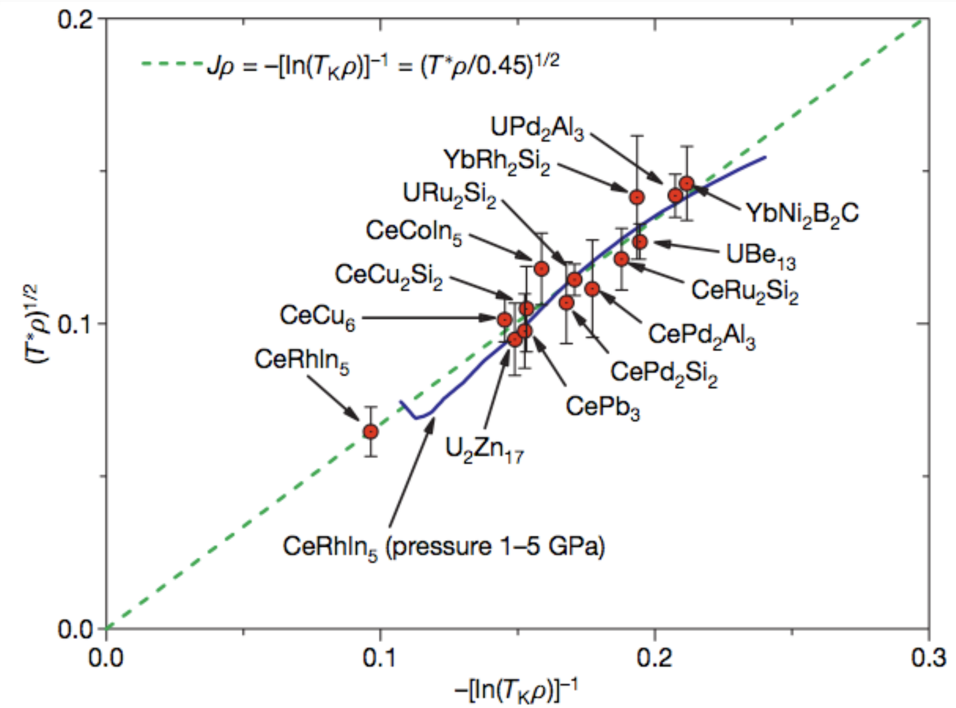


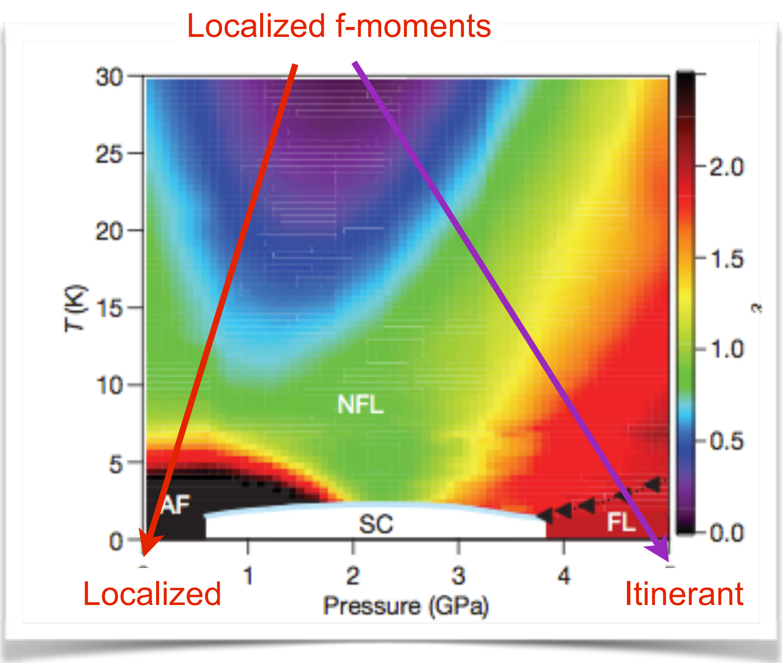
Figure 1 | Confirmation of T^* given by the intersite RKKY interaction for a variety of Kondo lattice materials. The solid line shows T^* (resistivity peak) of $CeRhIn_5$ under pressure from 1 GPa (lower left) to 5 GPa (upper right).

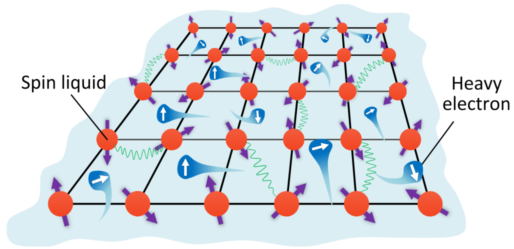
- **Non-Fermi liquid @ high T**
  - ✓ Onset of coherence (What is T*?)

Kondo or RKKY ?

In mean-field theory or DMFT, the onset of coherence is given by the Kondo scale!

Quantum criticality & Non-Fermi liquid





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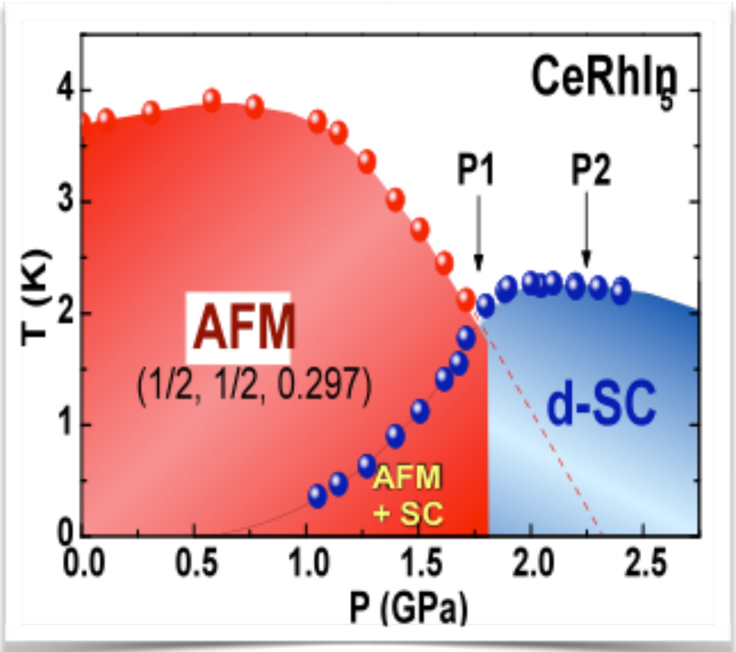
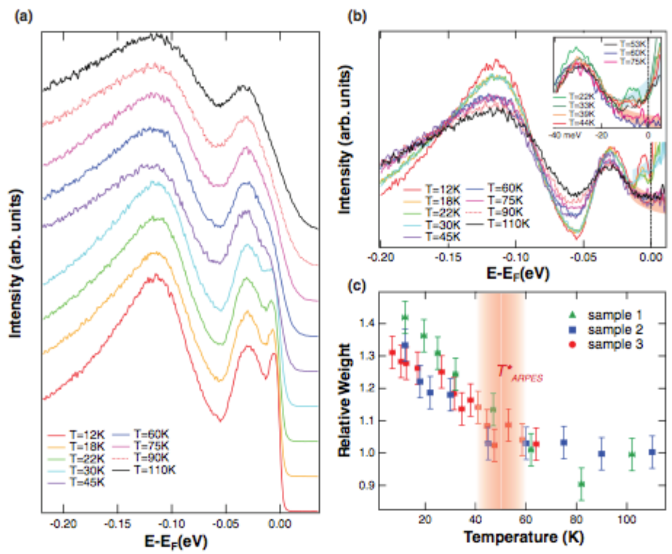
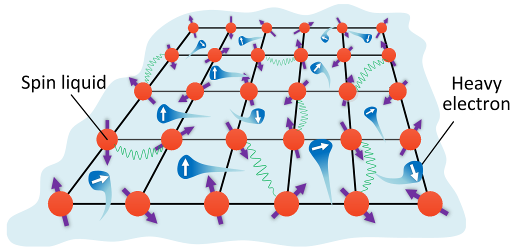


What is different above & below T*?

Local vs collective hybridization?



Quantum criticality & Non-Fermi liquid



Hence Kondo lattice physics is different from the single-ion Kondo physics!

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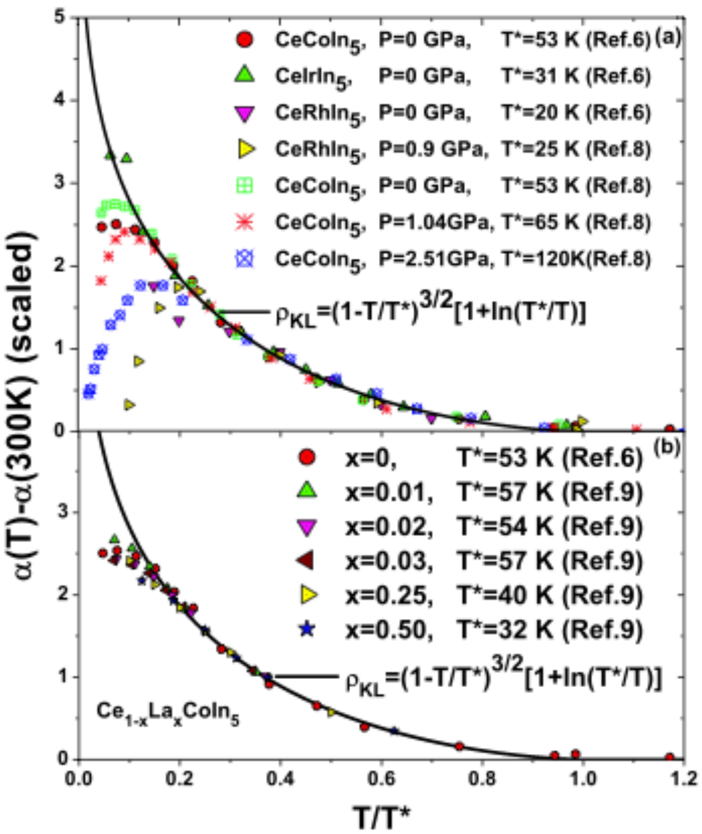
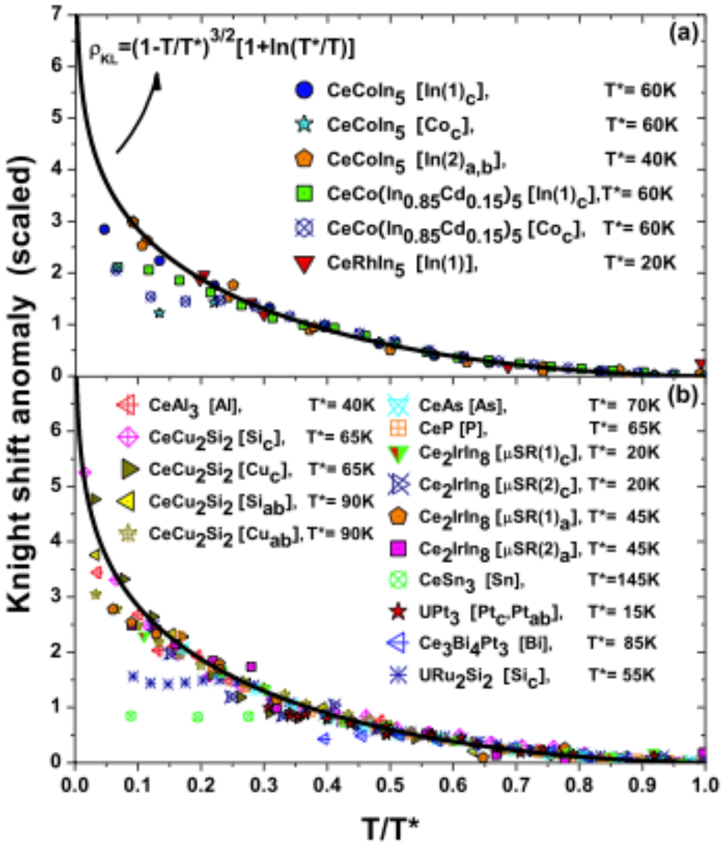
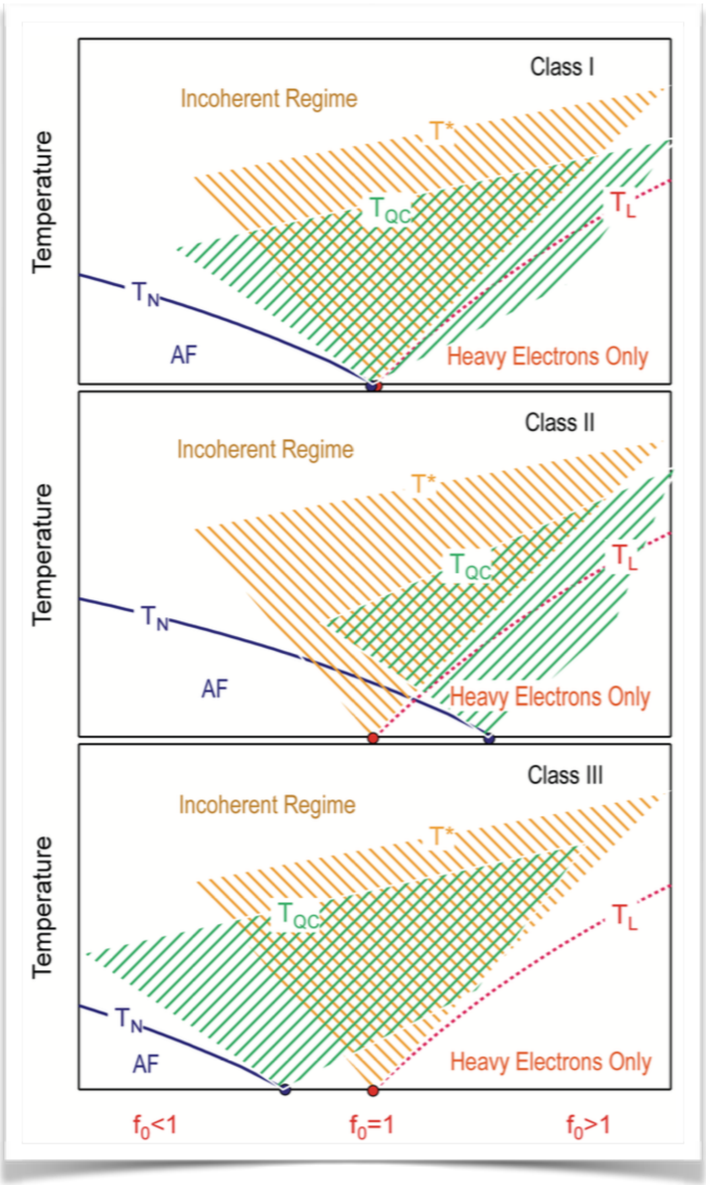
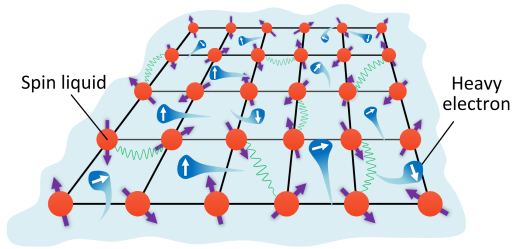
Different from Kondo scaling





Quantum criticality & Non-Fermi liquid





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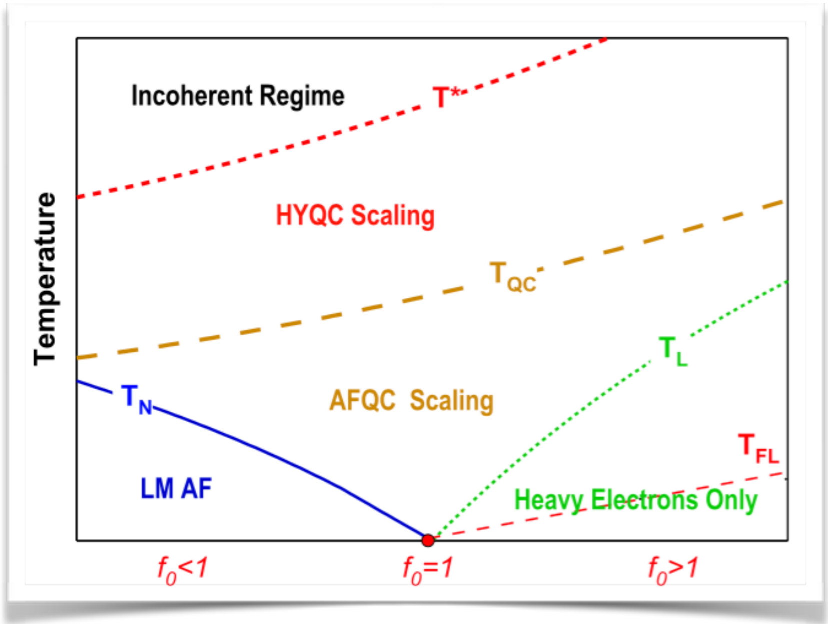
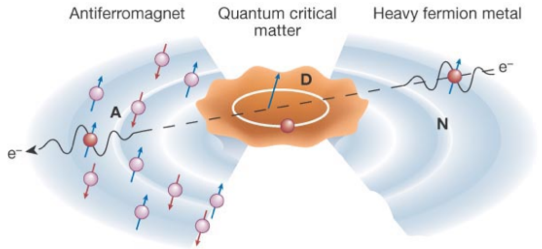


Heavy fermion physics determined by interplay of two types of QC fluctuations?

Quantum criticality & Non-Fermi liquid



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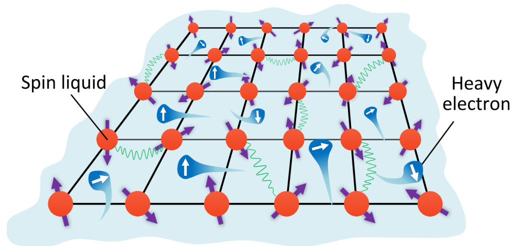

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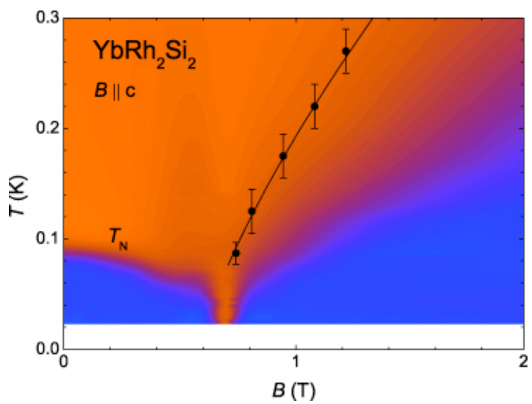
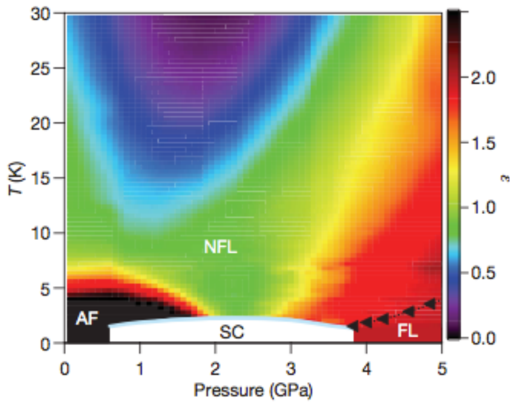
New Quantum Critical Matter

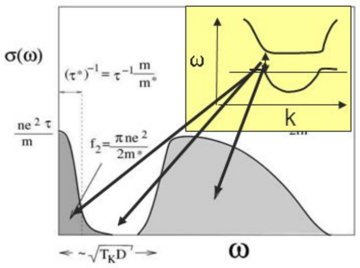
SC, Hidden order, TI …



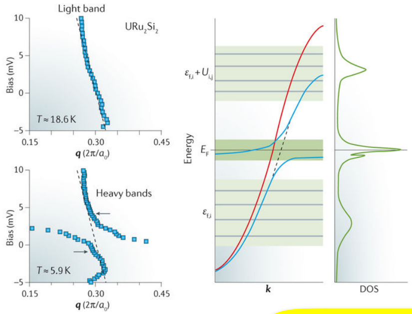


Schematic Understanding of Heavy Fermion Concepts

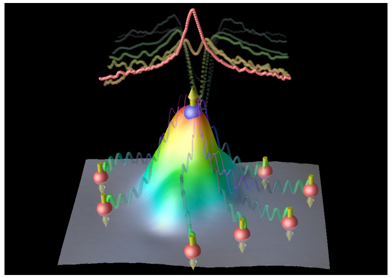
Optical



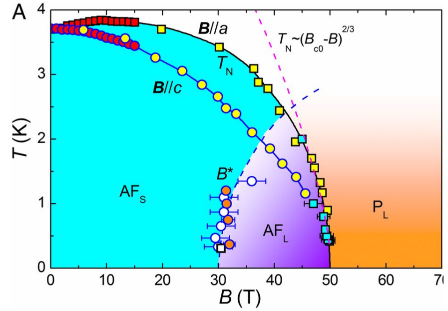
Mean-field

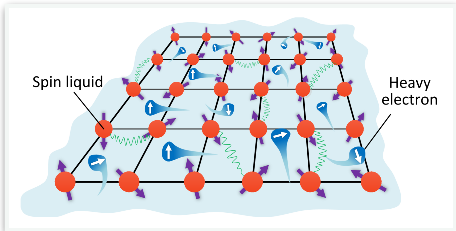


Kondo



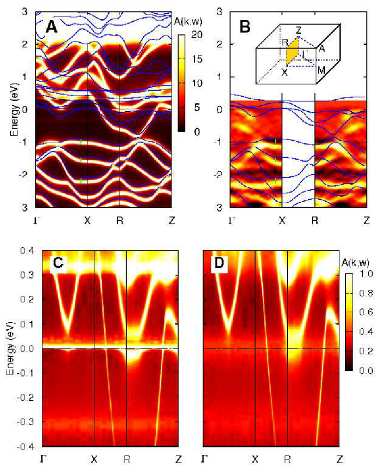
Kondo lattice



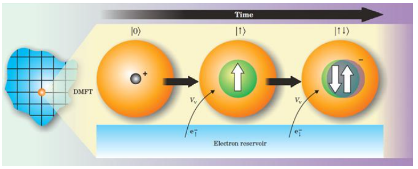


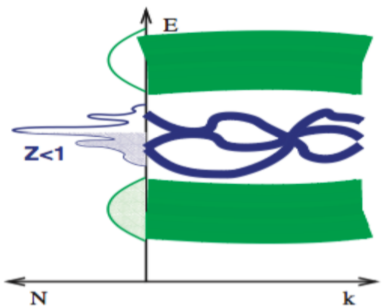
Local/Itinerant crossover

ARPES

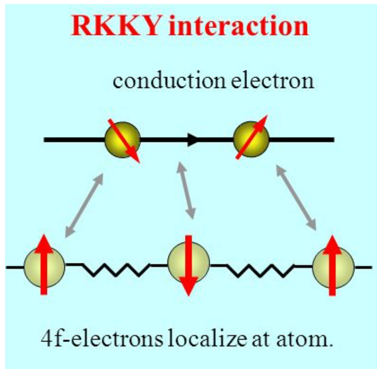


DMFT

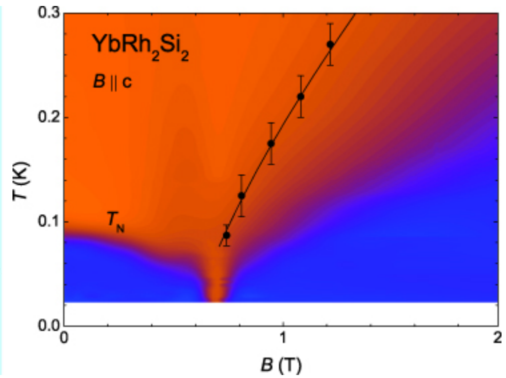




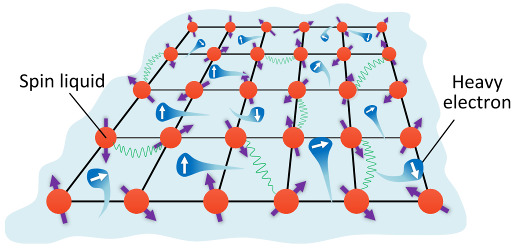
Magnetism



Quantum criticality

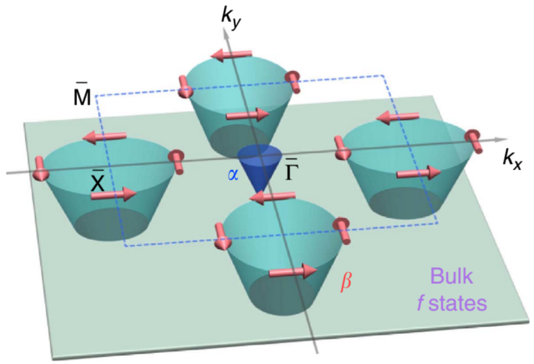


Future directions of heavy fermion research



- **Explore the basic nature of heavy fermion physics**

  We are approaching the final answer of the questions:
  local/itinerant duality & interplay of various QC fluctuations



- **Find new frontiers: "Heavy fermion +"**

  Topological, Weyl, frustration, new exotic phases …
  Thin film, superstructure, cold atoms, d-electron systems …

- **Implement state-of-the-art techniques**

  ARPES, STM, MBE, pump probe, ultrasound …